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Research Article

Well-Posedness of MultiCriteria Network Equilibrium Problem

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New notions of ϵ-equilibrium flow and ξ_{k_0}-ϵ-equilibrium flow of multicriteria network equilibrium problem are introduced; an equivalent relation between vector ϵ-equilibrium pattern flow and ξ_{k_0}-ϵ-equilibrium flow is established. Then, the well-posedness of multicriteria network equilibrium problem is discussed.

1. Introduction

For a long time, real-valued functions have played a central role in network equilibrium problems. Recently, motivated by applications to real-world situations, much attention has been attracted to multicriteria network equilibrium problems, that is, equilibrium problems with vector-valued cost functions. Different concepts of vector equilibrium flow have been introduced and the existence of such flows has been investigated by various authors (refer, first, to [1], in which Chen and Yen generalized Wardrop's scalar equilibrium principle to vector equilibrium principle, which asserts that users only choose Pareto optimal or efficient routes to travel on, and, among the others, to [2–6]).

Traffic equilibrium problem always depends on some parameters, because people's intuitive judgment plays a central role in route choices and it is impossible to have a precise estimation of the trip cost available to many paths. Therefore, some of the factors involved in transportation networks may be regarded as perturbing parameters. We were motivated to study the behavior of perturbations of multicriteria network equilibrium problems and cope with well-posed issues in the framework of traffic equilibrium problems. The notion of well-posedness, which can be useful for numerical purposes, is based on the behavior of either minimizing or maximizing sequences (Tychonov well-posedness). These notions for scalar optimization have been extensively investigated in many papers (cf., e.g., Dontchev and Zolezzi [7] in which a good list of basic references can be found). In the last decades, some extensions of this concept for vector optimization problems appeared; see [8–10] and the references therein. In the paper, we introduce the concept of well-posedness of multicriteria network equilibrium problems. As such, we obtain a sufficient condition that multicriteria network equilibrium problems are well-posed.

We now outline the remainder of the paper. Section 2 is devoted to the detailed description of the traffic network model. In Section 3, we introduce new notions of parametric equilibrium flows and obtain an equivalent relation between ϵ-equilibrium pattern flow and ξ_{k_0}-ϵ-equilibrium flow. Finally, we discuss the well-posedness of multicriteria network equilibrium problem.

2. Preliminaries

Consider a traffic network $[I, L]$, where I denotes the set of origin and destination (O/D) pairs and L is the set of directed arcs. We will suppose that I has l members. Let $i = (x, y) \in I$, which connects origin point x with destination point y. Then a sequence

$$(x, x_1), (x_1, x_2), \ldots, (x_{k-1}, y) \tag{1}$$

of contiguous links in L is called a path (or chain) from x to y. Denote by k the set of routes from x to y which traverse no link twice. Let K_i be the set of paths that connect an O/D pair $i \in I$. Then K_i is a finite set and

$$K = \bigcup_{i \in I} K_i \tag{2}$$

is a finite set too. We will suppose that K has m members.

For a given path $k \in P_i$, let v_k denote the traffic flow on this path and let $v = (v_1, v_2, \ldots, v_m) \in R^m$ be a flow of network. We will assume, for the rest of this paper, that the demand of network is fixed for each O/D pair; that is, $\sum_{k \in K_i} v_k = d_i$, where d_i is a given demand for each O/D pair i. A flow $v \geq 0$ satisfying the demand is called a feasible flow. D is clearly a convex and compact set in R^m. Let there be given a vector of demands $d = (d_1, d_2, \ldots, d_l)$.

We will consider that the network system maintains an expected amount of flow in every path. That is, for every O/D pair $i \in I$ and every path $k \in K_i$, an expected flow $m_k(v) > 0$ is given. For every O/D pair $i \in I$ and every path $k \in K_i$, we assume that $v_k \geq m_k(v)$. Let $m(v) = (m_k(v) : k \in K_i,\ i \in I)$. Suppose that the set-valued mapping $Q : D \to 2^D$ is defined as $Q(v) = \{u \in D : u \geq m(v)\}$.

Let Y be a Hausdorff topological vector space ordered by a pointed, closed convex cone $S \subset Y$ with k_0, $e \in \operatorname{int} S$. We denote by "$\leq$" the ordering induced by S; that is,

$$x \leq y \quad \text{iff } y - x \in S, \qquad x < y \quad \text{iff } y - x \in \operatorname{int} S. \tag{3}$$

For every $i \in I$ and $k \in K_i$, we define the cost function of the path k as a vector-valued function $C_k(v) : R^m \to Y$; the mapping $C(v) = (C_k(v) : k \in K_i,\ i \in I)$ is called the cost function of the network.

Definition 1 (see [11]). A vector v is said to be an equilibrium pattern flow with a vector-valued cost function if and only if $v \geq m(v)$ and

$$C_r(v) - C_k(v) \in S \setminus \{0\} \Longrightarrow (v_r - m_r(v))\, e \notin \operatorname{int} S, \tag{4}$$

for each $i \in I$ and any $k, r \in K_i$.

Remark 2. Notice the fact that $e \in \operatorname{int} S$ and $v_r \geq m_r(v)$ and the relationship $(v_r - m_r(v))e \notin \operatorname{int} S$ in (4) actually is equivalent to $v_r - m_r(v) = 0$. Therefore, (4) is equivalent to

$$C_r(v) - C_k(v) \in S \setminus \{0\} \Longrightarrow v_r - m_r(v) = 0. \tag{5}$$

Lemma 3 (see [11]). *A vector flow $v \in D$ is an equilibrium pattern flow with a unilateral constraint if and only if v is a solution to the quasivariational inequality: to find $v \in Q(v)$ such that*

$$\left\langle c(v), (u - v)^T \right\rangle \geq 0, \quad \forall u \in Q(v). \tag{6}$$

We now introduce a concept of approximate equilibria for the family of network equilibrium problems. Let $v \in D$ be a flow of the network $[I, L]$, and let $\epsilon \geq 0$. For each $i \in I$, denote

$$G_{i,\epsilon}(v) = \{r \in K_i : (C_r(v) - \epsilon k_0 - S \setminus \{0\}) \cap \{C_k(v), k \in K_i\} = \emptyset\}. \tag{7}$$

Definition 4. A vector v is said to be an ϵ-equilibrium pattern flow with a vector-valued cost function if and only if $v \geq m(v)$ and

$$v_t \leq m_t(v) + \epsilon, \quad \forall t \in K_i \setminus G_{i,\epsilon}(v). \tag{8}$$

Remark 5. When $\epsilon = 0$, the ϵ-equilibrium pattern flow reduces to the equilibrium pattern flow as shown in Definition 1. Namely, a vector $v \in D$ is a 0-equilibrium pattern flow if and only if $v \in D$ is an equilibrium pattern flow.

In fact, assume that $v \in Q(v)$ is a 0-equilibrium pattern flow but not an equilibrium pattern flow. Then, there exist $i \in I$ and $k, r \in K_i$ satisfying $C_r(v) - C_k(v) \in S \setminus \{0\}$ and $v_r > m_r(v)$. However, by Definition 4 and $v \in D$ being a 0-equilibrium pattern flow, $r \in G_{i,0}(v)$. Hence, we have, for all $k \in K_i$, $C_k(v) \notin C_r(v) - S \setminus \{0\}$, which is a contradiction. Conversely, assume that $v \in Q(v)$ is an equilibrium pattern flow but not a 0-equilibrium pattern flow. Then, there exists $r \in K_i \setminus G_{i,0}(v)$ with $v_r > m_r(v)$. From the definition of $G_{i,0}(v)$, there exists $k \in K_i$ satisfying $C_k(v) \in C_r(v) - S \setminus \{0\}$. From (5), $v_r = m_r(v)$. This is a contradiction.

3. Well-Posedness of Multicriteria Network Equilibrium Problem

The following real-valued function is of fundamental importance to our current analysis. The original version is due to what Gerstewitz (Tammer) [12] published in German.

Definition 6. Let $k_0 \in \operatorname{int} S$ and $E \subset Y$. Gerstewitz's function $\xi_{k_0} : E \to R$ is defined by

$$\xi_{k_0}(y) = \min\{t \in R \mid y \in tk_0 - S\}, \quad \forall y \in E. \tag{9}$$

By Theorem 2.1 of [13] and Lemmas 3 and 4 of [14], we have the following results.

Lemma 7. *Let $k_0 \in \operatorname{int} S$ and $E \subset Y$. For each $l \in R$ and $y \in E$, we have the following results:*

(i) $\xi_{k_0}(y) < l \Leftrightarrow y \in lk_0 - \operatorname{int} S$;

(ii) $\xi_{k_0}(y) \leq l \Leftrightarrow y \in lk_0 - S$;

(iii) $\xi_{k_0}(y) \geq l \Leftrightarrow y \notin lk_0 - \operatorname{int} S$;

(iv) $\xi_{k_0}(y) > l \Leftrightarrow y \notin lk_0 - S$;

(v) $\xi_{k_0}(\cdot)$ *is a continuous and strictly monotone function; namely,*

$$\xi_{k_0}(y_1) > \xi_{k_0}(y_2), \quad \text{if } y_1, y_2 \in E,\ y_1 - y_2 \in \operatorname{int} S; \tag{10}$$

(vi) $\xi_{k_0}(\cdot)$ *is subadditive; namely,*

$$\xi_{k_0}(y_1 + y_2) \leq \xi_{k_0}(y_1) + \xi_{k_0}(y_2), \quad \forall y_1,\ y_2 \in E; \tag{11}$$

(vii) $\xi_{k_0}(lk_0) = l$, *for all* $l \in R$;

(viii) $\xi_{k_0}(y + lk_0) = l + \xi_{k_0}(y)$, *for all* $l \in R$, $y \in E$.

Definition 8 (see [11]). A vector v is said to be a ξ_{k_0}-equilibrium pattern flow if and only if $v \geq m(v)$ and

$$\xi_{k_0}(C_r(v)) - \xi_{k_0}(C_k(v)) > 0 \Longrightarrow v_r - m_r(v) = 0, \tag{12}$$

for each $i \in I$ and $k, r \in K_i$.

Now let $v \in D$ be a flow of network $[I, L]$, and let $\epsilon \geq 0$. For each $i \in I$, denote

$$\begin{aligned} g_i'(v) &= \min\left\{\xi_{k_0}(C_k(v)), k \in K_i\right\}, \\ G_{i,\epsilon}'(v) &= \left\{r \in K_i : \xi_{k_0}(C_r(v)) \leq g_i'(v) + \epsilon\right\} \subset K_i. \end{aligned} \tag{13}$$

Definition 9. Let $\epsilon \geq 0$. A vector v is said to be a ξ_{k_0}-ϵ-equilibrium pattern flow if and only if $v \geq m(v)$ and

$$v_t \leq m_t(v) + \epsilon, \quad \forall t \in K_i \setminus G_{i,\epsilon}'(v). \tag{14}$$

Theorem 10. *If $v^n \in Q(v^n)$ is a ξ_{k_0}-ϵ_n-equilibrium pattern flow, $v^n \to v$, and m, C are continuous mappings, then v is a ξ_{k_0}-equilibrium pattern flow.*

Proof. Let a sequence $v^n \in D$ be a ξ_{k_0}-ϵ_n-equilibrium pattern flow, let $v^n \to v$, and let $u \in Q(v)$ be arbitrarily chosen. From the continuity of m, compactness of D, and $v^n \in Q(v^n)$, we have $v \in Q(v)$. Moreover, it follows from the continuity of m and the compactness of D that, for every $u \in Q(v)$, there exists a sequence $\{u^n : u^n \geq m(v^n)\}$ satisfying $u^n \to u$.

Let $i \in I$ and $\epsilon_n \geq 0$; define

$$\begin{aligned} g_i(v^n) &= \min\left\{\xi_{k_0} \circ C_k(v^n), k \in K_i\right\}, \\ G_i(v^n) := G_{i,\epsilon_n}'(v^n) &= \left\{t \in K_i : \xi_{k_0} \circ C_t(v^n) \leq g_i(v^n) + \epsilon_n\right\}, \\ G_i^1(v^n) &= \left\{t \in G_i(v^n), u_t^n > v_t^n\right\}, \\ G_i^2(v^n) &= \left\{t \in G_i(v^n), u_t^n \leq v_t^n\right\}. \end{aligned} \tag{15}$$

Clearly, $G_i(v^n) = G_i^1(v^n) \cup G_i^2(v^n)$.

By Definition 9, we have

$$v_t^n \leq m_t(v^n) + \epsilon_n, \quad \forall t \in K_i \setminus G_i(v^n). \tag{16}$$

For $v^n \in D$ and $u^n \in Q(v^n)$, we have

$$\begin{aligned}
&\left\langle \xi_{k_0} \circ C(v^n), (u^n - v^n)^T \right\rangle \\
&= \sum_{i=1}^{l} \sum_{t \in K_i} \xi_{k_0} \circ C_t(v^n)(u_t^n - v_t^n) \\
&= \sum_{i=1}^{l} \left[\sum_{t \in G_i^1(v^n)} \xi_{k_0} \circ C_t(v^n)(u_t^n - v_t^n) + \sum_{t \in G_i^2(v^n)} \xi_{k_0} \circ C_t(v^n)(u_t^n - v_t^n) + \sum_{t \in K_i \setminus G_i(v^n)} \xi_{k_0} \circ C_t(v^n)(u_t^n - v_t^n) \right] \\
&\geq \sum_{i=1}^{l} \left[\sum_{t \in G_i^1(v^n)} g_i(v^n)(u_t^n - v_t^n) + \sum_{t \in G_i^2(v^n)} (g_i(v^n) + \epsilon_n)(u_t^n - v_t^n) + \sum_{t \in K_i \setminus G_i(v^n)} \xi_{k_0} \circ C_t(v^n)(u_t^n - m_t(v^n) - \epsilon_n) \right] \\
&\geq \sum_{i=1}^{l} \left[\sum_{t \in G_i^1(v^n)} g_i(v^n)(u_t^n - v_t^n) + \sum_{t \in G_i^2(v^n)} (g_i(v^n) + \epsilon_n)(u_t^n - v_t^n) + \sum_{t \in K_i \setminus G_i(v^n)} g_i(v^n)(u_t^n - m_t(v^n)) - \sum_{t \in K_i \setminus G_i(v^n)} \epsilon_n \xi_{k_0} \circ C_t(v^n) \right] \\
&= \sum_{i=1}^{l} \left[\sum_{t \in K_i} g_i(v^n) u_t^n - \sum_{t \in G_i(v^n)} g_i(v^n) v_t^n - \sum_{t \in K_i \setminus G_i(v^n)} g_i(v^n) m_t(v^n) + \sum_{t \in G_i^2(v^n)} \epsilon_n (u_t^n - v_t^n) - \sum_{t \in K_i \setminus G_i(v^n)} \epsilon_n \xi_{k_0} \circ C_t(v^n) \right] \\
&= \sum_{i=1}^{l} \left[\sum_{t \in K_i} g_i(v^n) u_t^n - \sum_{t \in K_i} g_i(v^n) v_t^n + \sum_{t \in K_i \setminus G_i(v^n)} g_i(v^n)(v_t^n - m_t(v^n)) + \sum_{t \in G_i^2(v^n)} \epsilon_n (u_t^n - v_t^n) - \sum_{t \in K_i \setminus G_i(v^n)} \epsilon_n \xi_{k_0} \circ C_t(v^n) \right] \\
&= \sum_{i=1}^{l} \left[g_i(v^n) \left(\sum_{t \in K_i} u_t^n - \sum_{t \in K_i} v_t^n \right) + \sum_{t \in K_i \setminus G_i(v^n)} g_i(v^n)(v_t^n - m_t(v^n)) + \sum_{t \in G_i^2(v^n)} \epsilon_n (u_t^n - v_t^n) - \sum_{t \in K_i \setminus G_i(v^n)} \epsilon_n \xi_{k_0} \circ C_t(v^n) \right].
\end{aligned} \tag{17}$$

Since $u^n \to u$, then, for every i, $\sum_{t\in K_i} u_t^n \to \sum_{t\in K_i} u_t$ when $n \to +\infty$. By $v^n, u \in D$, $\sum_{t\in K_i} u_t = \sum_{t\in K_i} v_t^n = d_i$. Thus, for each i,

$$\sum_{t\in K_i} u_t^n - \sum_{t\in K_i} v_t^n = \sum_{t\in K_i} u_t^n - d_i \longrightarrow 0. \tag{18}$$

Letting $n \to \infty$, we conclude that

$$\left\langle \xi_{k_0} \circ C(v), (u-v)^T \right\rangle \geq 0. \tag{19}$$

Therefore, from Lemma 3 (take $\xi_{k_0} \circ C = c$), v is a ξ_{k_0}-equilibrium pattern flow. □

Set $C_t : R^m \to Y$ in the following form:

$$C_t(v) = f_t(v) k_0, \quad \forall t \in K_i,\ i \in I, \tag{20}$$

where $f_t(v) : R^m \to R$.

Theorem 11. *Let $\epsilon \geq 0$ and C_t be defined as* (20) *for all $t \in K_i$, $i \in I$. v is an ϵ-equilibrium pattern flow with a vector-valued cost function if and only if v is a ξ_{k_0}-ϵ-equilibrium pattern flow.*

Proof. Assume that v is an ϵ-equilibrium pattern flow but not a ξ_{k_0}-ϵ-equilibrium pattern flow. Then, there exist an $\bar{i} \in I$ and a pair of $\bar{k}, \bar{r} \in K_{\bar{i}}$ satisfying

$$\xi_{k_0}(C_{\bar{r}}(v)) - \xi_{k_0}(C_{\bar{k}}(v)) + \epsilon > 0, \tag{21}$$

$$v_{\bar{r}} - m_{\bar{r}}(v) > \epsilon. \tag{22}$$

From $\xi_{k_0}(C_{\bar{r}}(v)) - \xi_{k_0}(C_{\bar{k}}(v)) - \epsilon > 0$, we have

$$\begin{aligned} &\xi_{k_0}(C_{\bar{k}}(v) - C_{\bar{r}}(v) + \epsilon k_0) \\ &\quad = f_{\bar{k}}(v) - f_{\bar{r}}(v) + \epsilon \\ &\quad = \xi_{k_0}(f_{\bar{k}}(v) k_0) - \xi_{k_0}(f_{\bar{r}}(v) k_0) + \epsilon \\ &\quad = \xi_{k_0}(C_{\bar{k}}(v)) - \xi_{k_0}(C_{\bar{r}}(v)) + \epsilon < 0. \end{aligned} \tag{23}$$

By Lemma 7,

$$C_{\bar{k}}(v) - C_{\bar{r}}(v) + \epsilon k_0 \in -\operatorname{int} S \subset -S \setminus \{0\}. \tag{24}$$

Thus,

$$C_{\bar{k}}(v) \in C_{\bar{r}}(v) - \epsilon k_0 - S \setminus \{0\}. \tag{25}$$

This implies that $\bar{r} \in K_{\bar{i}} \setminus G'_{\bar{i},\epsilon}(v)$. It follows from v being an ϵ-equilibrium pattern flow that

$$v_{\bar{r}} - m_{\bar{r}}(v) \leq \epsilon, \tag{26}$$

which contradicts (22).

Conversely, assume that v is a ξ_{k_0}-ϵ-equilibrium pattern flow. Then, for any $i \in I$ and every path $r \in K_i \setminus G_{i,\epsilon}(v)$, we want to deduce $v_r - m_r(v) \leq \epsilon$.

It follows from $r \in K_i \setminus G_{i,\epsilon}(v)$ that there exists $k \in K_i$ satisfying

$$C_r(v) - C_k(v) - \epsilon k_0 \in S \setminus \{0\}. \tag{27}$$

Then,

$$\xi_{k_0}(C_r(v) - C_k(v) - \epsilon k_0) = f_r(v) - f_k(v) - \epsilon \geq 0. \tag{28}$$

Since $C_r(v) - C_k(v) - \epsilon k_0 \neq 0$, we have $f_r(v) - f_k(v) - \epsilon \neq 0$. Therefore,

$$\begin{aligned} &\xi_{k_0}(C_r(v)) - \xi_{k_0}(C_k(v)) - \epsilon \\ &\quad = \xi_{k_0}(C_r(v) - C_k(v) - \epsilon k_0) > 0. \end{aligned} \tag{29}$$

Hence, $r \in K_i \setminus G'_{i,\epsilon}(v)$. By Definition 9, we have $v_r - m_r(v) \leq \epsilon$. □

Definition 12. Let $\{\epsilon_n : \epsilon_n \geq 0, \epsilon_n \to 0\}$ be a sequence, let $v^n \in D$ be an ϵ_n-equilibrium flow for each n, and let $v^n \to v$; the network $[I, L]$ is called well-posed if v is a traffic equilibrium flow.

Theorem 13. *Let C_t be defined as* (20)*. If m and f_t are continuous mappings, then the network $[I, L]$ is well-posed.*

Proof. Let $v^n \in D$ be an ϵ_n-equilibrium pattern flow. From Theorem 11, v^n is a ξ_{k_0}-ϵ_n-equilibrium pattern flow. From Theorem 10 and $v^n \to v$, v is a ξ_{k_0}-equilibrium pattern flow. By Theorem 11 (take $\epsilon = 0$), $v \in D$ is an equilibrium pattern flow for the vector network equilibrium problem. □

Conflict of Interests

The author declares that there is no conflict of interests regarding the publication of this paper.

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