Hindawi Publishing Corporation
Abstract and Applied Analysis
Volume 2014, Article ID 324912, 10 pages
http://dx.doi.org/10.1155/2014/324912

Research Article

Dynamics of an Almost Periodic Food Chain System with Impulsive Effects

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Received 5 June 2014; Accepted 6 July 2014; Published 21 July 2014

Academic Editor: Yonghui Xia

In order to obtain a more accurate description of the ecological system perturbed by human exploitation activities such as planting and harvesting, we need to consider the impulsive differential equations. Therefore, by applying the comparison theorem and the Lyapunov method of the impulsive differential equations, this paper gives some new sufficient conditions for the permanence and existence of a unique uniformly asymptotically stable positive almost periodic solution in a food chain system with almost periodic impulsive perturbations. The method used in this paper provides a possible method to study the permanence and existence of a unique uniformly asymptotically stable positive almost periodic solution of the models with impulsive perturbations in biological populations. Finally, an example and numerical simulations are given to illustrate that our results are feasible.

1. Introduction

Let $\mathbb{R}$ and $\mathbb{Z}$ denote the sets of real numbers and integers, respectively. Related to a continuous function f, we use the following notations:

$$f^l = \inf_{s\in\mathbb{R}} f(s), \qquad f^u = \sup_{s\in\mathbb{R}} f(s). \tag{1}$$

As was pointed out by Berryman [1], the dynamic relationship between predators and their prey has long been and will continue to be one of the dominant themes in both ecology and mathematical ecology due to its universal existence and importance. Food chain predator-prey system, as one of the most important predator-prey systems, has been extensively studied by many scholars; many excellent results were concerned with the persistent property and positive periodic solution of the system; see [2–8] and the references cited therein. Recently, Shen considered the following three species food chain predator-prey system with Holling type IV functional response:

$$\begin{aligned}
\dot{x}_1(t) &= x_1(t)\left[r_1(t) - a_1(t)x_1(t) - \frac{b_1(t)x_2(t)}{m_1 + x_1^2(t)}\right],\\
\dot{x}_2(t) &= x_2(t)\left[-r_2(t) + \frac{b_2(t)x_1(t)}{m_1 + x_1^2(t)}\right.\\
&\qquad\left. - a_2(t)x_2(t) - \frac{b_3(t)x_3(t)}{m_2 + x_2^2(t)}\right],\\
\dot{x}_3(t) &= x_3(t)\left[-r_3(t) + \frac{b_4(t)x_2(t)}{m_2 + x_2^2(t)} - a_3(t)x_3(t)\right],
\end{aligned} \tag{2}$$

where $x_i(t)$, $i = 1, 2, 3$, denotes the density of species X_i at time t, X_2 is the predator of the first species X_1, and X_3 is the predator of the second species X_2. By applying the comparison theorem of the differential equation and constructing the suitable Lyapunov function, sufficient conditions which guarantee the permanence and the global attractivity of the system are obtained.

Considering the exploited predator-prey system (harvesting or stocking) is very valuable, for it involves the human activities. It can be referred to [9], in which the human activities always happen in a short time or instantaneously. The continuous action of human is then removed from the model and replaced with an impulsive perturbation. These models are subject to short-term perturbations which are often assumed to be in the form of impulsive in the modelling process. Consequently, impulsive differential equations provide a natural description of such systems [10–13]. Then, in [14], Zhang and Tan studied the following Holling II functional responses food chain system with periodic constant impulsive perturbation of predator:

$$\begin{aligned}
\dot{x}_1(t) &= x_1(t)\left[1 - x_1(t) - \frac{b_1 x_2(t)}{1 + x_1(t)}\right], \\
\dot{x}_2(t) &= x_2(t)\left[-r_2 + \frac{b_2 x_1(t)}{1 + x_1(t)} - \frac{b_3 x_3(t)}{1 + x_2(t)}\right], \\
\dot{x}_3(t) &= x_3(t)\left[-r_3 + \frac{b_4 x_2(t)}{1 + x_2(t)}\right], \quad t \neq nT, \\
\Delta x_1(nT) &= 0, \\
\Delta x_2(nT) &= h, \\
\Delta x_3(nT) &= 0, \quad n \in \{0, 1, \ldots\} = \mathbb{Z}^+,
\end{aligned} \tag{3}$$

where $h > 0$ is the release amount of top predator at $t = nT$ and T is the period of the impulsive effect. By using the Floquet theory of impulsive differential equation and small amplitude perturbation skills, we consider the local stability of prey and top predator eradication periodic solution.

In real world phenomenon, the environment varies due to the factors such as seasonal effects of weather, food supplies, mating habits, and harvesting. So, it is usual to assume the periodicity of parameters in system (2). However, if the various constituent components of the temporally nonuniform environment are with incommensurable (nonintegral multiples) periods, then one has to consider the environment to be almost periodic since the assumption of almost periodicity is more realistic, more important, and more general when we consider the effects of the environmental factors. In recent years, there have been many mathematical studies for the existence, uniqueness, and stability of positive almost periodic solution of biological models governed by differential equations in the literature (see [11, 15–25] and the references cited therein). Therefore, Bai and wang in [15] studied the following nonautonomous food chains system with Holling's type II functional response:

$$\begin{aligned}
\dot{x}_1(t) &= x_1(t)\left[r_1(t) - a_1(t)x_1(t) - \frac{b_1(t)x_2(t)}{m_1 + x_1(t)}\right], \\
\dot{x}_2(t) &= x_2(t)\left[-r_2(t) + \frac{b_2(t)x_1(t)}{m_1 + x_1(t)} - a_2(t)x_2(t)\right. \\
&\qquad \left. - \frac{b_3(t)x_3(t)}{m_2 + x_2(t)}\right], \\
\dot{x}_3(t) &= x_3(t)\left[-r_3(t) + \frac{b_4(t)x_2(t)}{m_2 + x_2(t)} - a_3(t)x_3(t)\right].
\end{aligned} \tag{4}$$

By applying the comparison theorem and the Lyapunov method of ordinary differential equations, some sufficient conditions which guarantee the permanence and existence of a unique uniformly asymptotically stable positive almost periodic solution of system (4) are obtained.

Stimulated by the above reason, this paper is concerned with the following almost periodic food chain system with almost periodic impulsive perturbations and general functional responses:

$$\begin{aligned}
\dot{x}_1(t) &= x_1(t)\left[r_1(t) - a_1(t)x_1(t) - \frac{b_1(t)x_2(t)}{m_1 + x_1^{\alpha_1}(t)}\right], \\
\dot{x}_2(t) &= x_2(t)\left[-r_2(t) + \frac{b_2(t)x_1(t)}{m_1 + x_1^{\alpha_1}(t)} - a_2(t)x_2(t)\right. \\
&\qquad \left. - \frac{b_3(t)x_3(t)}{m_2 + x_2^{\alpha_2}(t)}\right], \\
\dot{x}_3(t) &= x_3(t)\left[-r_3(t) + \frac{b_4(t)x_2(t)}{m_2 + x_2^{\alpha_2}(t)} - a_3(t)x_3(t)\right], \\
&\qquad t \neq \tau_k, \\
\Delta x_1(\tau_k) &= h_{1k}x_1(\tau_k), \\
\Delta x_2(\tau_k) &= h_{2k}x_2(\tau_k), \\
\Delta x_3(\tau_k) &= h_{3k}x_3(\tau_k), \quad k \in \mathbb{Z}^+,
\end{aligned} \tag{5}$$

where $r_i(t)$, $a_i(t)$, and $b_j(t)$, $i = 1, 2, 3$, $j = 1, 2, 3, 4$, are all continuous almost periodic functions which are bounded above and below by positive constants; $m_1, m_2, \alpha_1, \alpha_2$ are positive constants; $h_{1k}, h_{2k}, h_{3k} > -1$ are constants; $0 = \tau_0 < \tau_1 < \tau_2 < \cdots < \tau_k < \tau_{k+1} < \cdots$ are impulse points with $\lim_{k\to+\infty}\tau_k = +\infty$; and the set of sequences $\{\tau_k^j\}, \tau_k^j = \tau_{k+j} - \tau_k, k \in \mathbb{Z}^+, j \in \mathbb{Z}$, is uniformly almost periodic (see Definition 1 in Section 2).

Obviously, system (2)–(4) is special case of system (5).

The main purpose of this paper is to establish some new sufficient conditions which guarantee the permanence and existence of a unique uniformly asymptotically stable positive almost periodic solution of system (5) by using the comparison theorem and the Lyapunov method of the impulsive differential equations [10, 11] (see Theorems 11 and 14 in Sections 3 and 4).

The organization of this paper is as follows. In Section 2, we give some basic definitions and necessary lemmas which will be used in later sections. In Section 3, by using the comparison theorem of the impulsive differential equations [10], we give the permanence of system (5). In Section 4, we study the existence of a unique uniformly asymptotically stable positive almost periodic solution of system (5) by applying the Lyapunov method of the impulsive differential equations [11].

2. Preliminaries

Now, let us state the following definitions and lemmas, which will be useful in proving our main result.

By $\mathbb{I}$, $\mathbb{I} = \{\{\tau_k\} \in \mathbb{R} : \tau_k < \tau_{k+1}, k \in \mathbb{Z}, \lim_{k\to\pm\infty}\tau_k = \pm\infty\}$, we denote the set of all sequences that are unbounded and strictly increasing. Introduce the following notations.

For $J \subset \mathbb{R}^3$, $PC(J, \mathbb{R}^3)$ is the space of all piecewise continuous functions from J to $\mathbb{R}^3$ with points of discontinuity of the first kind τ_k, at which it is left continuous. By the basic theories of impulsive differential equations in [10, 11], system (5) has a unique solution $X(t) = X(t, X_0) \in PC([0, +\infty), \mathbb{R}^3)$.

Since the solution of system (5) is a piecewise continuous function with points of discontinuity of the first kind τ_k, $k \in \mathbb{Z}$, we adopt the following definitions for almost periodicity.

Definition 1 (see [11]). The set of sequences $\{\tau_k^j\}$, $\tau_k^j = \tau_{k+j} - \tau_k$, $k \in \mathbb{Z}$, $j \in \mathbb{Z}$, $\{\tau_k\} \in \mathbb{I}$, is said to be uniformly almost periodic if for arbitrary $\epsilon > 0$ there exists a relatively dense set of ϵ-almost periods common for any sequences.

Definition 2 (see [11]). The function $\varphi \in PC(\mathbb{R}, \mathbb{R})$ is said to be almost periodic, if the following hold.

(1) The set of sequences $\{\tau_k^j\}$, $\tau_k^j = \tau_{k+j} - \tau_k$, $k \in \mathbb{Z}$, $j \in \mathbb{Z}$, $\{\tau_k\} \in \mathbb{I}$, is uniformly almost periodic.

(2) For any $\epsilon > 0$, there exists a real number $\delta > 0$ such that if the points t' and t'' belong to one and the same interval of continuity of $\varphi(t)$ and satisfy the inequality $|t' - t''| < \delta$, then $|\varphi(t') - \varphi(t'')| < \epsilon$.

(3) For any $\epsilon > 0$, there exists a relatively dense set T such that if $\eta \in T$, then $|\varphi(t + \eta) - \varphi(t)| < \epsilon$ for all $t \in \mathbb{R}$ satisfying condition $|t - \tau_k| > \epsilon$, $k \in \mathbb{Z}$. The elements of T are called ϵ-almost periods.

Lemma 3 (see [11]). *Let $\{\tau_k\} \in \mathbb{I}$. Then, there exists a positive integer A such that, on each interval of length 1, one has no more than A elements of the sequence $\{\tau_k\}$; that is,*

$$i(s, t) \le A(t - s) + A, \tag{6}$$

where $i(s, t)$ is the number of the points τ_k in the interval (s, t).

Theoretically, one can investigate the existence, uniqueness, and stability of almost periodic solution for functional differential equations by using Lyapunov functional as follows [11, P_{109}].

Consider the system of impulsive differential equations as follows:

$$\begin{aligned} \dot{x}(t) &= f(t, x(t)), \quad t \ne \tau_k, \\ \Delta x(\tau_k) &= I_k x(\tau_k), \end{aligned} \tag{7}$$

where $t \in \mathbb{R}$, $\{\tau_k\} \in \mathbb{I}$, $f : \mathbb{R} \times D \to \mathbb{R}^n$, $I_k : D \to \mathbb{R}^n$, $k \in \mathbb{Z}$, and D is an open set in $\mathbb{R}^n$.

Introduce the following conditions.

(C_1) Function $f(t, x)$ is almost periodic in t uniformly with respect to $x \in D$.

(C_2) Sequence $\{I_k(x)\}$, $k \in \mathbb{Z}$, is almost periodic uniformly with respect to $x \in D$.

Lemma 4 (see [11, P_{109}]). *Suppose that there exists a Lyapunov functional $V(t, x, y)$ defined on $\mathbb{R}^+ \times D \times D$ satisfying the following conditions.*

(1) *$u(\|x - y\|) \le V(t, x, y) \le v(\|x - y\|)$, where $u, v \in \mathscr{P}$ with $\mathscr{P} = \{u : \mathbb{R}^+ \to \mathbb{R}^+ \mid u$ is continuous increasing function and $u(s) \to 0$ as $s \to 0\}$.*

(2) *$|V(t, \overline{x}, \overline{y}) - V(t, \widehat{x}, \widehat{y})| \le K(\|\overline{x} - \widehat{x}\| + \|\overline{y} - \widehat{y}\|)$, where $K > 0$ is a constant.*

(3) *For $t = \tau_k$, $V(t^+, x + I_k(x), y + I_k(y)) \le V(t, x, y)$; for $t \ne \tau_k$, $\dot{V}_{(2.2)}(t, x, y) \le -\gamma V(t, x, y)$, $\forall k \in \mathbb{Z}$, where $\gamma > 0$ is a constant.*

Moreover, one assumes that system (7) has a solution that remains in a compact set $S \subset D$. Then, system (7) has a unique almost periodic solution which is uniformly asymptotically stable.

3. Permanence

In this section, we establish a permanence result for system (5).

Lemma 5 (see [10]). *Assume that $x \in PC(\mathbb{R})$ with points of discontinuity at $t = \tau_k$ and is left continuous at $t = \tau_k$ for $k \in \mathbb{Z}^+$ and*

$$\begin{aligned} \dot{x}(t) &\le f(t, x(t)), \quad t \ne \tau_k, \\ x(\tau_k^+) &\le I_k(x(\tau_k)), \quad k \in \mathbb{Z}^+, \end{aligned} \tag{8}$$

where $f \in C(\mathbb{R} \times \mathbb{R}, \mathbb{R})$, $I_k \in C(\mathbb{R}, \mathbb{R})$, and $I_k(x)$ is nondecreasing in x for $k \in \mathbb{Z}^+$. Let $u^*(t)$ be the maximal solution of the scalar impulsive differential equation as

$$\begin{aligned} \dot{u}(t) &= f(t, u(t)), \quad t \ne \tau_k, \\ u(\tau_k^+) &= I_k(u(\tau_k)) \ge 0, \quad k \in \mathbb{Z}^+, \\ u(t_0^+) &= u_0 \end{aligned} \tag{9}$$

existing on $[t_0, \infty)$. Then, $x(t_0^+) \le u_0$ implies $x(t) \le u^*(t)$ for $t \ge t_0$.

Remark 6. If inequalities (8) in Lemma 5 are reversed and $u_*(t)$ is the minimal solution of system (9) existing on $[t_0, \infty)$, then $x(t_0^+) \ge u_0$ implies $x(t) \ge u_*(t)$ for $t \ge t_0$.

Lemma 7. *Assume that $a, b > 0$; then, the following impulsive logistic equation*

$$\begin{aligned} \dot{x}(t) &= x(t)[a - bx(t)], \quad t \ne \tau_k, \\ \Delta x(\tau_k^+) &= h_k x(\tau_k), \quad k \in \mathbb{Z}^+ \end{aligned} \tag{10}$$

has a unique globally asymptotically stable positive almost periodic solution x^* which can be expressed as follows:

$$\frac{\alpha}{e^{\xi A}b} \le x^*(t) = \left[b\int_{-\infty}^{t} W(t,s)\,ds\right]^{-1} \le \frac{a}{\eta b\left(1-e^{-a\theta}\right)}, \tag{11}$$

where A is defined as that in Lemma 3, $\xi := \ln\sup_{k\in\mathbb{Z}}(1/(1+h_k))$, $\alpha := a-\xi A$, $\theta := \inf_{k\in\mathbb{Z}}\tau_k^1$, $\eta := \inf_{k\in\mathbb{Z}}\prod_{j=1}^{2}(1/(1+h_{j+k}))$, and

$$W(t,s) = \begin{cases} e^{-a(t-s)}, & \tau_{k-1} < s < t < \tau_k; \\ \displaystyle\prod_{j=m}^{k+1}\frac{1}{1+h_j}e^{-a(t-s)}, & \tau_{m-1} < s \le \tau_m \\ & < \tau_k < t \le \tau_{k+1}. \end{cases} \tag{12}$$

Proof. Let $u = 1/x$; then, system (10) changes to

$$\begin{aligned} \frac{du(t)}{dt} &= -au(t) + b, \quad t \ne \tau_k, \\ \Delta u(\tau_k^+) &= -\frac{h_k}{1+h_k}u(\tau_k), \quad k\in\mathbb{Z}^+. \end{aligned} \tag{13}$$

We can easily obtain that system (13) has a unique almost periodic solution which can be expressed as follows:

$$u^*(t) = b\int_{-\infty}^{t} W(t,s)\,ds. \tag{14}$$

Then, system (10) has a unique almost periodic solution x^* which can be expressed by (11). By Lemma 3, we have

$$x^*(t) \ge \left[b\int_{-\infty}^{t} e^{\xi A}e^{-\alpha(t-s)}\,ds\right]^{-1} = \frac{\alpha}{e^{\xi A}b}. \tag{15}$$

On the other hand,

$$x^*(t) \le \left[b\int_{t-\theta}^{t} \eta e^{-a(t-s)}\,ds\right]^{-1} = \frac{a}{\eta b\left(1-e^{-a\theta}\right)}. \tag{16}$$

Suppose that $x(t)$ is another positive solution of system (10). Define a Lyapunov function as

$$V(t) = \left|\ln x^*(t) - \ln x(t)\right|, \quad \forall t\in\mathbb{R}. \tag{17}$$

For $t \ne \tau_k$, $k\in\mathbb{Z}^+$, calculating the upper right derivative of $V(t)$ along the solution of system (10), we have

$$D^+V(t) = -b\left|x^*(t) - x(t)\right|. \tag{18}$$

For $t = \tau_k$, $k\in\mathbb{Z}^+$, we have

$$\begin{aligned} V(\tau_k^+) &= \left|\ln x^*(\tau_k^+) - \ln x(\tau_k^+)\right| \\ &= \left|\ln\frac{(1+h_k)x^*(\tau_k)}{(1+h_k)x(\tau_k)}\right| \\ &= \left|\ln x^*(\tau_k) - \ln x(\tau_k)\right| = V(\tau_k). \end{aligned} \tag{19}$$

Therefore, V is nonincreasing. Integrating (18) from 0 to t leads to

$$V(t) + b\int_0^t \left|x(s) - x^*(s)\right| ds \le V(0) < +\infty, \quad \forall t\ge 0; \tag{20}$$

that is,

$$\int_0^{+\infty}\left|x(s)-x^*(s)\right| ds < +\infty, \tag{21}$$

which implies that

$$\lim_{s\to+\infty}\left|x(s)-x^*(s)\right| = 0. \tag{22}$$

Thus, the almost periodic solution of system (10) is globally asymptotically stable. This completes the proof. □

Let

$$\eta_i := \inf_{k\in\mathbb{Z}}\prod_{j=1}^{2}\frac{1}{1+h_{i(j+k)}}, \qquad \xi_i := \ln\sup_{k\in\mathbb{Z}}\frac{1}{1+h_{ik}}, \qquad i = 1,2,3. \tag{23}$$

Proposition 8. *Every solution $(x_1,x_2,x_2)^T$ of system* (5) *satisfies*

$$\limsup_{t\to\infty} x_i(t) \le M_i, \quad i = 1,2,3, \tag{24}$$

where M_1, M_2, and M_3 are defined as those in (27), (32), *and* (35), *respectively.*

Proof. From the first equation of system (5), we have

$$\begin{aligned} \dot{x}_1(t) &\le x_1(t)\left[r_1^u - a_1^l x_1(t)\right], \quad t\ne\tau_k, \\ x_1(\tau_k^+) &= (1+h_{1k})x_1(\tau_k), \quad k\in\mathbb{Z}^+. \end{aligned} \tag{25}$$

Consider the following auxiliary system:

$$\begin{aligned} \dot{z}_1(t) &= z_1(t)\left[r_1^u - a_1^l z_1(t)\right], \quad t\ne\tau_k, \\ z_1(\tau_k^+) &= (1+h_{1k})z_1(\tau_k), \quad k\in\mathbb{Z}^+. \end{aligned} \tag{26}$$

By Lemma 5, $x_1(t) \le z_1(t)$, where $z_1(t)$ is the solution of system (26) with $z_1(0^+) = x_1(0^+)$. By Lemma 7, system (26) has a unique globally asymptotically stable positive almost periodic solution z_1^* which can be expressed as follows:

$$\begin{aligned} z_1^*(t) &= \left[a_1^l\int_{-\infty}^{t} W_1(t,s)\,ds\right]^{-1} \le \left[a_1^l\int_{t-\theta}^{t} W_1(t,s)\,ds\right]^{-1} \\ &\le \frac{r_1^u}{\eta_1 a_1^l\left(1-e^{-r_1^u\theta}\right)} := M_1, \end{aligned} \tag{27}$$

where

$$W_1(t,s) = \begin{cases} e^{-r_1^u(t-s)}, & \tau_{k-1} < s < t < \tau_k; \\ \prod_{j=m}^{k+1} \frac{1}{1+h_{1j}} e^{-r_1^u(t-s)}, & \tau_{m-1} < s \le \tau_m \\ & < \tau_k < t \le \tau_{k+1}. \end{cases} \tag{28}$$

Then, for any constant $\epsilon > 0$, there exists $T_1 > 0$ such that $x_1(t) \le z_1(t) < z_1^*(t) + \epsilon \le M_1 + \epsilon$ for $t > T_1$. So,

$$\limsup_{t\to\infty} x_1(t) \le M_1. \tag{29}$$

For any $\epsilon > 0$, there exists $T_2 > 0$ such that

$$x_1(t) \le M_1 + \epsilon \quad \text{for } t \ge T_2. \tag{30}$$

From the second equation of system (5), we have

$$\begin{aligned} \dot{x}_2(t) &\le x_2(t)\left[-r_2^l + \frac{b_2^u(M_1+\epsilon)}{m_1} - a_2^l x_2(t)\right], \\ &\qquad t \ne \tau_k,\ t > T_2, \\ x_2(\tau_k^+) &= (1+h_{2k})\, x_2(\tau_k), \quad k \in \mathbb{Z}^+. \end{aligned} \tag{31}$$

Similar to the above argument as that in (29), one has

$$\limsup_{t\to\infty} x_2(t) \le \frac{m_1^{-1} b_2^u M_1 - r_2^l}{\eta_2 a_2^l\left[1 - e^{(r_2^l - m_1^{-1} b_2^u M_1)\theta}\right]} := M_2. \tag{32}$$

Then, there exists $T_3 > T_2$ such that

$$x_2(t) \le M_2 + \epsilon \quad \text{for } t \ge T_3. \tag{33}$$

By the third equation of system (5), we have

$$\begin{aligned} \dot{x}_3(t) &\le x_3(t)\left[-r_3^l + \frac{b_4^u(M_2+\epsilon)}{m_2} - a_3^l x_3(t)\right], \\ &\qquad t \ne \tau_k, \quad t > T_3, \\ x_3(\tau_k^+) &= (1+h_{3k})\, x_3(\tau_k), \quad k \in \mathbb{Z}^+. \end{aligned} \tag{34}$$

Similar to the above argument as that in (32), we have

$$\limsup_{t\to\infty} x_3(t) \le \frac{m_2^{-1} b_4^u M_2 - r_3^l}{\eta_3 a_3^l\left[1 - e^{(r_3^l - m_2^{-1} b_4^u M_2)\theta}\right]} := M_3. \tag{35}$$

This completes the proof. □

Proposition 9. *Let N_1, N_2, and N_3 be defined as those in* (42)–(48), *respectively. Then, every solution $(x_1, x_2, x_3)^T$ of system* (5) *satisfies*

$$\liminf_{t\to\infty} x_i(t) \ge N_i, \quad i = 1,2,3, \tag{36}$$

if the following condition holds:

(H_1) $r_1^l > m_1^{-1} b_1^u M_2 + \xi_1 A$, $b_2^l N_1/(m_1 + M_1^{\alpha_1}) > r_2^u + (b_3^u M_3/m_2) + \xi_2 A$, *and* $b_4^l N_2/(m_2 + M_2^{\alpha_2}) > r_3^u + \xi_3 A$.

Proof. According to Proposition 8, there exist $\epsilon > 0$ and $T_4 > 0$ such that

$$\begin{gathered} r_1^l - m_1^{-1} b_1^u (M_2 + \epsilon) - \xi_1 A \ge 0, \\ x_i(t) \le M_i + \epsilon \quad \text{for } t \ge T_4, \quad i = 1,2,3. \end{gathered} \tag{37}$$

From the first equation of system (5), we have

$$\begin{aligned} \dot{x}_1(t) &\ge x_1(t)\left[r_1^l - a_1^u x_1(t) - \frac{b_1^u(M_2+\epsilon)}{m_1}\right], \\ &\qquad t \ne \tau_k, \quad t > T_4, \\ x_1(\tau_k^+) &= (1+h_{1k})\, x_1(\tau_k), \quad k \in \mathbb{Z}^+. \end{aligned} \tag{38}$$

Consider the following auxiliary system:

$$\begin{aligned} \dot{p}_1(t) &= p_1(t)\left[r_1^l - m_1^{-1} b_1^u (M_2+\epsilon) - a_1^u x_1(t)\right], \\ &\qquad t \ne \tau_k, \quad t > T_4, \\ p_1(\tau_k^+) &= (1+h_{1k})\, p_1(\tau_k), \quad k \in \mathbb{Z}^+. \end{aligned} \tag{39}$$

By Remark 15, $x_1(t) \ge p_1(t)$ for $t > T_4$, where $p_1(t)$ is the solution of system (39) with $p_1(T_4^+) = x_1(T_4^+)$. By Lemma 7, system (39) has a unique globally asymptotically stable positive almost periodic solution p_1^* which can be expressed as follows:

$$\begin{aligned} p_1^*(t) &= \left[a_1^u \int_{-\infty}^{t} W_2(t,s)\, ds\right]^{-1} \\ &\ge \frac{r_1^l - m_1^{-1} b_1^u (M_2+\epsilon) - \xi_1 A}{e^{\xi_1 A} a_1^u} := N_1(\epsilon), \end{aligned} \tag{40}$$

$$W_2(t,s) = \begin{cases} e^{-[r_1^l - m_1^{-1} b_1^u (M_2+\epsilon) - \xi_1 A](t-s)}, & \tau_{k-1} < s < t < \tau_k; \\ \prod_{j=m}^{k+1} \frac{1}{1+h_{1j}} \\ \quad \times e^{-[r_1^l - m_1^{-1} b_1^u (M_2+\epsilon) - \xi_1 A](t-s)}, & \tau_{m-1} < s \le \tau_m \\ & < \tau_k < t \le \tau_{k+1}. \end{cases} \tag{41}$$

Similar to the above argument as that in (29), we have $\liminf_{t\to\infty} x_1(t) \ge N_1(\epsilon)$. By the arbitrariness of ϵ, it leads to

$$\liminf_{t\to\infty} x_1(t) \ge N_1 := \frac{r_1^l - m_1^{-1} b_1^u M_2 - \xi_1 A}{e^{\xi_1 A} a_1^u}. \tag{42}$$

Then, there exist $0 < \epsilon_1 \le \epsilon$ and $T_5 > T_4$ such that

$$\begin{gathered} \frac{b_2^l(N_1 - \epsilon_1)}{m_1 + (M_1+\epsilon_1)^{\alpha_1}} - r_2^u - \frac{b_3^u(M_3+\epsilon_1)}{m_2} - \xi_2 A \ge 0, \\ x_1(t) \ge N_1 - \epsilon_1 \quad \text{for } t \ge T_5. \end{gathered} \tag{43}$$

By the second equation of system (5), we have

$$\begin{aligned}
&\dot{x}_2(t)\\
&\geq x_2(t)\left[\frac{b_2^l(N_1-\epsilon_1)}{m_1+(M_1+\epsilon_1)^{\alpha_1}}-r_2^u-\frac{b_3^u(M_3+\epsilon_1)}{m_2}\right.\\
&\qquad\left.-a_2^l x_2(t)\right]\quad t\neq\tau_k,\quad t>T_2,\\
&x_2(\tau_k^+)=(1+h_{2k})x_2(\tau_k),\quad k\in\mathbb{Z}^+.
\end{aligned}\tag{44}$$

Similar to the above argument as that in (42), one has

$$\begin{aligned}
&\lim_{t\to\infty}\inf x_2(t)\\
&\geq\frac{\left[\left(b_2^l N_1/\left(m_1+M_1^{\alpha_1}\right)\right)-r_2^u-\left(b_3^u M_3/m_2\right)-\xi_2 A\right]}{e^{\xi_2 A}a_2^u}\\
&:=N_2.
\end{aligned}\tag{45}$$

Then, there exist $0<\epsilon_2\leq\epsilon_1$ and $T_6>T_5$ such that

$$\begin{gathered}
\frac{b_4^l(N_2-\epsilon_2)}{m_2+(M_2+\epsilon_2)^{\alpha_2}}-r_3^u-\xi_3 A\geq 0,\\
x_2(t)\geq N_2-\epsilon_2\quad\text{for } t\geq T_6.
\end{gathered}\tag{46}$$

In view of the third equation of system (5), we have

$$\begin{aligned}
&\dot{x}_3(t)\geq x_3(t)\left[\frac{b_4^l(N_2-\epsilon_2)}{m_2+(M_2+\epsilon_2)^{\alpha_2}}-r_3^u-a_3^u x_3(t)\right],\\
&\qquad\qquad t\neq\tau_k,\quad t>T_2,\\
&x_3(\tau_k^+)=(1+h_{3k})x_3(\tau_k),\quad k\in\mathbb{Z}^+.
\end{aligned}\tag{47}$$

Similar to the above argument as that in (45), one has

$$\lim_{t\to\infty}\inf x_3(t)\geq\frac{\left[\left(b_4^l N_2/\left(m_2+M_2^{\alpha_2}\right)\right)-r_3^u-\xi_3 A\right]}{e^{\xi_3 A}a_3^u}:=N_3.\tag{48}$$

This completes the proof. □

Remark 10. In view of (H_1) in Proposition 9, the values of impulse coefficients h_{ik} $(i=1,2,3)$ and the number of the impulse points τ_k in each interval of length 1 have negative effect on the permanence of system (5).

By Propositions 8 and 9, we have the following theorem.

Theorem 11. *Assume that (H_1) holds; then, system (5) is permanent.*

Remark 12. When h_{ik} $(i=1,2,3)\equiv 0$ in system (5), then Theorem 11 changes to the corresponding permanence result in Bai and Wang [15]. So, Theorem 11 extends the corresponding result in Bai and Wang [15]. Further, Theorem 11 gives the sufficient conditions for the permanence of system (5) with almost periodic impulsive perturbations. Therefore, Theorem 11 provides a possible method to study the permanence of the models with impulsive perturbations in biological populations.

Remark 13. From the proof of Propositions 8 and 9, we know that, under the conditions of Theorem 11, set $S=\{(x_1,x_2,x_3)^T\in\mathbb{R}^3: N_i\leq x_i\leq M_i, i=1,2,3\}$ is an invariant set of system (5).

4. Almost Periodic Solution

The main result of this paper is concerned with the existence of a unique uniformly asymptotically stable positive almost periodic solution for system (5).

Let

$$\begin{gathered}
c_1(t):=\max_{N_1\leq x\leq M_1}\frac{\alpha_1 x^{\alpha_1-1}M_2 b_1(t)}{(m_1+x^{\alpha_1})^2},\\
c_2(t):=\max_{N_1\leq x\leq M_1}\frac{\left|m_1+(1-\alpha_1)x^{\alpha_1}\right|b_2(t)}{(m_1+x^{\alpha_1})^2},\\
d_1(t):=\frac{b_1(t)}{m_1+N_1^{\alpha_1}},\\
d_2(t):=\max_{N_2\leq x\leq M_2}\frac{\alpha_2 x^{\alpha_2-1}M_3 b_3(t)}{(m_2+x^{\alpha_2})^2},\\
d_3(t):=\max_{N_2\leq x\leq M_2}\frac{\left|m_2+(1-\alpha_2)x^{\alpha_2}\right|b_4(t)}{(m_2+x^{\alpha_2})^2},\\
e(t):=\frac{b_3(t)}{m_2+N_2^{\alpha_2}},\quad\forall t\in\mathbb{R}.
\end{gathered}\tag{49}$$

Theorem 14. *Assume that (H_1) holds; suppose further that*

(H_2) *there exist positive constants λ_1, λ_2, λ_3, and μ such that*

$$\begin{gathered}
\inf_{t\in\mathbb{R}}\left[\lambda_1 a_1(t)-\lambda_1 c_1(t)-\lambda_2 c_2(t)\right]>\mu,\\
\inf_{t\in\mathbb{R}}\left[\lambda_3 a_3(t)-\lambda_2 e_1(t)\right]>\mu,\\
\inf_{t\in\mathbb{R}}\left[\lambda_2 a_2(t)-\lambda_1 d_1(t)-\lambda_2 d_2(t)-\lambda_3 d_3(t)\right]>\mu;
\end{gathered}\tag{50}$$

then, system (5) admits a unique positive almost periodic solution, which is uniformly asymptotically stable.

Proof. Suppose that $Z(t) = (z_1(t), z_2(t), z_3(t))^T$ and $Z^*(t) = (z_1^*(t), z_2^*(t), z_3^*(t))^T$ are any two solutions of system (5). Consider the product system of system (5) as

$$\begin{aligned}
\dot{z}_1(t) &= z_1(t)\left[r_1(t) - a_1(t)z_1(t) - \frac{b_1(t)z_2(t)}{m_1 + z_1^{\alpha_1}(t)}\right],\\
\dot{z}_2(t) &= z_2(t)\left[-r_2(t) + \frac{b_2(t)z_1(t)}{m_1 + z_1^{\alpha_1}(t)}\right.\\
&\qquad\left. -a_2(t)z_2(t) - \frac{b_3(t)z_3(t)}{m_2 + z_2^{\alpha_2}(t)}\right],\\
\dot{z}_3(t) &= z_3(t)\left[-r_3(t) + \frac{b_4(t)z_2(t)}{m_2 + z_2^{\alpha_2}(t)} - a_3(t)z_3(t)\right],\\
\dot{z}_1^*(t) &= z_1^*(t)\left[r_1(t) - a_1(t)z_1^*(t) - \frac{b_1(t)z_2^*(t)}{m_1 + z_1^{*\alpha_1}(t)}\right],\\
\dot{z}_2^*(t) &= z_2^*(t)\left[-r_2(t) + \frac{b_2(t)z_1^*(t)}{m_1 + z_1^{*\alpha_1}(t)} - a_2(t)z_2^*(t)\right.\\
&\qquad\left. - \frac{b_3(t)z_3^*(t)}{m_2 + z_2^{*\alpha_2}(t)}\right],\\
\dot{z}_3^*(t) &= z_3^*(t)\left[-r_3(t) + \frac{b_4(t)z_2^*(t)}{m_2 + z_2^{*\alpha_2}(t)} - a_3(t)z_3^*(t)\right],\\
&\qquad t \neq \tau_k,\\
\Delta z_1(\tau_k) &= h_{1k}z_1(\tau_k),\\
\Delta z_2(\tau_k) &= h_{2k}z_2(\tau_k),\\
\Delta z_3(\tau_k) &= h_{3k}z_3(\tau_k),\\
\Delta z_1^*(\tau_k) &= h_{1k}z_1^*(\tau_k),\\
\Delta z_2^*(\tau_k) &= h_{2k}z_2^*(\tau_k),\\
\Delta z_3^*(\tau_k) &= h_{3k}z_3^*(\tau_k), \quad k \in \mathbb{Z}.
\end{aligned} \tag{51}$$

Set $S_1 = \{(z_1, z_2, z_3)^T \in \mathbb{R}^3 : N_i \le z_i \le M_i, i = 1, 2, 3\}$, which is an invariant set of system (51) directly from Remark 13.

Construct a Lyapunov functional $V(t, Z, Z^*) = V(t, (z_1, z_2, z_3)^T, (z_1^*, z_2^*, z_3^*)^T)$ defined on $\mathbb{R}^+ \times S_1 \times S_1 \times S_1$ as follows:

$$V(t, Z, Z^*) = \sum_{i=1}^{3} \lambda_i \left|\ln z_i(t) - \ln z_i^*(t)\right|. \tag{52}$$

It is obvious that

$$\begin{aligned}
&V(t, Z, Z^*)\\
&\ge \min\{\lambda_1, \lambda_2, \lambda_3\} \sum_{i=1}^{3} \left|\ln z_i(t) - \ln z_i^*(t)\right|\\
&\ge \min\{\lambda_1, \lambda_2, \lambda_3\} \sum_{i=1}^{3} \frac{1}{M_i} \left|z_i(t) - z_i^*(t)\right| \ge \underline{\lambda}\left\|Z - Z^*\right\|,
\end{aligned} \tag{53}$$

where $\underline{\lambda} := \min\{\lambda_1, \lambda_2, \lambda_3\}\min\{M_1^{-1}, M_2^{-1}, M_3^{-1}\}$. Further, we have

$$\begin{aligned}
&V(t, Z, Z^*)\\
&\le \max\{\lambda_1, \lambda_2, \lambda_3\} \sum_{i=1}^{3} \left|\ln z_i(t) - \ln z_i^*(t)\right|\\
&\le \max\{\lambda_1, \lambda_2, \lambda_3\} \sum_{i=1}^{3} \frac{1}{N_i} \left|z_i(t) - z_i^*(t)\right| \le \overline{\lambda}\left\|Z - Z^*\right\|,
\end{aligned} \tag{54}$$

where $\overline{\lambda} := \max\{\lambda_1, \lambda_2, \lambda_3\}\max\{N_1^{-1}, N_2^{-1}, N_3^{-1}\}$; thus, (1) in Lemma 4 is satisfied.

Since

$$\begin{aligned}
&\left|V(t, Z, Z^*) - V\left(t, \overline{Z}, \overline{Z}^*\right)\right|\\
&= \sum_{i=1}^{3} \lambda_i \left|\ln z_i(t) - \ln z_i^*(t)\right|\\
&\quad - \sum_{i=1}^{3} \lambda_i \left|\ln \overline{z}_i(t) - \ln \overline{z}_i^*(t)\right|\\
&\le \overline{\lambda} \sum_{i=1}^{3} \left[\left|z_i(t) - z_i^*(t)\right| + \left|\overline{z}_i(t) - \overline{z}_i^*(t)\right|\right]\\
&= \overline{\lambda}\left[\left|Z(t) - Z^*(t)\right| + \left|\overline{Z}(t) - \overline{Z}^*(t)\right|\right],
\end{aligned} \tag{55}$$

(2) in Lemma 4 holds.

For $t \neq \tau_k, k \in \mathbb{Z}^+$, calculating the upper right derivative of $V(t)$ along the solution of system (51), we have

$$\begin{aligned}
D^+V(t) &= \sum_{i=1}^{3} \lambda_i \left[\frac{\dot{z}_i(t)}{z_i(t)} - \frac{\dot{z}_i^*(t)}{z_i^*(t)}\right] \operatorname{sgn}\left(z_i(t) - z_i^*(t)\right)\\
&= \lambda_1 \operatorname{sgn}\left(z_1(t)\right.\\
&\qquad \left. - z_1^*(t)\right)\left\{-a_1(t)\left[z_1(t) - z_1^*(t)\right]\right.\\
&\qquad \left. - \frac{b_1(t)z_2(t)}{m_1 + z_1^{\alpha_1}(t)} + \frac{b_1(t)z_2^*(t)}{m_1 + z_1^{*\alpha_1}(t)}\right\}\\
&\quad + \lambda_2 \operatorname{sgn}\left(z_2(t) - z_2^*(t)\right)\\
&\qquad \times \left\{-a_2(t)\left[z_2(t) - z_2^*(t)\right]\right.\\
&\qquad + \frac{b_2(t)z_1(t)}{m_1 + z_1^{\alpha_1}(t)} - \frac{b_2(t)z_1^*(t)}{m_1 + z_1^{*\alpha_1}(t)}\\
&\qquad \left. - \frac{b_3(t)z_3(t)}{m_2 + z_2^{\alpha_2}(t)} + \frac{b_3(t)z_3^*(t)}{m_2 + z_2^{*\alpha_2}(t)}\right\}\\
&\quad + \lambda_3 \operatorname{sgn}\left(z_3(t) - z_3^*(t)\right)
\end{aligned}$$

$$
\begin{aligned}
&\times \left\{-a_3(t)\left[z_3(t)-z_3^*(t)\right]\right. \\
&\qquad \left. + \frac{b_4(t) z_2(t)}{m_2+z_2^{\alpha_2}(t)} - \frac{b_4(t) z_2^*(t)}{m_2+z_2^{*\alpha_2}(t)}\right\} \\
&\leq -\left[\lambda_1 a_1(t)-\lambda_1 c_1(t)-\lambda_2 c_2(t)\right]\left|z_1(t)-z_1^*(t)\right| \\
&\quad -\left[\lambda_2 a_2(t)-\lambda_1 d_1(t)-\lambda_2 d_2(t)-\lambda_3 d_3(t)\right] \\
&\quad \times\left|z_2(t)-z_2^*(t)\right| \\
&\quad -\left[\lambda_3 a_3(t)-\lambda_2 e_1(t)\right]\left|z_3(t)-z_3^*(t)\right| \\
&\leq -\sum_{i=1}^{3} \frac{\mu}{\lambda_i N_i} \lambda_i\left|\ln z_i(t)-\ln z_i^*(t)\right| \\
&\leq -\min\left\{\frac{\mu}{\lambda_1 N_1}, \frac{\mu}{\lambda_2 N_2}, \frac{\mu}{\lambda_3 N_3}\right\} V(t, Z, Z^*).
\end{aligned} \tag{56}
$$

For $t=\tau_k$, $k \in \mathbb{Z}^+$, we have

$$
\begin{aligned}
V\left(\tau_k^+, Z\left(\tau_k^+\right), Z^*\left(\tau_k^+\right)\right) &= \sum_{i=1}^{3} \lambda_i\left|\ln z_i\left(\tau_k^+\right)-\ln z_i^*\left(\tau_k^+\right)\right| \\
&= \sum_{i=1}^{3} \lambda_i\left|\ln \frac{\left(1+h_{ik}\right) z_i\left(\tau_k\right)}{\left(1+h_{ik}\right) z_i^*\left(\tau_k\right)}\right| \\
&= \sum_{i=1}^{3} \lambda_i\left|\ln z_i\left(\tau_k\right)-\ln z_i^*\left(\tau_k\right)\right| \\
&= V\left(\tau_k, Z\left(\tau_k\right), Z^*\left(\tau_k\right)\right).
\end{aligned} \tag{57}
$$

In view of (56)-(57), (3) in Lemma 4 is satisfied.

By Lemma 4, system (5) admits a unique uniformly asymptotically stable positive almost periodic solution $(z_1(t), z_2(t), z_3(t))^T$. This completes the proof. □

Remark 15. When h_{ik} $(i=1,2,3) \equiv 0$ in system (5), then Theorem 14 changes to the corresponding permanence result in Bai and Wang [15]. So, Theorem 14 extends the corresponding result in Bai and Wang [15]. Further, Theorem 14 gives the sufficient conditions for the uniform asymptotical stability of a unique positive almost periodic solution of system (5), in which α_1 and α_2 are allowed to be any real-valued positive number. Therefore, Theorem 14 provides a possible method to study the existence, uniqueness, and stability of positive almost periodic solution of the models with impulsive perturbations in biological populations.

5. An Example and Numerical Simulations

Example 1. Consider the following food chain system with impulsive perturbations:

$$
\begin{aligned}
\dot{x}_1(t) &= x_1(t)\left[2+\cos\left(\sqrt{2}t\right)-10 x_1(t) - \frac{0.1\cos\left(\sqrt{3}t\right) x_2(t)}{1+x_1(t)}\right], \\
\dot{x}_2(t) &= x_2(t)\left[-0.1\sin\left(\sqrt{2}t\right) + \frac{\left(5+\cos\left(\sqrt{2}t\right)\right) x_1(t)}{m_1+x_1(t)} - 10 x_2(t) - \frac{0.1 x_3(t)}{1+x_2(t)}\right], \\
\dot{x}_3(t) &= x_3(t)\left[-0.1\cos\left(\sqrt{5}t\right) + \frac{\left(6+\sin\left(\sqrt{3}t\right)\right) x_2(t)}{1+x_2(t)} - 10 x_3(t)\right], \\
&\qquad t \neq \tau_k, \\
\Delta x_1\left(\tau_k\right) &= 0.1 x_1\left(\tau_k\right), \\
\Delta x_2\left(\tau_k\right) &= 0.2 x_2\left(\tau_k\right), \\
\Delta x_3\left(\tau_k\right) &= 0.3 x_3\left(\tau_k\right), \quad k \in \{0,1,\ldots\} = \mathbb{Z}^+, \\
\theta &= \inf_{k\in\mathbb{Z}} \tau_k^1 = 1;
\end{aligned} \tag{58}
$$

then, system (58) is permanent and admits a unique uniformly asymptotically stable positive almost periodic solution.

Proof. Corresponding to system (2), $r_1(t) = 2+\cos(\sqrt{2}t)$, $r_2(t) = 0.1\sin(\sqrt{2}t)$, $r_3(t) = 0.1\cos(\sqrt{5}t)$, $a_1 = a_2 = a_3 \equiv 10$, $b_1(t) = 0.1\cos(\sqrt{3}t)$, $b_2(t) = 5+\cos(\sqrt{2}t)$, $b_3(t) = 0.1$, $b_4(t) = 6+\sin(\sqrt{3}t)$, $m_1 = m_2 = \alpha_1 = \alpha_2 = 1$, $h_{1k} \equiv 0.1$, $h_{2k} \equiv 0.2$, $h_{3k} \equiv 0.3$, $k \in \mathbb{Z}^+$. Taking $\lambda_1 = \lambda_2 = \lambda_3 = 1$, the result is easy to obtain from Theorems 11 and 14; we would omit it (see Figures 1, 2, 3, 4, 5, and 6). This completes the proof. □

6. Conclusion

By using the comparison theorem and the Lyapunov method of the impulsive differential equations, sufficient conditions

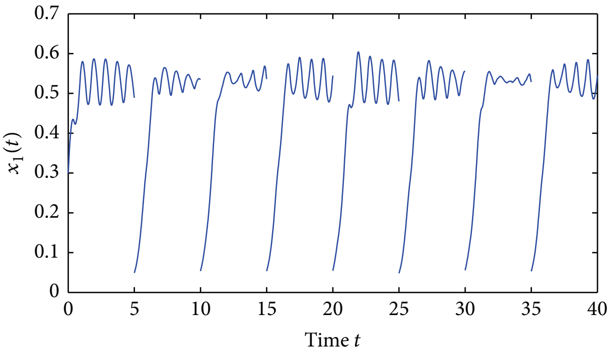

FIGURE 1: State variable x_1 of Example 1.

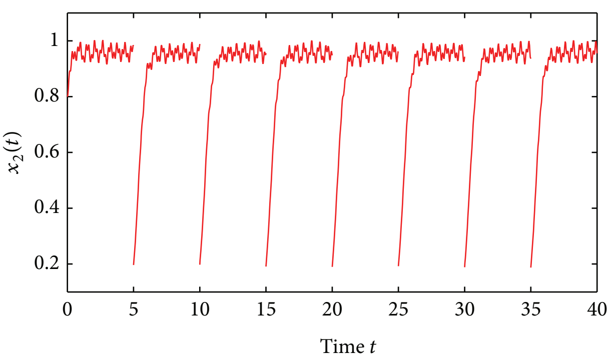

FIGURE 2: State variable x_2 of Example 1.

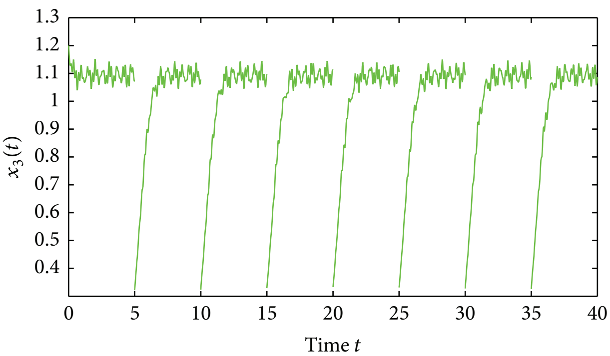

FIGURE 3: State variable x_3 of Example 1.

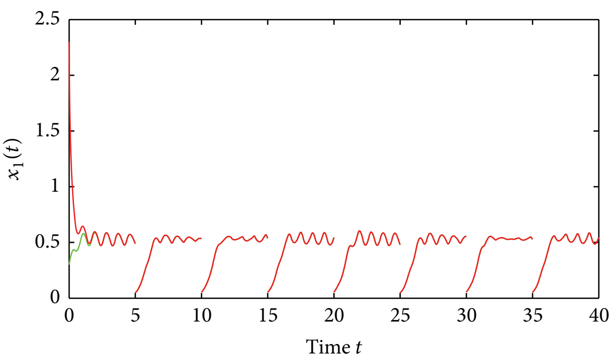

FIGURE 4: Stability of state variable x_1 of Example 1.

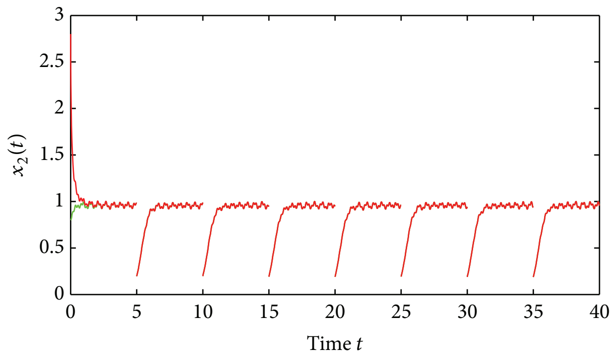

FIGURE 5: Stability of state variable x_2 of Example 1.

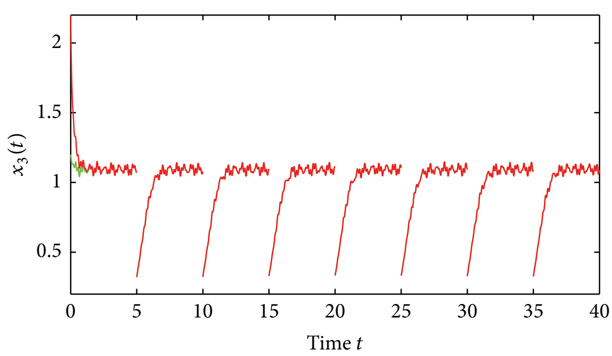

FIGURE 6: Stability of state variable x_3 of Example 1.

are obtained which guarantee the permanence and existence of a unique uniformly asymptotically stable positive almost periodic solution of a food chain system with almost periodic impulsive perturbations. Proposition 9 and Theorem 14 imply that the values of impulse coefficients h_{ik} $(i = 1, 2, 3)$ and the number of the impulse points τ_k in each interval of length 1 are harm for the permanence and existence of a unique uniformly asymptotically stable positive almost periodic solution of the model. The main results obtained in this paper are completely new and the method used in this paper provides a possible method to study the permanence and existence of a unique uniformly asymptotically stable positive almost periodic solution of the models with impulsive perturbations in biological populations.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

Acknowledgments

The authors would like to thank the reviewers for their valuable comments and constructive suggestions, which considerably improve the presentation of this paper. This work was supported by the Scientific Research Fund of Yunnan Provincial Education Department.

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