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Erratum

Erratum to "Seminormal Structure and Fixed Points of Cyclic Relatively Nonexpansive Mappings"

Moosa Gabeleh[1] and Naseer Shahzad[2]

[1] *Department of Mathematics, Ayatollah Boroujerdi University, Boroujerd, Iran*
[2] *Department of Mathematics, King Abdulaziz University, P.O. Box 80203, Jeddah 21589, Saudi Arabia*

Correspondence should be addressed to Naseer Shahzad; nshahzad@kau.edu.sa

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In this paper we point out some corrections needed in [1].

Recently, a geometric notion of *seminormal structure* has been introduced as follows.

Definition 1 (see [1]). A convex pair (A, B) in a Banach space X is said to have seminormal structure if, for any bounded, closed, and convex pair $(K_1, K_2) \subseteq (A, B)$ with $\delta(K_1, K_2) > 0$, there exits $(p, q) \in K_1 \times K_2$ such that

$$\max\left\{\delta_p(K_2), \delta_q(K_1)\right\} < \delta(K_1, K_2). \tag{1}$$

It has been remarked in [1] that the pair (A, A) has seminormal structure if and only if A has normal structure in the sense of Brodskiĭ and Mil'man [2]. We revise this remark as follows. If the pair (A, A) has seminormal structure, then A has normal structure in the sense of Brodskiĭ and Mil'man. Indeed, if the set A has normal structure, then (A, A) may not have seminormal structure. We illustrate this with the following example.

Example 2. Let $X := \mathbb{R}$ with the usual metric and let $A := [0, 1]$. Then A has normal structure because A is a nonempty, bounded, closed, and convex subset of the uniformly convex Banach space X. Suppose $K_1 := \{0\}$ and $K_2 := \{1\}$. Then $\max\{\delta_p(K_2), \delta_q(K_1)\} = \delta(K_1, K_2)$; that is, (A, A) does not have seminormal structure.

The following notion has also been given in [1].

Definition 3 (see [1]). A nonempty, bounded, closed, and convex pair (A, B) of a normed linear space is said to have property (D) provided that for each nonempty, closed, and convex pair $(E, F) \subseteq (A, B)$ one has

$$\min\{\operatorname{diam}(E), \operatorname{diam}(F)\} \le \delta(E, F). \tag{2}$$

In [1], the following proposition has been obtained to derive Corollary 5 (see Corollary 12 in [1]).

Proposition 4 (see Proposition 11 in [1]). *Let (A, B) be a nonempty, bounded, closed, and convex pair in a uniformly convex Banach space X such that (A, B) has the property (D). Then (A, B) has seminormal structure.*

Corollary 5 (see Corollary 12 in [1]). *Let (A, B) be a nonempty, bounded, closed, and convex pair in a uniformly convex Banach space X such that (A, B) has the property (D). Assume that $T : A \cup B \to A \cup B$ is a cyclic relatively nonexpansive mapping. Then T has a fixed point.*

In the following, we give a counterexample to Proposition 4 which suggests that the result of Corollary 5 should be revised.

Example 6. Let $X := \mathbb{R}$ with the usual metric and let $A := [0, 1]$ and $B := [2, 3]$. It is clear that (A, B) has the property (D). Now, consider $K_1 := \{0\}$ and $K_2 := \{3\}$ and suppose $(p, q) = (0, 3)$. Then

$$\max\left\{\delta_p(K_2), \delta_q(K_1)\right\} = \delta(K_1, K_2); \tag{3}$$

that is, (A, B) does not have seminormal structure.

Using an argument similar to that in the proof of Proposition 11 in [1], we are able to correct Corollary 5 as follows.

Corollary 7. *Let (A, B) be a nonempty, bounded, closed, and convex pair in a uniformly convex Banach space X such that (A, B) has the property (D). If $T : A \cup B \to A \cup B$ is a cyclic relatively nonexpansive mapping, then either $A \cap B$ is nonempty and T has a fixed point in $A \cap B$ or T has a best proximity point.*

Proof. Suppose $\mathscr{F}$ denotes the collection of all nonempty, closed, and convex pairs $(E, F) \subseteq (A, B)$ such that T is cyclic on $E \cup F$ and there exists a pair $(p, q) \in E \times F$ for which $\|p - q\| = \operatorname{dist}(A, B)$. Note that $(A_0, B_0) \in \mathscr{F}$. By using Zorn's lemma we can see that $\mathscr{F}$ has a minimal element, say (K_1, K_2). If $\delta(K_1, K_2) = 0$, then $A \cap B$ is a nonempty, bounded, closed, and convex subset of a uniformly convex Banach space X and $T : A \cap B \to A \cap B$ is a nonexpansive mapping. Thus T has a fixed point and we are finished. So, we assume that $\delta(K_1, K_2) > 0$. We now consider the following cases.

Case 1. If $\min\{\operatorname{diam}(K_1), \operatorname{diam}(K_2)\} = 0$, we may assume that $K_1 = \{x^*\}$. Consequently, there exists $y^* \in K_2$ such that $\|x^* - y^*\| = \operatorname{dist}(A, B)$. Since T is a cyclic relatively nonexpansive mapping, we have

$$\left\|x^* - Tx^*\right\| = \left\|Ty^* - Tx^*\right\| \leq \left\|y^* - x^*\right\| = \operatorname{dist}(A, B). \quad (4)$$

This implies that T has a best proximity point.

Case 2. If $\min\{\operatorname{diam}(K_1), \operatorname{diam}(K_2)\} > 0$, by an argument similar to that in Proposition 11 of [1], we conclude that there exists a pair $(p, q) \in K_1 \times K_2$ such that $\max\{\delta_p(K_2), \delta_q(K_1)\} < \delta(K_1, K_2)$. By analogous proof of Theorem 8 in [1], we obtain that $\delta(K_1, K_2) = 0$, which is a contradiction. □

References

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