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Research Article

Nonlocal Boundary Value Problem for Nonlinear Impulsive q_k-Integrodifference Equation

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A nonlinear impulsive integrodifference equation within the frame of q_k-quantum calculus is investigated by applying using fixed point theorems. The conditions for existence and uniqueness of solutions are obtained.

1. Introduction

Recently, by introducing and applying the fractional difference operators to real world problems (see, e.g., [1–7] and the references therein) we revitalized the importance of the quantum calculus [8]. However the real world phenomena are usually described by complex model based involving different types of operators. In this way we hope to understand deeper the dynamics of complex or hypercomplex systems and to reveal their hidden aspects.

On this line of thought in this paper, we study the existence and uniqueness of solutions for nonlinear q_k-integrodifference equation with nonlocal boundary condition and impulses:

$$\begin{gathered} D_{q_k} u(t) = f(t, u(t)) + {}_{t_k}I_{q_k} g(t, u(t)), \\ 0 < q_k < 1, \ t \in J', \\ \Delta u(t_k) = I_k(u(t_k)), \quad k = 1, 2, \ldots, m, \\ u(0) = h(u) + u_0, \quad u_0 \in \mathbb{R}, \end{gathered} \tag{1}$$

where D_{q_k}, ${}_{t_k}I_{q_k}$ are q_k-derivatives and q_k-integrals ($k = 0, 1, 2, \ldots, m$), respectively. $f, g \in C(J \times \mathbb{R}, \mathbb{R})$, $I_k, h \in C(\mathbb{R}, \mathbb{R})$, $J = [0, T] (T > 0)$, $0 = t_0 < t_1 < \cdots < t_k < \cdots < t_m < t_{m+1} = T$, $J' = J \setminus \{t_1, t_2, \ldots, t_m\}$, $\Delta u(t_k) = u(t_k^+) - u(t_k^-)$, where $u(t_k^+)$ and $u(t_k^-)$ denote the right and the left limits of $u(t)$ at $t = t_k (k = 1, 2, \ldots, m)$, respectively.

2. Preliminaries

Let us set $J_0 = [0, t_1]$, $J_1 = (t_1, t_2], \ldots, J_{m-1} = (t_{m-1}, t_m]$, $J_m = (t_m, T]$ and introduce the space:

$$\begin{aligned} PC(J, \mathbb{R}) = \{ u : J \longrightarrow \mathbb{R} \mid u \in C(J_k), \ k = 0, 1, \ldots, m, \\ \text{and } u(t_k^+) \text{ exist}, \ k = 1, 2, \ldots, m \}, \end{aligned} \tag{2}$$

with the norm $\|u\| = \sup_{t \in J} |u(t)|$. Then, $PC(J, \mathbb{R})$ is a Banach space.

For convenience, let us recall some basic concepts of q_k-calculus [9].

For $0 < q_k < 1$ and $t \in J_k$, we define the q_k-derivatives of a real valued continuous function f as

$$\begin{aligned} D_{q_k} f(t) &= \frac{f(t) - f(q_k t + (1 - q_k) t_k)}{(1 - q_k)(t - t_k)}, \\ D_{q_k} f(t_k) &= \lim_{t \to t_k} D_{q_k} f(t). \end{aligned} \tag{3}$$

Higher order q_k-derivatives are given by

$$D^0_{q_k} f(t) = f(t), \qquad D^n_{q_k} f(t) = D_{q_k} D^{n-1}_{q_k} f(t), \quad n \in \mathbb{N},\ t \in J_k. \tag{4}$$

The q_k-integral of a function f is defined by

$${}_{t_k}I_{q_k} f(t) := \int_{t_k}^{t} f(s)\, d_{q_k}s = (1-q_k)(t-t_k) \times \sum_{n=0}^{\infty} q_k^n f(q_k^n t + (1-q_k^n) t_k), \quad t \in J_k, \tag{5}$$

provided the series converges. If $a \in (t_k, t)$ and f is defined on the interval (t_k, t), then

$$\int_a^t f(s)\, d_{q_k}s = \int_{t_k}^t f(s)\, d_{q_k}s - \int_{t_k}^a f(s)\, d_{q_k}s. \tag{6}$$

Observe that

$$\begin{aligned} D_{q_k}\left({}_{t_k}I_{q_k} f(t)\right) &= D_{q_k} \int_{t_k}^t f(s)\, d_{q_k}s = f(t), \\ {}_{t_k}I_{q_k}\left(D_{qk} f(t)\right) &= \int_{t_k}^t D_{qk} f(s)\, d_{q_k}s = f(t), \\ {}_{a}I_{q_k}\left(D_{qk} f(t)\right) &= \int_a^t D_{qk} f(s)\, d_{q_k}s = f(t) - f(a), \\ & a \in (tk, t). \end{aligned} \tag{7}$$

For $t \in J_k$, the following reversing order of q_k-integration holds

$$\int_{t_k}^t \int_{t_k}^s f(r)\, d_{q_k}r d_{q_k}s = \int_{t_k}^t \int_{q_k r + (1-q_k)t_k}^t f(r)\, d_{q_k}s d_{q_k}r. \tag{8}$$

Note that if $t_k = 0$ and $q_k = q$ in (3) and (5), then $D_{q_k} f = D_q f$, ${}_{t_k}I_{q_k} f = {}_0I_q f$, where D_q and ${}_0I_q$ are the well-known q-derivative and q-integral of the function $f(t)$ defined by

$$\begin{aligned} D_q f(t) &= \frac{f(t) - f(qt)}{(1-q)t}, \\ {}_0I_q f(t) &= \int_0^t f(s)\, d_q s = \sum_{n=0}^{\infty} t(1-q) q^n f(tq^n). \end{aligned} \tag{9}$$

Lemma 1. *For given $y_{q_k} \in C(J, \mathbb{R})$, the function $u \in PC(J, \mathbb{R})$ is a solution of the impulsive q_k-integrodifference equation*

$$\begin{aligned} D_{q_k} u(t) &= y_{q_k}(t), \quad 0 < q_k < 1,\ t \in J', \\ \Delta u(t_k) &= I_k(u(t_k)), \quad k = 1, 2, \ldots, m, \\ u(0) &= h(u) + u_0, \quad u_0 \in \mathbb{R}, \end{aligned} \tag{10}$$

if and only if u satisfies the q_k-integral equation

$$u(t) = \begin{cases} \int_0^t y_{q_0}(s)\, d_{q_0}s + h(u) + u_0, & t \in J_0; \\ \int_{t_k}^t y_{q_k}(s)\, d_{q_k}s + \sum_{i=0}^{k-1} \int_{t_i}^{t_{i+1}} y_{q_i}(s)\, d_{q_i}s \\ \quad + \sum_{i=1}^{k} I_i(u(t_i)) + h(u) + u_0, & t \in J_k. \end{cases} \tag{11}$$

Proof. Let u be a solution of q_k-difference equation (10). For $t \in J_0$, applying the operator ${}_0I_{q_0}$ on both sides of $D_{q_0}u(t) = y_{q_0}(t)$, we have

$$u(t) = u(0) + {}_0I_{q_0} y_{q_0}(t) = u(0) + \int_0^t y_{q_0}(s)\, d_{q_0}s. \tag{12}$$

Thus,

$$u(t_1^-) = u(0) + \int_0^{t_1} y_{q_0}(s)\, d_{q_0}s. \tag{13}$$

Similarly, for $t \in J_1$, applying the operator ${}_{t_1^+}I_{q_1}$ on both sides of $D_{q_1}u(t) = y_{q_1}(t)$, then

$$u(t) = u(t_1^+) + \int_{t_1}^t y_{q_1}(s)\, d_{q_1}s. \tag{14}$$

In view of $\Delta u(t_1) = u(t_1^+) - u(t_1^-) = I_1(u(t_1))$, it holds

$$\begin{aligned} u(t) = u(0) &+ \int_{t_1}^t y_{q_1}(s)\, d_{q_1}s + \int_0^{t_1} y_{q_0}(s)\, d_{q_0}s \\ &+ I_1(u(t_1)), \quad \forall t \in J_1. \end{aligned} \tag{15}$$

Repeating the above process, we can get

$$\begin{aligned} u(t) = u(0) &+ \int_{t_k}^t y_{q_k}(s)\, d_{q_k}s + \sum_{i=0}^{k-1} \int_{t_i}^{t_{i+1}} y_{q_i}(s)\, d_{q_i}s \\ &+ \sum_{i=1}^{k} I_i(u(t_i)), \quad t \in J_k. \end{aligned} \tag{16}$$

Using the boundary value condition given in (10), it follows

$$\begin{aligned} u(t) = &\int_{t_k}^t y_{q_k}(s)\, d_{q_k}s + \sum_{i=0}^{k-1} \int_{t_i}^{t_{i+1}} y_{q_i}(s)\, d_{q_i}s \\ &+ \sum_{i=1}^{k} I_i(u(t_i)) + h(u) + u_0, \quad t \in J_k. \end{aligned} \tag{17}$$

Conversely, assume that u satisfies the impulsive q_k-integral equation (11); applying D_{q_k} on both sides of (11) and substituting $t = 0$ in (11), then (10) holds. This completes the proof. □

3. Main Results

Letting $y_{q_k}(t) = f(t,u(t)) + {}_{t_k}I_{q_k} g(t,u(t))$, in view of Lemma 1, we introduce an operator $Q : PC(J,\mathbb{R}) \to PC(J,\mathbb{R})$ as

$$(Qu)(t) = \int_{t_k}^{t} \left[f(s,u(s)) + \int_{t_k}^{s} g(r,u(r))\, d_{q_k} r \right] d_{q_k} s + \sum_{i=0}^{k-1} \int_{t_i}^{t_{i+1}} \left[f(s,u(s)) + \int_{t_i}^{s} g(r,u(r))\, d_{q_i} r \right] \times (s)\, d_{q_i} s + \sum_{i=1}^{k} I_i(u(t_i)) + h(u) + u_0. \tag{18}$$

By reversing the order of integration, we obtain

$$\begin{aligned}(Qu)(t) = &\int_{t_k}^{t} [f(s,u(s)) + [(t-t_k) - q_k(s-t_k)] \times g(s,u(s))]\, d_{q_k} s \\ &+ \sum_{i=0}^{k-1} \int_{t_i}^{t_{i+1}} [f(s,u(s)) + [(t_{i+1}-t_i) - q_i(s-t_i)] \times g(s,u(s))]\, d_{q_i} s \\ &+ \sum_{i=1}^{k} I_i(u(t_i)) + h(u) + u_0.\end{aligned} \tag{19}$$

Then, the impulsive q_k-integrodifference equation (1) has a solution if and only if the operator equation $u = Qu$ has a fixed point.

In order to prove the existence of solutions for (1), we need the following known result [10].

Theorem 2. *Let E be a Banach space. Assume that $T : E \to E$ is a completely continuous operator and the set $V = \{x \in E \mid x = \mu Tx, 0 < \mu < 1\}$ is bounded. Then T has a fixed point in E.*

Theorem 3. *Assume the following.*

(H_1) *There exist nonnegative bounded functions $M_i(t)$ ($i = 1,2,3,4$) such that*

$$|f(t,u)| \le M_1(t) + M_2(t)|u|, \qquad |g(t,u)| \le M_3(t) + M_4(t)|u|, \tag{20}$$

for any $t \in J$, $u \in \mathbb{R}$.

(H_2) *There exist positive constants $\overline{L}, \widetilde{L}$ such that*

$$|I_k(u)| \le \overline{L}, \quad |h(u)| \le \widetilde{L}, \tag{21}$$

for any $u \in \mathbb{R}$, $k = 1,2,\ldots,m$.

Then problem (1) *has at least one solution provided*

$$\sup_{t\in J} \left[TM_2(t) + M_4(t) \sum_{i=0}^{m} \frac{(t_{i+1}-t_i)^2}{1+q_i} \right] < 1. \tag{22}$$

Proof. Firstly, we prove the operator $Q : PC(J,\mathbb{R}) \to PC(J,\mathbb{R})$ is completely continuous. Clearly, continuity of the operator Q follows from the continuity of f, g, I_k, and h. Let $\Omega \subset PC(J,\mathbb{R})$ be bounded. Then $\forall t \in J$, $u \in \Omega$; there exist positive constants L_i ($i = 1,2,3,4$) such that $|f(t,u)| \le L_1$, $|g(t,u)| \le L_2$, $|I_k(u)| \le L_3$, $|h(u)| \le L_4$. Thus

$$\begin{aligned}|(Qu)(t)| \le &\int_{t_k}^{t} [|f(s,u(s))| + [(t-t_k) - q_k(s-t_k)] \times |g(s,u(s))|]\, d_{q_k} s \\ &+ \sum_{i=0}^{k-1} \int_{t_i}^{t_{i+1}} [|f(s,u(s))| + [(t_{i+1}-t_i) - q_i(s-t_i)] \times |g(s,u(s))|]\, d_{q_i} s \\ &+ \sum_{i=1}^{k} |I_i(u(t_i))| + |h(u)| + |u_0| \\ \le &\int_{t_k}^{t} [L_1 + [(t-t_k) - q_k(s-t_k)] L_2]\, d_{q_k} s \\ &+ \sum_{i=0}^{k-1} \int_{t_i}^{t_{i+1}} [L_1 + [(t_{i+1}-t_i) - q_i(s-t_i)] L_2]\, d_{q_i} s \\ &+ \sum_{i=1}^{k} L_3 + L_4 + |u_0| \\ \le &L_1(t-t_k) + L_2 \frac{(t-t_k)^2}{1+q_k} \\ &+ \sum_{i=0}^{k-1} \left[L_1(t_{i+1}-t_i) + L_2 \frac{(t_{i+1}-t_i)^2}{1+q_i} \right] \\ &+ mL_3 + L_4 + |u_0| \\ \le &TL_1 + L_2 \sum_{i=0}^{m} \frac{(t_{i+1}-t_i)^2}{1+q_i} + mL_3 \\ &+ L_4 + |u_0| := \mathscr{L}\ (\text{constant}).\end{aligned} \tag{23}$$

This implies $\|Qu\| \le \mathscr{L}$.

Furthermore, for any $t', t'' \in J_k$ $(k = 0, 1, 2, \ldots, m)$ satisfying $t' < t''$, we have

$$\begin{aligned}
\left|(Qu)(t'') - (Qu)(t')\right| &\le \left| \int_{t_k}^{t''} \left[\left|f(s,u(s))\right| + \left[\left(t''-t_k\right) - q_k\left(s-t_k\right)\right] \times \left|g(s,u(s))\right|\right] d_{q_k}s \right. \\
&\quad \left. - \int_{t_k}^{t'} \left[\left|f(s,u(s))\right| + \left[\left(t'-t_k\right) - q_k\left(s-t_k\right)\right] \times \left|g(s,u(s))\right|\right] d_{q_k}s \right| \\
&\le \int_{t'}^{t''} \left|f(s,u(s)) + \left[\left(t''-t_k\right) - q_k\left(s-t_k\right)\right] \times g(s,u(s))\right| d_{q_k}s \\
&\quad + \int_{t_k}^{t'} \left(t''-t'\right) \times \left|g(s,u(s))\right| d_{q_k}s \\
&\le \int_{t'}^{t''} \left[\left|f(s,u(s))\right| + \left(t''-t_k\right) \times \left|g(s,u(s))\right|\right] d_{q_k}s \\
&\quad + L_2\left(t'-t_k\right)\left(t''-t'\right) \\
&\le \left[L_1 + L_2\left(t''-t_k\right)\right]\left(t''-t'\right) + L_2\left(t'-t_k\right)\left(t''-t'\right).
\end{aligned} \tag{24}$$

As $t' \to t''$, the right hand side of the above inequality tends to zero. Thus, $Q(\Omega)$ is relatively compact. As a consequence of Arzela Ascoli's theorem, Q is a compact operator. Therefore, Q is a completely continuous operator.

Define the set $\mathscr{W} = \{u \in PC(J,\mathbb{R}) \mid u = \lambda Qu, 0 < \lambda < 1\}$.

Next, we show $\mathscr{W}$ is bounded. Let $u \in \mathscr{W}$; then $u = \lambda Qu$, $0 < \lambda < 1$. For any $t \in J$, by conditions (H_1) and (H_2), we have

$$\begin{aligned}
|u(t)| &= \lambda\left|(Qu)(t)\right| \\
&\le \int_{t_k}^{t} \left[\left|f(s,u(s))\right| + \left[\left(t-t_k\right) - q_k\left(s-t_k\right)\right] \times \left|g(s,u(s))\right|\right] d_{q_k}s \\
&\quad + \sum_{i=0}^{k-1} \int_{t_i}^{t_{i+1}} \left[\left|f(s,u(s))\right| + \left[\left(t_{i+1}-t_i\right) - q_i\left(s-t_i\right)\right] \times \left|g(s,u(s))\right|\right] d_{q_i}s \\
&\quad + \sum_{i=1}^{k} \left|I_i\left(u\left(t_i\right)\right)\right| + |h(u)| + \left|u_0\right| \\
&\le \int_{t_k}^{t} \left[M_1(s) + M_2(s)|u(s)| + \left[\left(t-t_k\right) - q_k\left(s-t_k\right)\right] \times \left[M_3(s) + M_4(s)|u(s)|\right]\right] d_{q_k}s \\
&\quad + \sum_{i=0}^{k-1} \int_{t_i}^{t_{i+1}} \left[M_1(s) + M_2(s)|u(s)| + \left[\left(t_{i+1}-t_i\right) - q_i\left(s-t_i\right)\right] \times \left[M_3(s) + M_4(s)|u(s)|\right]\right] d_{q_i}s \\
&\quad + \sum_{i=1}^{k} \overline{L} + \widetilde{L} + \left|u_0\right| \\
&\le \left(M_1 + M_2\|u\|\right)\left(t-t_k\right) + \left(M_3 + M_4\|u\|\right) \times \frac{\left(t-t_k\right)^2}{1+q_k} \\
&\quad + \sum_{i=0}^{k-1}\left[\left(M_1 + M_2\|u\|\right)\left(t_{i+1}-t_i\right) + \left(M_3 + M_4\|u\|\right)\frac{\left(t_{i+1}-t_i\right)^2}{1+q_i}\right] \\
&\quad + m\overline{L} + \widetilde{L} + \left|u_0\right| \\
&\le \left(M_1 + M_2\|u\|\right)T + \left(M_3 + M_4\|u\|\right) \times \sum_{i=0}^{m}\frac{\left(t_{i+1}-t_i\right)^2}{1+q_i} + m\overline{L} + \widetilde{L} + \left|u_0\right| \\
&\le M_1T + M_3\sum_{i=0}^{m}\frac{\left(t_{i+1}-t_i\right)^2}{1+q_i} + m\overline{L} \\
&\quad + \widetilde{L} + \left|u_0\right| + \left[M_2T + M_4\sum_{i=0}^{m}\frac{\left(t_{i+1}-t_i\right)^2}{1+q_i}\right]\|u\|,
\end{aligned} \tag{25}$$

which implies

$$\begin{aligned}
\|u\| &\le \frac{M_1T + M_3\sum_{i=0}^{m}\left(\left(t_{i+1}-t_i\right)^2/\left(1+q_i\right)\right) + m\overline{L} + \widetilde{L} + \left|u_0\right|}{1 - \left[M_2T + M_4\sum_{i=0}^{m}\left(\left(t_{i+1}-t_i\right)^2/\left(1+q_i\right)\right)\right]} \\
&:= \text{constant}.
\end{aligned} \tag{26}$$

So, the set $\mathscr{W}$ is bounded. Thus, Theorem 2 ensures the impulsive q_k-integrodifference equation (1) has at least one solution. □

Corollary 4. *Assume the following.*

(H_3) *There exist nonnegative constants L_i ($i = 1,2,3,4$) such that*

$$\left|f(t,u)\right| \le L_1, \qquad \left|g(t,u)\right| \le L_2, \qquad \left|I_k(u)\right| \le L_3, \qquad |h(u)| \le L_4, \tag{27}$$

for any $t \in J$, $u \in \mathbb{R}$, $k = 1,2,\ldots,m$.

Then problem (1) has at least one solution.

Theorem 5. *Assume the following.*

(H_4) *There exist nonnegative bounded functions $M(t)$ and $N(t)$ such that*

$$\left|f(t,u) - f(t,v)\right| \le M(t)|u-v|, \qquad \left|g(t,u) - g(t,v)\right| \le N(t)|u-v|, \tag{28}$$

for $t \in J, u, v \in \mathbb{R}$.

(H_5) *There exist positive constants K, G such that*

$$\left|I_k(u) - I_k(v)\right| \le K|u-v|, \qquad |h(u) - h(v)| \le G|u-v|, \tag{29}$$

for $u, v \in \mathbb{R}$ and $k = 1,2,\ldots,m$.

(H_6) $\mathscr{K} = \sup_{t\in J}[M(t)t + mK + G + N(t)\sum_{i=0}^{m}((t_{i+1}-t_i)^2/(1+q_i))] < 1$.

Then problem (1) has a unique solution.

Proof. Denote $\sup_{t\in J}|M(t)| = M$, $\sup_{t\in J}|N(t)| = N$. For $\forall u, v \in PC(J,\mathbb{R})$, by ($H_4$) and ($H_5$), we have

$$\begin{aligned}
&|(Qu)(t) - (Qv)(t)| \\
&\le \int_{t_k}^{t}\Big[\left|f(s,u(s)) - f(s,v(s))\right| + \left[(t-t_k) - q_k(s-t_k)\right] \times \left|g(s,u(s)) - g(s,v(s))\right|\Big] d_{q_k}s \\
&\quad + \sum_{i=0}^{k-1}\int_{t_i}^{t_{i+1}}\Big[\left|f(s,u(s)) - f(s,v(s))\right| + \left[(t_{i+1}-t_i) - q_i(s-t_i)\right] \times \left|g(s,u(s)) - g(s,v(s))\right|\Big] d_{q_i}s \\
&\quad + \sum_{i=1}^{k}\left|I_i(u(t_i)) - I_i(v(t_i))\right| + |h(u) - h(v)| \\
&\le \int_{t_k}^{t}\left[M(s) + \left[(t-t_k) - q_k(s-t_k)\right]N(s)\right] \times |(u-v)(s)|\, d_{q_k}s \\
&\quad + \sum_{i=0}^{k-1}\int_{t_i}^{t_{i+1}}\left[M(s) + \left[(t_{i+1}-t_i) - q_i(s-t_i)\right] \times N(s)\right]|(u-v)(s)|\, d_{q_i}s \\
&\quad + \sum_{i=1}^{k}K\left|(u-v)(t_i)\right| + G|u-v| \\
&\le \Bigg[M(t-t_k) + N\frac{(t-t_k)^2}{1+q_k} + \sum_{i=0}^{k-1}\left[M(t_{i+1}-t_i) + N\frac{(t_{i+1}-t_i)^2}{1+q_i}\right] + mK + G\Bigg]\|u-v\| \\
&\le \left[Mt + mK + G + N\sum_{i=0}^{m}\frac{(t_{i+1}-t_i)^2}{1+q_i}\right]\|u-v\| \\
&\le \mathscr{K}\|u-v\|.
\end{aligned} \tag{30}$$

As $\mathscr{K} < 1$ by (H_6), then $\|Qu - Qv\| < \|u-v\|$. Therefore, Q is a contractive map. Thus, the conclusion of the Theorem 5 follows by Banach contraction mapping principle. □

4. Example

Consider the following nonlinear q_k-integrodifference equation with impulses

$$\begin{aligned}
D_{1/(2+k)}u(t) &= 8 + 3\sqrt{t} + \ln\left(1 + 5t^3 + \frac{t^2}{5}|u(t)|\right) + \int_{1/(1+2k)}^{t}\left[10s + \frac{s^3}{3}\sin u(s)\right] d_{1/(2+k)}s, \\
&\qquad t \in [0,1],\ t \ne \frac{1}{1+2k}, \\
\Delta u\left(\frac{1}{1+2k}\right) &= \cos\left(u\left(\frac{1}{1+2k}\right)\right), \qquad k = 1,2,\ldots,6, \\
u(0) &= 5 + e^{-u^2(1/2)}.
\end{aligned} \tag{31}$$

Obviously, $q_k = 1/(2+k)$ $(k = 0, 1, 2, \ldots, 6)$, $t_k = 1/(1+2k)$ $(k = 1, 2, \ldots, 6)$, $f(t,u) = 8 + 3\sqrt{t} + \ln(1 + 5t^3 + (t^2/5)|u|)$, $g(t,u) = 10t + (t^3/3)\sin u$, $I_k(u) = \cos u$, and $h(u) = e^{-u^2}$.

By a simple calculation, we can get

$$\left|f(t,u)\right| \le 8 + 3\sqrt{t} + 5t^3 + \frac{t^2}{5}|u|,$$
$$\left|g(t,u)\right| \le 10t + \frac{t^3}{3}|u|, \tag{32}$$
$$\left|I_k(u)\right| \le 1, \qquad |h(u)| \le 1.$$

Take $M_1(t) = 8 + 3\sqrt{t} + 5t^3$, $M_2(t) = t^2/5$, $M_3(t) = 10t$, $M_4(t) = t^3/3$, and $\overline{L} = \widetilde{L} = 1$. Then all conditions of Theorem 3 hold. By Theorem 3, nonlinear impulsive q_k-integrodifference (31) has at least one solution.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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