Hindawi Publishing Corporation
Abstract and Applied Analysis
Volume 2014, Article ID 512037, 9 pages
http://dx.doi.org/10.1155/2014/512037

Research Article

Event-Based H_∞ Filter Design for Sensor Networks with Missing Measurements

Jinliang Liu,[1,2] Shumin Fei,[2] and Engang Tian[3]

[1] *Department of Applied Mathematics, Nanjing University of Finance and Economics, Nanjing, Jiangsu 210023, China*
[2] *School of Automation, Southeast University, Nanjing 210096, China*
[3] *Institute of Information and Control Engineering Technology, Nanjing Normal University, Nanjing, Jiangsu 210042, China*

Correspondence should be addressed to Shumin Fei; smfei@seu.edu.cn

Received 26 December 2013; Accepted 16 January 2014; Published 24 February 2014

Academic Editor: Ming Liu

In order to save network resources and network bandwidth, this paper proposed an event triggered mechanism based on sampled-data information, which has some advantages over existing ones. Considering the missing sensor measurements and the network-induced delay in the transmission, we construct a new event-based H_∞ filtering by taking the effect of sensor faults with different failure rates. By using the Lyapunov stability theory and the stochastic analysis theory, sufficient criteria are derived for the existence of a solution to the algorithm of the event-based filter design. Finally, an example is exploited to illustrate the effectiveness of the proposed method.

1. Introduction

The application of network technologies is becoming increasingly important in many areas for its predominant advantages (such as low cost, simple installation and maintenance, and high reliability). However, it is known that implementing a communication network can induce multiple channel transmission, packet dropout, and so on. This has motivated much attention to the research. Various techniques have been proposed to deal with the above issues, such as time triggered communication scheme [1, 2] and event triggered communication scheme [3–7]. In general, under a time triggered communication scheme, a fixed sampling interval should be selected under worse conditions such as external disturbances and time delay. However, such situation rarely occurs. Hence, time triggered communication scheme can lead to transmit much unnecessary information and inefficient utilization of limited network resources. Comparing with time triggered scheme, the event triggered scheme can save the network resources such as network bandwidth while maintaining the control performance. The adoption of the event triggered scheme has drawn a great deal of interest to the researchers. The authors in [3] firstly proposed a kind of event triggered scheme which decided whether the newly sampled signal should be transmitted to the controller and invested the controller design problem. In [8], the authors took the sensor and actuator faults into consideration and studied the reliable control design for networked control system under event triggered scheme. The authors in [9] were concerned with the control design problem of event triggered networked systems with both state and control input quantization. In [10], the authors discussed the event-based fault detection for the networked systems with communication delay and nonlinear perturbation.

On the other hand, the filtering problem has been a hot topic over the past decades. A large number of outstanding results have been published [9, 11–18]. For example, the researchers in [9] studied the problem of event-based H_∞ filtering for networked systems with communication delay. Most of them are based on an assumption that sensors are working without any flaws. However, the distortion of the sensor usually occurs due to the internal noise or external disturbance. Therefore, it is necessary to discuss the situation when the filter cannot receive the value of the process accurately. Fortunately, much effort has been put into this issue. The authors in [19] were concerned with reliable

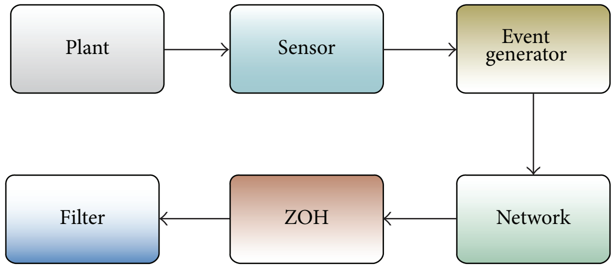

Figure 1: The structure of an event triggered filter design system.

H_∞ filter design for sampled-data systems with probabilistic sensor signal distortion. In [20], the authors investigated reliable H_∞ filter design for T-S fuzzy model based networked control systems with random sensor failure.

To the best of our knowledge, the filter design of event triggering network-based systems with random sensor failures is still an open problem, which motivates our present paper. The main contributions of the obtained results are as follows: (I) the insertion of the event triggering generator saves the network resources and network bandwidth. (II) A new kind of event triggering network-based systems with probabilistic sensor failures and network induced delay, which has not been investigated in the existing literatures, is proposed.

This paper is outlined as follows. Section 2 presents the modeling. Section 3 presents our main stability theorem and develops a filter design method. In Section 4, an example is given to illustrate the effectiveness of the proposed method.

$\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote the n-dimensional Euclidean space and the set of $n \times m$ real matrices; the superscript "T" stands for matrix transposition; I is the identity matrix of appropriate dimension; $\|\cdot\|$ stands for the Euclidean vector norm or the induced matrix 2-norm as appropriate; the notation $X > 0$ (resp., $X \geq 0$), for $X \in \mathbb{R}^{n\times n}$ means that the matrix X is real symmetric positive definite (resp., positive semidefinite), when x is a stochastic variable. For a matrix B and two symmetric matrices A and C, $\left[\begin{smallmatrix} A & * \\ B & C \end{smallmatrix}\right]$ denotes a symmetric matrix, where $*$ denotes the entries implied by symmetry.

2. System Description

As shown in Figure 1, our aim in this paper is to investigate an event-based reliable filtering design problem by taking the effect of sensor faults. Suppose the plant model is governed by

$$\begin{aligned} x(k+1) &= Ax(k) + Bw(k), \\ y(k) &= Cx(k), \\ z(k) &= Lx(k), \end{aligned} \tag{1}$$

where $x(k) \in \mathbb{R}^n$ is the state vector, $y(k) \in \mathbb{R}^m$ is the measured output, $z(k)$ is the signal to be estimated, $w(k)$ is the process noise belonging to $\mathbb{L}_2(0,\infty)$, A, B, C, and L are known constant matrices with appropriate dimensions.

Remark 1. Considering the network induced delay, the transmission time of measured output $y(k)$ from sensor to filt cannot be neglected. The input of the filter is not $y(k)$, but $\hat{y}(k)$, in fact, $\hat{y}(k) = y(k+\tau(k))$. $\tau(k)$ is the network induced delay, and $\tau(k) \in [0, \tau^M)$, where τ^M is a positive real number.

For the network-based system in described in Figure 1, we propose the following filter:

$$\begin{aligned} x_f(k+1) &= A_f x_f(k) + B_f \hat{y}(k), \\ z_f(k) &= C_f x_f(k), \end{aligned} \tag{2}$$

where $x_f(k)$ is the filter state, $\hat{y}(k)$ is the input of the filter, A_f, B_f, C_f are the filter matrices of appropriate dimensions.

If we take the missing sensor measurements into consideration, (2) can be described as

$$\begin{aligned} x_f(k+1) &= A_f x_f(k) + B_f \Xi \hat{y}(k), \\ z_f(k) &= C_f x_f(k), \end{aligned} \tag{3}$$

where $\Xi = \operatorname{diag}\{\Xi_1, \Xi_2, \ldots, \Xi_m\}$, $\Xi_i \in [0, \theta_1]$ $(i = 1, 2, \ldots)$ $(\theta_1 > 1)$ being m unrelated random variables, and the mathematical expectation and variance of Ξ_i are α_i and σ_i^2, respectively.

Remark 2. When $\alpha_i = 1$, it means the sensor i works normally. When $\alpha_i = 0$, it means the sensor i completely failed and the signal transmitted by sensor i is lost. When $\Xi_i \in [0, 1]$, it means the signal at the filter is smaller or greater than it actually is [20].

In order to reduce the load of network transmission and save the network resources such as network bandwidth, it is necessary to introduce an event triggered mechanism. As is shown in Figure 1, an event generator is constructed between the sensor and filter, which is used to decide whether the measured output should be sent to the filter. We adopt the following judgement algorithm:

$$\begin{aligned} &[\mathbb{E}\{\Xi y(k)\} - \mathbb{E}\{\Xi y(s_i)\}]^T \Omega [\mathbb{E}\{\Xi y(k)\} - \mathbb{E}\{\Xi y(s_i)\}] \\ &\quad \leq \sigma [\mathbb{E}\{\Xi y(k)\}]^T \Omega [\mathbb{E}\{\Xi y(k)\}], \end{aligned} \tag{4}$$

where $\Omega \in \mathbb{R}^m \times m$ is a symmetric positive definite matrix, $\sigma \in [0, 1)$, $\Xi = \operatorname{diag}\{\Xi_1, \Xi_2, \ldots\}$, and $\Xi_i \in [0, \theta_1]$ $(i = 1, 2, \ldots)$

are m unrelated random variables. Only when the expectation of a certain function of current sampled value $y(k)$ and the previously transmitted one $y(s_i)$ violate (4), it can be sent out to the filter.

Remark 3. Under the event triggering (4), the release times are assumed to be $s_0, s_1, s_2, \ldots$. Due to the delay in the network transmission, the measured output will arrive at the filter at the instants $s_0 + \tau(s_0), s_1 + \tau(s_1), s_2 + \tau(s_2), \ldots$, respectively.

Based on the above analysis, considering the behavior of ZOH, the input of the filter is

$$\Xi \hat{y}(k) = \Xi \hat{y}(s_i), \quad k \in [s_i + \tau(s_i), s_{i+1} + \tau(s_{i+1}) - 1]. \tag{5}$$

Similar to [4, 6, 11], for technical convenience, consider the following two cases.

Case 1. When $s_i + 1 + \tau^M \geq s_{i+1} + \tau(s_{i+1}) - 1$, define a function $d(k)$ as

$$d(k) = k - s_i, \quad k \in [s_i + \tau(s_i), s_{i+1} + \tau(s_{i+1}) - 1]. \tag{6}$$

Obviously,

$$\tau(s_i) \leq d(k) \leq (s_{i+1} - s_i) + \tau(s_{i+1}) - 1 \leq 1 + \tau^M. \tag{7}$$

Case 2. When $s_i + 1 + \tau^M \leq s_{i+1} + \tau(s_{i+1}) - 1$, consider the following two intervals:

$$[s_i + \tau(s_i), s_i + \tau^M], \qquad [s_i + \tau^M + l, s_i + \tau^M + l + 1]. \tag{8}$$

From $\tau(k) \leq \tau^M$, we can deduce that there must exist d satisfying

$$s_i + d + \tau^M < s_{i+1} + \tau(s_{i+1}) - 1 \leq s_i + d + 1 + \tau^M. \tag{9}$$

Moreover, $y(s_i)$ and $y(s_i + l)$ $l = 1, 2, \ldots, d$ satisfy (4). Set

$$\begin{aligned} I_0 &= [s_i + \tau(s_i), s_i + \tau^M + 1), \\ I_l &= [s_i + \tau^M + l, s_i + \tau^M + l + 1), \\ I_d &= [s_i + d + \tau^M, s_{i+1} + \tau(s_{i+1}) - 1], \end{aligned} \tag{10}$$

where $l = 1, 2, \ldots, d - 1$. Clearly, we have

$$[s_i + \tau(s_i), s_{i+1} + \tau(s_{i+1}) - 1] = \bigcup_{i=0}^{i=d} I_i. \tag{11}$$

Define $d(k)$ as

$$d(k) = \begin{cases} k - s_i, & k \in I_0, \\ k - s_i - l, & k \in I_l, \ l = 1, 2, \ldots, d - 1, \\ k - s_i - d, & k \in I_d. \end{cases} \tag{12}$$

Then, one can easily get

$$\begin{aligned} \tau(s_i) \leq d(k) \leq 1 + \tau^M \triangleq d^M, &\quad k \in I_0, \\ \tau(s_i) \leq \tau^M \leq d(k) \leq d^M, &\quad k \in I_l, \ l = 1, 2, \ldots, d - 1, \\ \tau(s_i) \leq \tau^M \leq d(k) \leq d^M, &\quad k \in I_d. \end{aligned} \tag{13}$$

Due to $s_{i+1} + \tau(s_{i+1}) - 1 \leq s_i + d + 1 + \tau^M$, the third row in (13) holds. Obviously,

$$\tau(s_i) \leq \tau^M \leq d(k) \leq d^M, \quad k \in I_d. \tag{14}$$

In Case 1, for $k \in [s_i + \tau(s_i), s_{i+1} + \tau(s_{i+1}) - 1]$, define $e_i(k) = 0$. When it comes to Case 2, define

$$\overline{\Xi} e_i(k) = \begin{cases} 0, & k \in I_0, \\ \overline{\Xi} y(s_i) - \overline{\Xi} y(s_i + l), & k \in I_l, \ l = 1, 2, \ldots, d - 1, \\ \overline{\Xi} y(s_i) - \overline{\Xi} y(s_i + d), & k \in I_d. \end{cases} \tag{15}$$

It can be deduced from the definition of $\overline{\Xi} e_i(k)$ and the event triggering scheme (4); for $k \in [s_i + \tau(s_i), s_{i+1} + \tau(s_{i+1}) - 1]$, the following inequality holds

$$e_i^T(k) \overline{\Xi}^T \Omega \overline{\Xi} e_i(k) \leq \sigma y^T(k - d(k)) \overline{\Xi}^T \Omega \overline{\Xi} y(k - d(k)). \tag{16}$$

Remark 4. From (15), it can be easily obtained that

$$e_i(k) = \begin{cases} 0, & k \in I_0, \\ y(s_i) - y(s_i + l), & k \in I_l, \ l = 1, 2, \ldots, d - 1, \\ y(s_i) - y(s_i + d), & k \in I_d. \end{cases} \tag{17}$$

Employing $d(k)$ $e_i(k)$, the input of the filter $\Xi \hat{y}(k)$ can be rewritten as

$$\begin{aligned} \Xi \hat{y}(k) = \Xi y(s_i) = \Xi (y(k - d(k)) + \Xi e_i(k)), \\ k \in [s_i + \tau(s_i), s_{i+1} + \tau(s_{i+1}) - 1]. \end{aligned} \tag{18}$$

Obviously,

$$\begin{aligned} \hat{y}(k) = y(s_i) = (y(k - d(k)) + e_i(k)), \\ k \in [s_i + \tau(s_i), s_{i+1} + \tau(s_{i+1}) - 1]. \end{aligned} \tag{19}$$

Combining (19) and (3), we can get

$$\begin{aligned} x_f(k+1) &= A_f x_f(k) + B_f \Xi (C x(k - d(k)) + e_i(k)), \\ z_f(k) &= C_f x_f(k). \end{aligned} \tag{20}$$

Define $\eta(k) = \begin{bmatrix} x(k) \\ x_f(k) \end{bmatrix}$, $e(k) = z(k) - z_f(k)$; the following filtering-error system based on (1) and (20) can be obtained as

$$\begin{aligned} \eta(k+1) &= \overline{A}\eta(k) + Dx(k-d(k)) + D_k x(k-d(k)) \\ &\quad + \overline{B}e_i(k) + \overline{B}_k e_i(k) + \overline{B}_1 w(k), \\ e(k) &= \overline{L}\eta(k), \end{aligned} \tag{21}$$

where $\overline{A} = \begin{bmatrix} A & 0 \\ 0 & A_f \end{bmatrix}$, $D = \begin{bmatrix} 0 \\ B_f\overline{\Xi}C \end{bmatrix}$, $D_k = \begin{bmatrix} 0 \\ B_f(\Xi-\overline{\Xi})C \end{bmatrix}$, $\overline{B} = \begin{bmatrix} 0 \\ B_f\overline{\Xi} \end{bmatrix}$, $\overline{B}_k = \begin{bmatrix} 0 \\ B_f(\Xi-\overline{\Xi}) \end{bmatrix}$, $\overline{B}_1 = \begin{bmatrix} B \\ 0 \end{bmatrix}$, $\overline{L} = [L \ \ -C_f]$.

Remark 5. The event triggering scheme (4) can be applied to the situation when the sensor have failures. Besides, the effect of the network environment is also taken into consideration. From the modeling process, we can see that the system (21) is more general.

Before giving the main results in the next section, the following lemmas will be introduced, which will be helpful in deriving the main results.

Lemma 6 (see [21]). *For any vectors x, $y \in \mathbb{R}^n$, and positive definite matrix $Q \in \mathbb{R}^{n\times n}$, the following inequality holds:*

$$2x^T y \le x^T Qx + y^T Q^{-1} y. \tag{22}$$

Lemma 7 (see [22]). *Ω_1, Ω_2, and Ω are matrices with appropriate dimensions, $d(k) \in [0, d^M]$; then*

$$d(k)\Omega_1 + \left(d^M - d(k)\right)\Omega_2 + \Omega < 0, \tag{23}$$

if and only if the following two inequalities hold

$$\begin{aligned} d^M\Omega_1 + \Omega &< 0, \\ d^M\Omega_2 + \Omega &< 0. \end{aligned} \tag{24}$$

3. Main Results

In this section, we will invest a new approach to guarantee the filter error system (21) to be globally asymptotically stable. A sufficient condition is established for (21). Then, the explicit filter design method in (20) is given.

Theorem 8. *For given scalars α_i, μ_i $(i = 1, \ldots, m)$, $\rho \in [0, 1)$, $0 \le d(k) \le d^M$, and γ, under the event triggered communication scheme (4), the augmented system (21) is asymptotically stable with an H_∞ performance index γ for the disturbance attention, if there exist positive definite matrices P, Q, R and matrices N, M with appropriate dimensions, such that*

$$\Omega(s) = \begin{bmatrix} \Omega_{11} + \Gamma + \Gamma^T & * & * & * \\ \Omega_{21} & \Omega_{22} & * & * \\ \Omega_{31} & 0 & \Omega_{33} & * \\ \Omega_{41}(s) & 0 & 0 & -R \end{bmatrix} < 0, \quad s = 1, 2, \tag{25}$$

where

$$\begin{gathered}
\Omega_{11} = \begin{bmatrix} P\overline{A} + \overline{A}^P - 2P + H^TQH & * & * & * & * \\ D^TP & 0 & * & * & * \\ 0 & 0 & -Q & * & * \\ \overline{B}^TP & 0 & 0 & -\overline{\Xi}^T\Omega\overline{\Xi} & * \\ \overline{B}_1^TP & 0 & 0 & 0 & -\gamma^2 I \end{bmatrix}, \\
\Gamma = [NH \ \ M-N \ \ -M \ \ 0 \ \ 0], \\
\Omega_{21} = \begin{bmatrix} P(\overline{A}-I) & PD & 0 & P\overline{B} & P\overline{B}_1 \\ \sqrt{d^M}RH(\overline{A}-I) & \sqrt{d^M}RHD & 0 & \sqrt{d^M}RH\overline{B} & \sqrt{d^M}RH\overline{B}_1 \\ \overline{L} & 0 & 0 & 0 & 0 \\ 0 & \sqrt{\sigma}\Omega\overline{\Xi}C & 0 & 0 & 0 \end{bmatrix}, \\
\Omega_{22} = \operatorname{diag}\{-P, -R, -I, -\Omega\}, \\
\Omega_{31} = \begin{bmatrix} 0 & \delta_1 P\widehat{D}_1 & 0 & 0 & 0 \\ 0 & \vdots & 0 & 0 & 0 \\ 0 & \delta_m P\widehat{D}_m & 0 & 0 & 0 \\ 0 & 0 & 0 & \delta_1 P\widehat{B}_1 & 0 \\ 0 & 0 & 0 & \vdots & 0 \\ 0 & 0 & 0 & \delta_m P\widehat{B}_m & 0 \end{bmatrix}, \\
\Omega_{41}(1) = \sqrt{d^M}N^T, \qquad \Omega_{41}(2) = \sqrt{d^M}M^T, \\
N^T = \left[N_1^T \ \ N_2^T \ \ N_3^T \ \ N_4^T \ \ N_5^T\right], \\
M^T = \left[M_1^T \ \ M_2^T \ \ M_3^T \ \ M_4^T \ \ M_5^T\right], \\
\widehat{D}_i = \begin{bmatrix} 0 \\ B_fE_iC \end{bmatrix}, \quad \widehat{B}_i = \begin{bmatrix} 0 \\ B_fE_i \end{bmatrix}, \quad (i = 1, 2, \ldots, m), \\
E_i = \operatorname{diag}\{\underbrace{0, \ldots, 0}_{i-1}, 1, \underbrace{0, \ldots, 0}_{m-i}\}, \qquad H = [I \ \ 0].
\end{gathered} \tag{26}$$

Proof. Set $\delta(k) = x(k+1) - x(k)$, $\overline{\eta}(k) = \eta(k+1) - \eta(k)$; choose the Lyapunov functional candidate

$$\begin{aligned} V(k) &= \eta^T(k)P\eta(k) + \sum_{k-d^M}^{k-1} x^T(i)Qx(i) \\ &\quad + \sum_{i=-d^M}^{-1}\sum_{j=k+i}^{k-1} \delta^T(j)R\delta(j). \end{aligned} \tag{27}$$

Calculating the difference of $V(k)$ along the solution of (27) and taking the mathematical expectation, we obtain

$$\begin{aligned}
\mathbb{E}\{\Delta V(k)\} = {} & 2\eta^T(k) P\left[\left(\overline{A}-I\right)\eta(k) + Dx(k-d(k)) + \overline{B}e_i(k) + \overline{B}_1 w(k)\right] + \mathscr{A}^T P \mathscr{A} \\
& + \sum_{i=1}^{m} \sigma_i^2 x^T(k-d(k)) \begin{bmatrix} 0 \\ B_f E_i C \end{bmatrix}^T \\
& \times P \begin{bmatrix} 0 \\ B_f E_i C \end{bmatrix} x(k-d(k)) \\
& + \sum_{i=1}^{m} \sigma_i^2 e_i^T(k) \begin{bmatrix} 0 \\ B_f E_i \end{bmatrix}^T P \begin{bmatrix} 0 \\ B_f E_i \end{bmatrix} e_i(k) \\
& + \eta^T(k) H^T Q H \eta(k) \\
& - x^T\left(k-d^M\right) Q x\left(k-d^M\right) \\
& + \mathbb{E}\left\{d^M \delta^T(k) R \delta(k)\right\} - \sum_{i=k-d^M}^{k-1} \delta^T(k) R \delta(k),
\end{aligned} \tag{28}$$

where $\mathscr{A} = (\overline{A}-I)\eta(k) + Dx(k-d(k)) + \overline{B}e_i(k) + \overline{B}_1 w(k)$ and

$$\begin{aligned}
\mathbb{E}\left\{d^M \delta^T(k) R \delta(k)\right\} &= \mathbb{E}\left\{d^M \overline{\eta}^T(k) H^T R H \overline{\eta(k)}\right\} \\
&= d^M \mathscr{A}^T H^T R H \mathscr{A}.
\end{aligned} \tag{29}$$

Then by employing free weight matrix method [23, 24], we have

$$\begin{aligned}
2\xi^T(k) M\left[x(k-d(k)) - x\left(k-d^M\right)\right] - \sum_{i=k-d^M}^{k-d(k)-1} \delta(i) = 0, \\
2\xi^T(k) N\left[x(k) - x(k-d(k))\right] - \sum_{i=k-d^M}^{k-1} \delta(i) = 0,
\end{aligned} \tag{30}$$

where $\xi^T(k) = [\eta^T(k)\ x^T(k-d(k))\ x^T(k-d^M)\ e_i^T(k)\ w^T(k)]^T$.

By Lemma 6, we can easily get

$$\begin{aligned}
-2\xi^T(k) M \sum_{i=k-d^M}^{k-d(k)-1} \delta(i) \le {} & \left(d^M - d(k)\right) \xi^T(k) M R^{-1} M^T \xi(k) \\
& + \sum_{i=k-d^M}^{k-d(k)-1} \delta^T(i) R \delta(i),
\end{aligned} \tag{31}$$

$$\begin{aligned}
-2\xi^T(k) N \sum_{i=k-d^M}^{k-1} \delta(i) \le {} & d(k) \xi^T(k) N R^{-1} N^T \xi(k) \\
& + \sum_{i=k-d(k)}^{k-1} \delta^T(i) R \delta(i).
\end{aligned} \tag{32}$$

Combine (28)–(31) and (16), we have

$$\begin{aligned}
& \mathbb{E}\{\Delta V(k)\} - \gamma^2 w^T(k) w(k) + e^T(k) e(k) \\
& \le \xi^T(k)\left[\Omega_{11} + \Gamma + \Gamma^T\right]\xi(k) \\
& \quad + \left(d^M - d(k)\right)\xi^T(k) M R^{-1} M^T \xi(k) \\
& \quad + d(k)\xi^T(k) N R^{-1} N^T \xi(k) \\
& \quad + \mathscr{A}^T P \mathscr{A} + \sum_{i=1}^{m} \sigma_i^2 x^T(k-d(k)) \begin{bmatrix} 0 \\ B_f E_i C \end{bmatrix}^T \\
& \quad \times P \begin{bmatrix} 0 \\ B_f E_i C \end{bmatrix} x(k-d(k)) \\
& \quad + \sum_{i=1}^{m} \sigma_i^2 e_i^T(k) \begin{bmatrix} 0 \\ B_f E_i \end{bmatrix}^T P \begin{bmatrix} 0 \\ B_f E_i \end{bmatrix} e_i(k) \\
& \quad + \sigma x^T(k-d(k)) C^T \overline{\Xi}^T \Omega \overline{\Xi} C x(k-d(k)) \\
& \quad + d^M \mathscr{A}^T H^T R H \mathscr{A} + \eta^T(k) \overline{L}^T \overline{L} \eta(k).
\end{aligned} \tag{33}$$

Subsequently, by the well known Schur complement and Lemma 7, from (25), we can deduce

$$\mathbb{E}\{\Delta V(k)\} - \gamma^2 w^T(k) w(k) + e^T(k) e(k) \le 0. \tag{34}$$

Similar to the method in [25], the filter error system (21) is asymptotically stable. □

Based on Theorem 8, a design method of the reliable filter in the form of (20) is given in Theorem 9.

Theorem 9. *For given parameters α, σ_i $(i = 1, 2, \ldots, m)$, $\rho \in [0, 1)$, and $0 \le d(k) \le d^M$, the filter error system (21) is asymptotically stable with H_∞ performance level γ, if there exist positive definite matrices X, Q, $\widehat{R}$, and A_f, B_f, C_f, N_{10}, N_{11}, M_{10}, M_{11}, M_i, and N_i $(i = 2, 3, 4, 5)$ with appropriate dimensions, such that*

$$\widehat{\Omega}(s) = \begin{bmatrix} \widehat{\Omega}_{11} + \widehat{\Gamma} + \widehat{\Gamma}^T & * & * & * & * \\ \widehat{\Omega}_{21} & \widehat{\Omega}_{22} & * & * & * \\ \widehat{\Omega}_{31} & 0 & \widehat{\Omega}_{33} & * & * \\ \widehat{\Omega}_{41} & 0 & 0 & \widehat{\Omega}_{44} & * \\ \widehat{\Omega}_{51}(s) & B_w & 0 & 0 & -R \end{bmatrix} < 0, \quad s = 1, 2, \tag{35}$$

$$P_1 - \overline{P}_3 > 0, \tag{36}$$

where

$$\widehat{\Omega}_{11}=\begin{bmatrix} P_1A+A^TP_1-2P_1+Q & * & * & * & * & * \\ \overline{P}_3A+\overline{A}_f^T-2\overline{P}_3 & A_f+A_f^T-2P_3 & * & * & * & * \\ C^T\overline{\Xi}^T\overline{B}_f^T & C^T\overline{\Xi}^T\overline{B}_f^T & 0 & * & * & * \\ 0 & 0 & 0 & -Q & * & * \\ \overline{\Xi}^T\overline{B}_f^T & \overline{\Xi}^T\overline{B}_f^T & 0 & 0 & -\overline{\Xi}^T\Omega\overline{\Xi} & * \\ B^TP_1 & B^T\overline{P}_3 & 0 & 0 & 0 & -\gamma^2 I \end{bmatrix},$$

$$\widehat{\Omega}_{21}=\begin{bmatrix} P_1(A-I) & \overline{A}_f-\overline{P}_3 & \overline{B}_f\overline{\Xi}C & 0 & \overline{B}_f\overline{\Xi} & P_1B \\ \overline{P}_3(A-I) & \overline{A}_f-\overline{P}_3 & \overline{B}_f\overline{\Xi}C & 0 & \overline{B}_f\overline{\Xi} & \overline{P}_3B \end{bmatrix}, \qquad \widehat{\Omega}_{22}=\begin{bmatrix} -P_1 & * \\ -\overline{P}_3 & -\overline{P}_3 \end{bmatrix},$$

$$\widehat{\Omega}_{31}=\begin{bmatrix} \sqrt{d^M}R(A-I) & 0 & 0 & 0 & 0 & \sqrt{d^M}RB \\ L & -C_f & 0 & 0 & 0 & 0 \\ 0 & \sqrt{\sigma}\Omega\overline{\Xi}C & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\widehat{\Omega}_{41}=\begin{bmatrix} 0 & 0 & \delta_1\overline{B}_fE_1C & 0 & 0 & 0 \\ 0 & 0 & \delta_1\overline{B}_fE_1C & 0 & 0 & 0 \\ 0 & 0 & \vdots & 0 & 0 & 0 \\ 0 & 0 & \delta_m\overline{B}_fE_mC & 0 & 0 & 0 \\ 0 & 0 & \delta_m\overline{B}_fE_mC & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \delta_1\overline{B}_fE_1 & 0 \\ 0 & 0 & 0 & 0 & \delta_1\overline{B}_fE_1 & 0 \\ 0 & 0 & 0 & 0 & \vdots & 0 \\ 0 & 0 & 0 & 0 & \delta_m\overline{B}_fE_m & 0 \\ 0 & 0 & 0 & 0 & \delta_m\overline{B}_fE_m & 0 \end{bmatrix}, \tag{37}$$

$$\widehat{\Gamma}=\begin{bmatrix} N_{10} & 0 & M_{10}-N_{10} & -M_{10} & 0 & 0 \\ N_{11} & 0 & M_{11}-N_{11} & -M_{11} & 0 & 0 \\ N_2 & 0 & M_2-N_2 & -M_2 & 0 & 0 \\ N_3 & 0 & M_3-N_3 & -M_3 & 0 & 0 \\ N_4 & 0 & M_4-N_4 & -M_4 & 0 & 0 \\ N_5 & 0 & M_5-N_5 & -M_5 & 0 & 0 \end{bmatrix},$$

$$\widehat{\Omega}_{33}=\operatorname{diag}\{-R,-I,-\Omega\}, \qquad \widehat{\Omega}_{44}=\operatorname{diag}\Big\{\underbrace{\widehat{\Omega}_{22},\ldots,\widehat{\Omega}_{22}}_{2m}\Big\},$$

$$\Omega_{51}(1)=\sqrt{d^M}\overline{N}^T, \qquad \Omega_{51}(2)=\sqrt{d^M}\overline{M}^T,$$

$$\overline{N}^T=\begin{bmatrix} N_{10}^T & N_{11}^T & N_2^T & N_3^T & N_4^T & N_5^T \end{bmatrix},$$

$$\overline{M}^T=\begin{bmatrix} M_{10}^T & M_{11}^T & M_2^T & M_3^T & M_4^T & M_5^T \end{bmatrix}.$$

The filter parameters are given by

$$A_f=\overline{A}_f\overline{P}_3^{-1},\quad B_f=\overline{B}_f,\quad C_f=\overline{C}_f\overline{P}_3^{-1}. \tag{38}$$

Proof. Since $\overline{P}_3>0$, there exist nonsingular matrix P_2 and symmetrical matrix $P_3>0$ satisfying $\overline{P}_3=P_2^TP_3^{-1}P_2$.

Define

$$P=\begin{bmatrix} P_1 & P_2^T \\ P_2 & P_3 \end{bmatrix}, \qquad J=\begin{bmatrix} I & 0 \\ 0 & P_2^TP_3^{-1} \end{bmatrix}. \tag{39}$$

Now premultiply and postmultiply Equation (25) with $\Upsilon = \operatorname{diag}\{J, \underbrace{I, I, \ldots, I}_{4}, J, I, I, I, \underbrace{J, J, \ldots, J}_{2m}, I\}$ and Υ^T, and define new variables as

$$\begin{gathered}\overline{A}_f = \widehat{A}_f \overline{P}_3, \qquad \widehat{A}_f = P_2^T A_f P_2^{-T}, \\ \overline{B}_f = P_2^T B_f, \\ \overline{C}_f = \widehat{C}_f \overline{P}_3, \qquad \widehat{C}_f = C_f P_2^{-T}, \\ N_1^T J^T = \begin{bmatrix} \overline{N}_{10}^T & \overline{N}_{11}^T \end{bmatrix}, \qquad M_1^T J^T = \begin{bmatrix} \overline{M}_{10}^T & \overline{M}_{11}^T \end{bmatrix}.\end{gathered} \tag{40}$$

We can obtain (35). Therefore, (35) holds, only if (25) holds. From Theorem 8, the filter error system (21) is asymptotic stable with H_∞ performance level γ. □

Similar to the analysis of [25], the filter parameters in (20) can be obtained as (38).

4. Simulation Examples

Consider a specific network controlled system of Equation (21) under a structure:

$$\begin{gathered}A = \begin{bmatrix} 0.1 & 0.4 \\ -0.4 & 0.1 \end{bmatrix}, \qquad B = \begin{bmatrix} -0.7 \\ 0.2 \end{bmatrix}, \\ C = \begin{bmatrix} 0 & 1 \end{bmatrix}, \qquad L = \begin{bmatrix} 1 & 1 \end{bmatrix}.\end{gathered} \tag{41}$$

Assume $0 \le d(k) \le 4$ and the failure rates of the sensors are $\alpha_1 = 0.8$ and $\sigma_1 = 0.05$.

According to Theorem 9, when H_∞ performance level $\gamma = 0.8$, the following parameters can be obtained from the solution of (35) and (36) by using the LMI technique:

$$\begin{gathered}\overline{P}_3 = \begin{bmatrix} 0.7521 & 0.5801 \\ 0.5801 & 1.4216 \end{bmatrix}, \qquad \overline{A}_f = \begin{bmatrix} -0.0837 & 0.3567 \\ -0.3238 & 0.2977 \end{bmatrix}, \\ \overline{B}_f = \begin{bmatrix} -0.0036 \\ -0.0019 \end{bmatrix}, \qquad \overline{C}_f = \begin{bmatrix} -0.6268 & -0.6181 \end{bmatrix}.\end{gathered} \tag{42}$$

From (38), the corresponding filter parameters can be obtained as

$$\begin{gathered}A_f = \begin{bmatrix} -0.4448 & 0.4325 \\ -0.8641 & 0.5620 \end{bmatrix}, \qquad B_f = \begin{bmatrix} -0.0036 \\ -0.0019 \end{bmatrix}, \\ C_f = \begin{bmatrix} -0.7268 & -0.1382 \end{bmatrix}\end{gathered} \tag{43}$$

and the parameter in the event triggering scheme (4) is $\Omega = 0.0239$.

Suppose the initial condition $x(0) = \begin{bmatrix} 0.2 & 0.1 \end{bmatrix}^T$ and external disturbance

$$w(k) = \begin{cases} 0.05 & 5s \le k \le 15s \\ 0 & \text{else.} \end{cases} \tag{44}$$

Based on the designed filter above, the response of the error $e(k)$ and the probabilistic failure Ξ are given in Figures 2 and 3, respectively. Figure 4 describes the release instants and release interval. It is easy to see from Figures 2–4 that the filter design method in this paper is effectiveness.

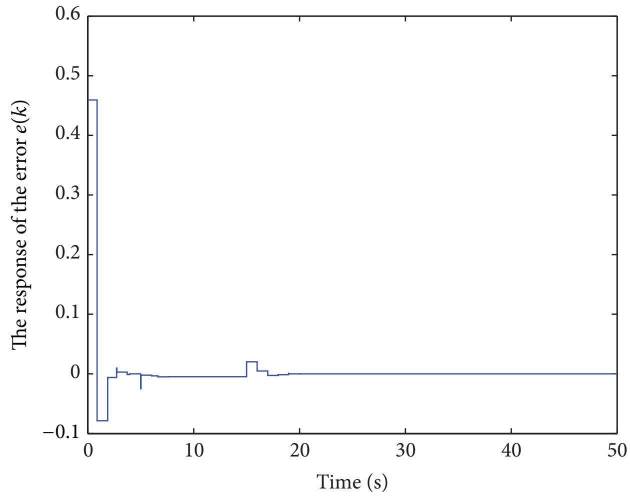

Figure 2: The response of the error $e(k)$.

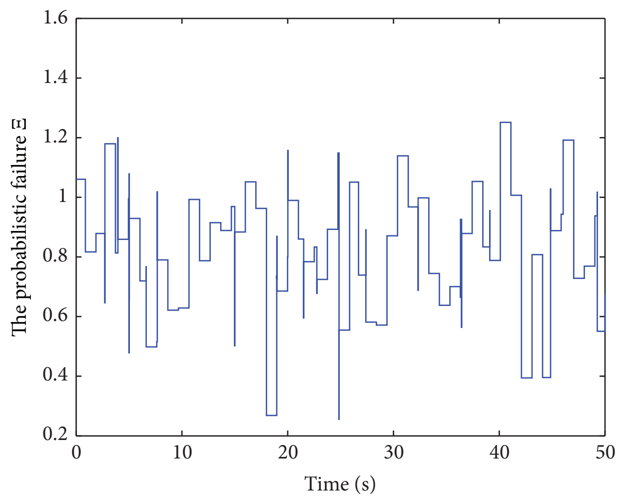

Figure 3: The probabilistic failure Ξ.

5. Conclusion

This paper investigates a H_∞ filter design for a class of network-based systems under an event triggered mechanism. In particular, the system under study is a more general sensor failure model. Considering the uncertain time delay, the uncertain network environment and probabilistic missing sensor measurements, we introduce an event triggered mechanism into the system. By using the free-weighting matrix method and the LMI techniques, the fundamental stability conditions are obtained and the filter design methods are developed. Finally, a numerical example is given to demonstrate the effectiveness of the proposed designed method.

We would like to point out that it is possible to extend our main results to the nonlinear systems such as T-S fuzzy systems, and complex network systems. This will also be one of our future research issues.

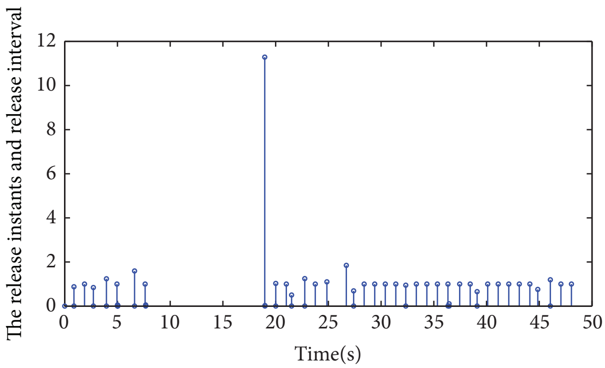

Figure 4: The release instants and release interval.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

Acknowledgments

This work is partly supported by the National Natural Science Foundation of China (no. 11226240 and no. 61273115), National Center for International Joint Research on E-Business Information Processing under Grant no. 2013B01035, the Natural Science Foundation of Jiangsu Province of China (nos. BK2012469, BK2012211, and BK2012847), the Natural Science Foundation of the Jiangsu Higher Education Institutions of China (no. 12KJD120001 and no. 13KJB110010), and a Project Funded by the Priority Academic Program Development of Jiangsu Higher Education Institutions (PAPD).

References

[1] H. Yang, Y. Xia, and P. Shi, "Stabilization of networked control systems with nonuniform random sampling periods," *International Journal of Robust and Nonlinear Control*, vol. 21, no. 5, pp. 501–526, 2011.

[2] Q.-L. Han, "A discrete delay decomposition approach to stability of linear retarded and neutral systems," *Automatica*, vol. 45, no. 2, pp. 517–524, 2009.

[3] D. Yue, E. Tian, and Q. Han, "A delay system method for designing event-triggered controllers of networked control systems," *IEEE Transactions on Automatic Control*, vol. 58, no. 2, pp. 475–481, 2013.

[4] C. Peng and T. Yang, "Event-triggered communication and H_∞ control co-design for networked control systems," *Automatica*, vol. 49, no. 5, pp. 1326–1332, 2013.

[5] A. Molin and S. Hirche, "On the optimality of certainty equivalence for event-triggered control systems," *IEEE Transactions on Automatic Control*, vol. 58, no. 2, pp. 470–474, 2013.

[6] C. Peng, Q. Han, and D. Yue, "To transmit or not to transmit: a discrete event-triggered communication scheme for networked takagisugeno fuzzy systems," *IEEE Transactions on Fuzzy Systems*, vol. 21, no. 1, pp. 164–170, 2013.

[7] X. Yin and D. Yue, "Event-triggered tracking control for heterogeneous multi-agent systems with markov communication delays," *Journal of the Franklin Institute*, vol. 350, no. 3, pp. 651–669, 2013.

[8] J. Liu and D. Yue, "Event-triggering in networked systems with probabilistic sensor and actuator faults," *Information Sciences*, vol. 240, no. 10, pp. 145–160, 2013.

[9] S. Hu and D. Yue, "Event-based H_∞ filtering for networked system with communication delay," *Signal Processing*, vol. 92, no. 9, pp. 2029–2039, 2012.

[10] J. Liu and D. Yue, "Event-based fault detection for networked systems with communication delay and nonlinear perturbation," *Journal of the Franklin Institute*, vol. 350, no. 9, pp. 2791–2807, 2013.

[11] M. Liu, P. Shi, L. Zhang, and X. Zhao, "Fault-tolerant control for nonlinear markovian jump systems via proportional and derivative sliding mode observer technique," *IEEE Transactions on Circuits and Systems I*, vol. 58, no. 11, pp. 2755–2764, 2011.

[12] M. Liu and P. Shi, "Sensor fault estimation and tolerant control for *itô* stochastic systems with a descriptor sliding mode approach," *Automatica*, vol. 49, no. 5, pp. 1242–1250, 2013.

[13] P. Shi, X. Luan, and F. Liu, "H_∞ filtering for discrete-time systems with stochastic incomplete measurement and mixed delays," *IEEE Transactions on Industrial Electronics*, vol. 59, no. 6, pp. 2732–2739, 2012.

[14] F. Li and X. Zhang, "Delay-range-dependent robust H_∞ filtering for singular lpv systems with time variant delay," *International Journal of Innovative Computing, Information and Control*, vol. 9, no. 1, pp. 339–353, 2013.

[15] Y. Xu, H. Su, Y. Pan, and Z. Wu, "Robust H_∞ filtering for networked stochastic systems with randomly occurring sensor nonlinearities and packet dropouts," *Signal Processing*, vol. 93, no. 7, pp. 1794–1803, 2013.

[16] J. Liu, Z. Gu, and S. Hu, "H_∞ filtering for Markovian jump systems with time-varying delays," *International Journal of Innovative Computing, Information and Control*, vol. 7, no. 3, pp. 1299–1310, 2011.

[17] C. Peng, D. Yue, E. Tian, and Z. Gu, "Observer-based fault detection for networked control systems with network quality of services," *Applied Mathematical Modelling*, vol. 34, no. 6, pp. 1653–1661, 2010.

[18] C. Peng, Y.-C. Tian, and D. Yue, "Output feedback control of discrete-time systems in networked environments," *IEEE Transactions on Systems, Man, and Cybernetics A*, vol. 41, no. 1, pp. 185–190, 2011.

[19] Z. Gu, E. Tian, and J. Liu, "Reliable H_∞ filter design for sampled-data systems with consideration of probabilistic sensor signal distortion," *IET Signal Processing*, vol. 7, no. 5, pp. 420–426, 2013.

[20] E. Tian and D. Yue, "Reliable H_∞ filter design for T-S fuzzy model-based networked control systems with random sensor failure," *International Journal of Robust and Nonlinear Control*, vol. 23, no. 1, pp. 15–32, 2013.

[21] Y. Wang, L. Xie, and C. E. de Souza, "Robust control of a class of uncertain nonlinear systems," *Systems and Control Letters*, vol. 19, no. 2, pp. 139–149, 1992.

[22] D. Yue, E. Tian, Y. Zhang, and C. Peng, "Delay-distribution-dependent stability and stabilization of T-S fuzzy systems with probabilistic interval delay," *IEEE Transactions on Systems, Man, and Cybernetics B*, vol. 39, no. 2, pp. 503–516, 2009.

[23] Y. He, M. Wu, J.-H. She, and G.-P. Liu, "Parameter-dependent Lyapunov functional for stability of time-delay systems with polytopic-type uncertainties," *IEEE Transactions on Automatic Control*, vol. 49, no. 5, pp. 828–832, 2004.

[24] D. Yue, Q.-L. Han, and J. Lam, "Network-based robust H_∞ control of systems with uncertainty," *Automatica*, vol. 41, no. 6, pp. 999–1007, 2005.

[25] X.-M. Zhang and Q.-L. Han, "Delay-dependent robust H_∞ filtering for uncertain discrete-time systems with time-varying delay based on a finite sum inequality," *IEEE Transactions on Circuits and Systems II*, vol. 53, no. 12, pp. 1466–1470, 2006.