

Research Article

Finite-Time Distributed Energy-to-Peak Control for Uncertain Multiagent Systems

Qu Chenggang,¹ Cao Xibin,¹ Hamid Reza Karimi,² Zhang Zhuo,³ and Zhang Zexu³

¹ Research Institute of Satellite Technology, Harbin Institute of Technology, Harbin 150080, China

² Department of Engineering, Faculty of Engineering and Science, University of Agder, 4898 Grimstad, Norway

³ Deep Space Exploration Research Center, Harbin Institute of Technology, Harbin 150080, China

Correspondence should be addressed to Zhang Zexu; zexuzhang@hit.edu.cn

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This paper investigates the finite-time distributed L_2 - L_∞ consensus control problem of multiagent systems with parameter uncertainties. The relative states of neighboring agents are used to construct the control law and some agents know their own states. By substituting the control input into multiagent systems, an augmented closed-loop system is obtained. Then, we analyze its finite-time boundedness (FTB) and finite-time L_2 - L_∞ performance. A sufficient condition for the existence of the designed controller is given with the form of linear matrix inequalities (LMIs). Finally, simulation results are described.

1. Introduction

The coordination control problems of multiagent systems have attracted increasing attentions from various fields, such as formation flights [1, 2], multiple robots formation control [3, 4], air traffic control [5], and multivehicle systems cooperative control [6, 7]. For the reason that centralized control is too expensive or even infeasible to accomplish, the distributed control protocol has been studied extensively in recent years.

The consensus problem for multiagent systems with switching communication topologies is studied in [8, 9], heterogeneous multiple agents are researched in [10], the distributed control problem for high-order multiagent systems is investigated in [11], the nonlinear uncertain multiagent systems are studied in [12], networked control problem for multiple agents with a leader is described in [13], multiagent systems with input time delays are investigated in [14], and, for the case that state information cannot be measured, a distributed output-feedback control for multiagent systems with Markov jumping is studied in [15]. However, to the best knowledge of the authors, there is no article investigating the finite-time energy-to-peak consensus problem for multiagent systems with parameter uncertainties.

For the reason that the forms of external disturbances are not exactly known and the existence of parameter uncertainties in systems, the robust control method is proposed. There are mainly three robust control theories. The first one is energy-to-energy control, which is the well-known H_∞ control [16–19], where the external disturbance can be any form but energy bounded. The second theory is peak-to-peak control, in which worse cases of performance variables are required to be minimized under the peak bounded external disturbances; it can be seen that the conservatism of peak-to-peak control is much less than the H_∞ control. The last method is the energy-to-peak one, which is also called L_2 - L_∞ in discrete-time and L_2 - L_∞ in continuous-time systems. In energy-to-peak control, the external disturbances are energy bounded and the peak values of performance variables are required to be minimized, which is less conservative than the H_∞ control. The energy-to-peak problem for Markovian jump systems is investigated in [20], the robust energy-to-peak FIR equalization for time-varying communication channels is described in [21], and energy-to-peak filtering is designed in [22, 23].

In practical engineering, the main concern is the behavior of the system in a fixed finite-time interval. In such case, the finite-time stability (FTS) is introduced. A system is said

to be finite-time stable if, in a given time interval, the state values will not exceed a certain upper bound under the initial conditions. The FTS method is applied to restrain the large state values in the initial period of time caused by the large initial conditions.

The FTS problem for switched systems is studied in [24], input-output FTS method is proposed in [25], and a time-varying system FTS problem is analyzed in [26]; the FTS is investigated for the singular impulsive systems in [27]. See for more details the references of the aforementioned literatures.

In this paper, we investigate the finite-time L_2 - L_∞ control problem for multiagent systems with parameter uncertainties. The rest of this paper is organized as follows. Some preliminaries on graph theory and problem formulation are given in Section 2. The finite-time distributed L_2 - L_∞ consensus control problem for multiagent systems with parameter uncertainties is investigated in Section 3. Numerical simulations are shown in Section 4. Section 5 concludes the paper.

In this paper, the following notations will be used: $R^{n \times n}$ denotes the set of $n \times n$ real matrix. The superscripts -1 and T mean the inverse and the transpose of a matrix, respectively. The real matrix $P > 0$ stands for the fact that P is positive definite. $\lambda_{\min}(P)$ and $\lambda_{\max}(P)$, respectively, refer to the minimum and maximum eigenvalues of the matrix P . $\text{diag}\{A_1, \dots, A_N\}$ is a block-diagonal matrix. $A \otimes B$ is the Kronecker product of matrices A and B . Further, some algorithms of Kronecker product applied are described as follows:

$$\begin{aligned} (A \otimes B)(C \otimes D) &= (AC) \otimes (BD), \\ (A \otimes I_n)(I_m \otimes B) &= (I_m \otimes B)(A \otimes I_n), \\ (A \otimes B)^T &= A^T \otimes B^T, \quad (A \otimes B)^{-1} = A^{-1} \otimes B^{-1}. \end{aligned} \quad (1)$$

2. Preliminaries

2.1. Graph Theory. An undirected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ is a pair $(\mathcal{V}, \mathcal{E})$, where $\mathcal{V} = \{v_1, v_2, \dots, v_N\}$ is the set of nodes and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is the set of pairs of nodes, called edges. Two nodes v_i and v_j are called neighbouring agents, if (v_i, v_j) is an edge of graph \mathcal{G} . A path on \mathcal{G}_f from node v_i to node v_j is a sequence of edges $(v_i, v_{k_1}), (v_{k_1}, v_{k_2}), \dots, (v_{k_l}, v_j)$. A graph is called connected graph if there exists a path from v_i to v_j for all pairs of (v_i, v_j) .

The adjacency matrix $\mathcal{A} = [a_{ij}] \in R^{N \times N}$ associated with undirected graph \mathcal{G} is defined by $a_{ii} = 0$ and $a_{ij} = a_{ji} = 1$ if $(v_i, v_j) \in \mathcal{E}$ and $a_{ij} = a_{ji} = 0$ otherwise. The Laplacian matrix $\mathcal{L} = [l_{ij}] \in R^{N \times N}$ is defined as $l_{ii} = \sum_{j=1}^N a_{ij}$ and $l_{ij} = -a_{ij}$ for $i \neq j$.

Define a diagonal matrix $\mathcal{D} = \text{diag}\{d_1, \dots, d_N\} \in R^{N \times N}$: $d_i = 1$ if the i th agent knows its own state and 0 otherwise. Define a new matrix called modified Laplacian matrix $\widehat{\mathcal{L}} = \mathcal{L} + \mathcal{D}$.

Assumption 1. The undirected topology graph \mathcal{G} is connected and at least one agent is able to achieve its own state.

Lemma 2 (see [28]). *If the graph \mathcal{G} satisfies Assumption 1, then matrix $\widehat{\mathcal{L}} = \mathcal{L} + \mathcal{D}$ is positive definite.*

2.2. Problem Formulation. Consider the following N linear multiagent systems with parameter uncertainties:

$$\begin{aligned} \dot{x}_i &= (A + \Delta A_i)x_i + Bu_i + Dw_i, \quad z_i = x_i, \\ i &= 1, \dots, N, \end{aligned} \quad (2)$$

where the subscripts i denote the i th agent. $x_i \in R^n$ stands for the state variable, $u_i \in R^p$ is the control input, $w_i \in R^m$ means the external disturbance, and $z_i \in R^n$ refers to the required performance variable. A , B , and D are constant matrices with the compatible dimensions; matrix ΔA_i denotes the parameter uncertainty which is unknown and described as $\Delta A_i = E\Sigma_i(t)F$, where $\Sigma_i(t)$ stands for the time-varying uncertainty and satisfies the following condition:

$$\Sigma_i^T(t)\Sigma_i(t) \leq \delta^2 I \quad i = 1, \dots, N \quad (3)$$

with the constant parameter $\delta > 0$, where E and F are constant matrices which are used to characterize the structure of the uncertainty.

Therefore, it is called that the multi-agent systems have the same nominal dynamics which are the certain parts, but the different uncertainties with each other.

Similar to [28, 29], in this paper, we aim to make multiple agents achieve the consensus and their states converge to zero thus, the controller is described as

$$u_i = cK \left[\sum_{j=1}^N a_{ij} (x_i - x_j) + d_i x_i \right], \quad i = 1, \dots, N, \quad (4)$$

where the scalar $c > 0$ denotes the coupling strength, K stands for the relative state-feedback control gain matrix to be determined, a_{ij} , $i, j \in \{1, \dots, N\}$, is the element of adjacency matrix \mathcal{A} associated with the communication topology graph, and \mathcal{G} , d_i are constant scalars which satisfy $d_i = 1$, $i = 1, \dots, q$, and $d_i = 0$, $i = q + 1, \dots, N$; that is to say, agents $1, \dots, q$ are able to achieve their own states, but agents $q + 1, \dots, N$ cannot get their own states.

We define the following augmented variables:

$$x = \begin{bmatrix} x_1 \\ \vdots \\ x_N \end{bmatrix}, \quad w = \begin{bmatrix} w_1 \\ \vdots \\ w_N \end{bmatrix}, \quad z = \begin{bmatrix} z_1 \\ \vdots \\ z_N \end{bmatrix}. \quad (5)$$

Then, the closed-loop system consisting of (2) and (4) can be rewritten as

$$\begin{aligned} \dot{x} &= \left[I_N \otimes A + c\widehat{\mathcal{L}} \otimes BK + (I_N \otimes E)\Sigma(I_N \otimes F) \right] x \\ &\quad + (I_N \otimes D)w, \\ z &= x, \end{aligned} \quad (6)$$

where $\widehat{\mathcal{L}} = \mathcal{L} + \mathcal{D}$ and $\Sigma = \text{diag}\{\Sigma_1, \dots, \Sigma_N\}$. It is obvious that matrix $\widehat{\mathcal{L}}$ is positive definite based on Assumption 1 and Lemma 2.

In order to analyze the finite-time L_2 - L_∞ performance of the closed-loop system (6) with uncertainties, some definitions and a lemma are described as follows.

Definition 3. The finite-time energy-to-peak gain of the transfer function matrix is described by

$$\Gamma_{\text{ep}(wz)} = \sqrt{\frac{\sup_{t \in (0, T]} [z^T(t) z(t)]}{\int_0^T w^T(t) w(t) dt}}. \quad (7)$$

Definition 4 (finite-time boundedness (FTB) [30]). A system is called FTB with respect to (c_1, c_2, \bar{w}, T) , where $0 < c_1 < c_2$, if the state variables satisfy

$$x^T(t) x(t) \leq c_2, \quad \forall t \in (0, T] \quad (8)$$

with the conditions

$$x^T(0) x(0) \leq c_1, \quad \int_0^T w^T(t) w(t) dt \leq \bar{w}. \quad (9)$$

Definition 5. The closed-loop system (6) is said to be FTB with a finite-time L_2 - L_∞ performance γ , if the following conditions hold.

- (1) System (6) is FTB with respect to (c_1, c_2, \bar{w}, T) .
- (2) Under zero initial conditions, the L_2 - L_∞ performance satisfies the following condition:

$$\sup_{t \in (0, T]} [z^T(t) z(t)] < \gamma^2 \int_0^T w^T(t) w(t) dt. \quad (10)$$

That is, $\Gamma_{\text{ep}(wz)} < \gamma$.

In addition, based on Definition 3, it is derived from [31] that

$$\Gamma_{\text{ep}(wz)} = \frac{1}{2\pi} \lambda_{\max} \left(\int_0^T T_{wz}(j\omega) T_{wz}^T(j\omega) d\omega \right), \quad (11)$$

where T_{wz} is the transfer function matrix of closed-loop system (6); note that the right side of (11) is the finite-time H_2 norm of the transfer function matrix.

Lemma 6 (see [32]). Let $\Theta = \Theta^T$; let \bar{E} and \bar{F} be real matrices with appropriate dimensions; $\Sigma(k)$ satisfies $\Sigma^T(k) \Sigma(k) \leq I$. Then the following condition:

$$\Theta + \bar{E} \Sigma(k) \bar{F} + (\bar{E} \Sigma(k) \bar{F})^T < 0, \quad (12)$$

holds if there exists a positive scalar $\varepsilon > 0$ such that the following inequality is satisfied:

$$\begin{bmatrix} \Theta & \varepsilon \bar{E} & \bar{F}^T \\ * & -\varepsilon I & 0 \\ * & * & -\varepsilon I \end{bmatrix} < 0. \quad (13)$$

Theorem 7. For a given scalar $\gamma > 0$, system (6) is FTB with the finite-time L_2 - L_∞ performance γ , if and only if the following N systems are finite-time bounded and the finite-time L_2 - L_∞ norm of their transfer function matrices $\Gamma_{\text{ep}(\hat{w}_i \hat{z}_i)} < \gamma$:

$$\begin{aligned} \dot{\hat{x}}_i &= (A + \Delta A_i + c \lambda_i BK) \hat{x}_i + D \hat{w}_i, & \hat{z}_i &= \hat{x}_i, \\ & & i &= 1, \dots, N, \end{aligned} \quad (14)$$

where λ_i is the i th eigenvalue of $\widehat{\mathcal{L}}$.

Proof. Let $U \in R^{N \times N}$ be a unitary matrix such that $U^T \widehat{\mathcal{L}} U = \Lambda = \text{diag}\{\lambda_1, \dots, \lambda_N\}$. Define new variables $\hat{x} = (U^T \otimes I_n)x$, $\hat{w} = (U^T \otimes I_m)w$, and $\hat{z} = (U^T \otimes I_n)z$, where

$$\hat{x} = \begin{bmatrix} \hat{x}_1 \\ \vdots \\ \hat{x}_N \end{bmatrix}, \quad \hat{w} = \begin{bmatrix} \hat{w}_1 \\ \vdots \\ \hat{w}_N \end{bmatrix}, \quad \hat{z} = \begin{bmatrix} \hat{z}_1 \\ \vdots \\ \hat{z}_N \end{bmatrix}. \quad (15)$$

Therefore, we have

$$\begin{aligned} \dot{\hat{x}} &= (U^T \otimes I_n) [I_N \otimes A + c \widehat{\mathcal{L}} \otimes BK + (I_N \otimes E) \Sigma (I_N \otimes F)] \\ &\quad \times (U \otimes I_n) \hat{x} + (U^T \otimes I_n) (I_N \otimes D) (U \otimes I_m) \hat{w} \\ &= [I_N \otimes A + c \Lambda \otimes BK + (U^T \otimes I_n) (I_N \otimes E \Sigma_i F) \\ &\quad \times (U \otimes I_n)] \hat{x} + (I_N \otimes D) \hat{w} \\ &= [I_N \otimes A + c \Lambda \otimes BK + (I_N \otimes E \Sigma_i F)] \hat{x} + (I_N \otimes D) \hat{w}. \end{aligned} \quad (16)$$

Hence, the closed-loop system (6) can be regarded as

$$\begin{aligned} \dot{\hat{x}} &= [I_N \otimes A + c \Lambda \otimes BK + (I_N \otimes E \Sigma_i F)] \hat{x} + (I_N \otimes D) \hat{w}, \\ \hat{z} &= \hat{x}. \end{aligned} \quad (17)$$

Note that the system in (17) is composed of the N systems in (14).

It can be seen that the system (17) is obtained by applying the elementary transformation to system (6). Thus the finite-time stability (boundedness) of systems (6) and (17) is equivalent.

It is concluded from (6), (14), and (17) that

$$T_{\hat{w}\hat{z}} = \text{diag} \{T_{\hat{w}_1 \hat{z}_1}, \dots, T_{\hat{w}_N \hat{z}_N}\} = (U^T \otimes I_m) T_{wz} (U^T \otimes I_n). \quad (18)$$

Via the finite-time L_2 - L_∞ norm described in (11), it implies that

$$\Gamma_{\text{ep}(wz)} = \Gamma_{\text{ep}(\hat{w}\hat{z})} = \max_{i=1, \dots, N} \Gamma_{\text{ep}(\hat{w}_i \hat{z}_i)}. \quad (19)$$

Therefore, it is obvious that $\Gamma_{\text{ep}(wz)} < \gamma$ if and only if $\Gamma_{\text{ep}(\hat{w}_i \hat{z}_i)} < \gamma$, $i \in \{1, \dots, N\}$. This completes the proof. \square

Remark 8. It is shown in Theorem 7 that the distributed finite-time energy-to-peak control problem of system (6) is equivalent to the finite-time energy-to-peak control problems of a set of decoupled linear systems having the same dimensions as a single agent with the assumption that the nominal dynamics of agents are identical.

In the next section, we are going to derive the sufficient condition for the existence of the control input and give the expression of feedback gain matrix K . Select the coupling strength $c \geq c_{\text{th}}$, where

$$c_{\text{th}} = \frac{\tau}{\min_{i=1,\dots,N}(\lambda_i)}. \quad (20)$$

3. Finite-Time Energy-to-Peak Control

Theorem 9. *The closed-loop network in (6) with a distributed controller described in (4) is finite-time bounded with a finite-time energy-to-peak performance γ , if there exist positive definite symmetric matrix \mathcal{Q} , scalars $\tau > 0$, $\alpha > 0$, and $\varepsilon > 0$ such that*

$$\begin{bmatrix} -\alpha\mathcal{Q} + A\mathcal{Q} + \mathcal{Q}A^T - \tau BB^T & D & \mathcal{Q} & \varepsilon E & \delta\mathcal{Q}F^T \\ D^T & -\gamma^2 I & 0 & 0 & 0 \\ \mathcal{Q} & 0 & -I & 0 & 0 \\ \varepsilon E^T & 0 & 0 & -\varepsilon I & 0 \\ \delta\mathcal{Q}F & 0 & 0 & 0 & -\varepsilon I \end{bmatrix} < 0, \quad (21)$$

$$\begin{bmatrix} -e^{-\alpha T}\mathcal{Q} & \mathcal{Q} \\ \mathcal{Q} & -I \end{bmatrix} < 0. \quad (22)$$

In addition, the feedback gain matrix is $K = -(1/2)B^T\mathcal{Q}^{-1}$.

Proof. Based on the Schur Complement Lemma, inequality (21) can be regarded as

$$\begin{aligned} & -\alpha\mathcal{Q} + A\mathcal{Q} + \mathcal{Q}A^T - \tau BB^T + \gamma^{-2}DD^T + \mathcal{Q}^2 + \varepsilon EE^T \\ & + \varepsilon^{-1}\delta^2\mathcal{Q}F^T F\mathcal{Q} < 0. \end{aligned} \quad (23)$$

Thus, by using (20), we have

$$\begin{aligned} & -\alpha\mathcal{Q} + A\mathcal{Q} + \mathcal{Q}A^T - c\lambda_i BB^T + \gamma^{-2}DD^T + \mathcal{Q}^2 + \varepsilon EE^T \\ & + \varepsilon^{-1}\delta^2\mathcal{Q}F^T F\mathcal{Q} < 0. \end{aligned} \quad (24)$$

We choose the control gain matrix as $K = -(1/2)B^T\mathcal{Q}^{-1}$; then we can obtain

$$\begin{aligned} & -\alpha\mathcal{Q} + A\mathcal{Q} + \mathcal{Q}A^T + c\lambda_i\mathcal{Q}K^T B^T + c\lambda_i BK\mathcal{Q} + \gamma^{-2}DD^T \\ & + \mathcal{Q}^2 + \varepsilon EE^T + \varepsilon^{-1}\delta^2\mathcal{Q}F^T F\mathcal{Q} < 0. \end{aligned} \quad (25)$$

By using Schur Complement Lemma again, the above inequality is equivalent to

$$\begin{bmatrix} \Omega_{11} & D & \mathcal{Q} & \varepsilon E & \delta\mathcal{Q}F^T \\ D^T & -\gamma^2 I & 0 & 0 & 0 \\ \mathcal{Q} & 0 & -I & 0 & 0 \\ \varepsilon E^T & 0 & 0 & -\varepsilon I & 0 \\ \delta\mathcal{Q}F & 0 & 0 & 0 & -\varepsilon I \end{bmatrix} < 0, \quad (26)$$

where

$$\Omega_{11} = -\alpha\mathcal{Q} + \mathcal{Q}(A + c\lambda_i BK)^T + (A + c\lambda_i BK)\mathcal{Q}. \quad (27)$$

By applying Lemma 6, we can obtain the following inequality if the inequality (26) holds:

$$\begin{bmatrix} \Pi_{11} & D & \mathcal{Q} \\ D^T & -\gamma^2 I & 0 \\ \mathcal{Q} & 0 & -I \end{bmatrix} < 0, \quad (28)$$

where

$$\begin{aligned} \Pi_{11} &= -\alpha\mathcal{Q} + \mathcal{Q}(A + c\lambda_i BK + E\Sigma_i F)^T \\ & + (A + c\lambda_i BKE\Sigma_i F)\mathcal{Q}. \end{aligned} \quad (29)$$

By multiplying the left and right sides of inequality (28) by $\text{diag}\{\mathcal{Q}^{-1}, I, I\}$, we have

$$\begin{bmatrix} \bar{\Pi}_{11} & \mathcal{Q}^{-1}D & I \\ D^T\mathcal{Q}^{-1} & -\gamma^2 I & 0 \\ I & 0 & -I \end{bmatrix} < 0, \quad (30)$$

where

$$\begin{aligned} \bar{\Pi}_{11} &= -\alpha\mathcal{Q}^{-1} + (A + c\lambda_i BK + E\Sigma_i F)^T\mathcal{Q}^{-1} \\ & + \mathcal{Q}^{-1}(A + c\lambda_i BKE\Sigma_i F). \end{aligned} \quad (31)$$

Let $P = \mathcal{Q}^{-1}$; we can get

$$\begin{bmatrix} \bar{\Pi}_{11} & PD & I \\ D^T P & -\gamma^2 I & 0 \\ I & 0 & -I \end{bmatrix} < 0, \quad (32)$$

where

$$\begin{aligned} \bar{\Pi}_{11} &= -\alpha P + (A + c\lambda_i BK + E\Sigma_i F)^T P \\ & + P(A + c\lambda_i BKE\Sigma_i F). \end{aligned} \quad (33)$$

It is obvious that P is also a positive definite symmetric matrix. By using Schur Complement Lemma, inequality (32) is equivalent to

$$\begin{bmatrix} \Xi_{11} & PD \\ D^T P & -\gamma^2 I \end{bmatrix} < 0, \quad (34)$$

where

$$\begin{aligned} \Xi_{11} = & -\alpha P + I + (A + c\lambda_i BK + E\Sigma_i F)^T P \\ & + P(A + c\lambda_i BKE\Sigma_i F). \end{aligned} \quad (35)$$

We choose the Lyapunov function for the N systems in (14) as follows:

$$V_i(t) = \hat{x}_i^T(t) P \hat{x}_i(t), \quad i = 1, \dots, N. \quad (36)$$

The derivate of the aforementioned Lyapunov function is

$$\begin{aligned} \dot{V}_i &= \hat{x}_i^T P \dot{\hat{x}}_i + \dot{\hat{x}}_i^T P \hat{x}_i \\ &= \hat{x}_i^T [(A + c\lambda_i BK + E\Sigma_i F)^T P + P(A + c\lambda_i BK + E\Sigma_i F)] \\ &\quad \times \hat{x}_i + \hat{x}_i^T P D \hat{w}_i + \hat{w}_i^T D^T P \hat{x}_i. \end{aligned} \quad (37)$$

Therefore, it is derived from the inequality (34) that

$$\dot{V}_i + \hat{x}_i^T \hat{x}_i - \alpha \hat{x}_i^T P \hat{x}_i - \gamma^2 \hat{w}_i^T \hat{w}_i < 0. \quad (38)$$

That is,

$$\dot{V}_i < \alpha V_i + \gamma^2 \hat{w}_i^T \hat{w}_i - \hat{z}_i^T \hat{z}_i. \quad (39)$$

Then we have

$$\dot{V}_i < \alpha V_i + \gamma^2 \hat{w}_i^T \hat{w}_i. \quad (40)$$

Hence, the above inequality can be regarded as

$$\frac{d}{dt} [e^{-\alpha t} V_i(t)] < e^{-\alpha t} \gamma^2 \hat{w}_i^T(t) \hat{w}_i(t). \quad (41)$$

By integrating the aforementioned inequality from 0 to t , we can obtain

$$\begin{aligned} e^{-\alpha t} V_i(t) &< V_i(0) + \gamma^2 \int_0^t e^{-\alpha s} \hat{w}_i^T(s) \hat{w}_i(s) ds \\ &\leq V_i(0) + \gamma^2 \int_0^t \hat{w}_i^T(s) \hat{w}_i(s) ds \\ &\leq \hat{x}_i^T(0) P \hat{x}_i(0) + \gamma^2 \int_0^T \hat{w}_i^T(t) \hat{w}_i(t) dt \\ &= \lambda_{\max}(P) \hat{x}_i^T(0) \hat{x}_i(0) + \gamma^2 \int_0^T \hat{w}_i^T(t) \hat{w}_i(t) dt. \end{aligned} \quad (42)$$

For the initial conditions $\hat{x}_i^T(0) \hat{x}_i(0) \leq c_1$ and finite-time energy-bounded disturbance conditions $\int_0^T \hat{w}_i^T(t) \hat{w}_i(t) dt \leq \bar{\omega}$ hold, the above inequality can be rewritten as

$$V_i(t) < e^{\alpha t} c_1 \lambda_{\max}(P) + \gamma^2 \bar{\omega} \leq e^{\alpha T} c_1 \lambda_{\max}(P) + \gamma^2 \bar{\omega}. \quad (43)$$

For the reason that $V_i(t) \geq \lambda_{\min}(P) \hat{x}_i^T(t) \hat{x}_i(t)$, it concludes that

$$\hat{x}_i^T(t) \hat{x}_i(t) < \frac{e^{\alpha T} c_1 \lambda_{\max}(P) + \gamma^2 \bar{\omega}}{\lambda_{\min}(P)} = c_2, \quad t \in (0, T]. \quad (44)$$

Therefore, the N systems with parameter uncertainties in (14) are all finite-time bounded with respect to $(c_1, c_2, \bar{\omega}, T)$.

Next, we are going to prove the finite-time L_2 - L_∞ performance of N systems (14) which contain parameter uncertainties. Multiplying the left and right sides of inequality (22) by $\text{diag}\{\mathcal{Q}^{-1}, I\}$ gives

$$\begin{bmatrix} -e^{-\alpha T} \mathcal{Q}^{-1} & I \\ I & -I \end{bmatrix} < 0, \quad (45)$$

where $\mathcal{Q}^{-1} = P$; then we have

$$\begin{bmatrix} -e^{-\alpha T} P & I \\ I & -I \end{bmatrix} < 0. \quad (46)$$

By using Schur Complement Lemma, the inequality (46) is equivalent to

$$I < e^{-\alpha T} P. \quad (47)$$

Therefore, we have

$$\hat{z}_i^T(t) \hat{z}_i(t) = \hat{x}_i^T(t) \hat{x}_i(t) < e^{-\alpha T} \hat{x}_i^T(t) P \hat{x}_i(t) = e^{-\alpha T} V_i(t). \quad (48)$$

By integrating the inequality (41) from 0 to t , we have

$$e^{-\alpha t} V_i(t) < V_i(0) + \gamma^2 \int_0^t e^{-\alpha s} \hat{w}_i^T(s) \hat{w}_i(s) ds. \quad (49)$$

Under the zero initial condition, which means that $V_i(0) = 0$, we will have

$$\begin{aligned} e^{-\alpha t} V_i(t) &< \gamma^2 \int_0^t e^{-\alpha s} \hat{w}_i^T(s) \hat{w}_i(s) ds \\ &\leq \gamma^2 \int_0^T e^{-\alpha t} \hat{w}_i^T(t) \hat{w}_i(t) dt \\ &\leq \gamma^2 \int_0^T \hat{w}_i^T(t) \hat{w}_i(t) dt. \end{aligned} \quad (50)$$

Therefore, by combing the inequalities (48) and (50), we can obtain

$$\begin{aligned} \hat{z}_i^T(t) \hat{z}_i(t) &< e^{-\alpha T} V_i(t) \leq e^{-\alpha t} V_i(t) \\ &< \gamma^2 \int_0^T \hat{w}_i^T(t) \hat{w}_i(t) dt \quad \forall t \in (0, T]. \end{aligned} \quad (51)$$

Hence, we have

$$\sup_{t \in (0, T]} [\hat{z}_i^T(t) \hat{z}_i(t)] < \gamma^2 \int_0^T \hat{w}_i^T(t) \hat{w}_i(t) dt. \quad (52)$$

Therefore, the N systems (14) are all finite-time bounded with a finite-time L_2 - L_∞ performance γ . Further, based on Theorem 7 the closed-loop network with parameter uncertainties in (6) is FTB with a finite-time L_2 - L_∞ performance γ , if and only if the systems (14) are all FTB with a finite-time L_2 - L_∞ performance γ .

Hence, system (6) is FTB with a finite-time L_2 - L_∞ performance γ under the control inputs (4), if the inequalities (21) and (22) hold. This completes the proof. \square

Corollary 10. *By applying Theorems 7 and 9, it can be seen that the minimum finite-time L_2 - L_∞ performance index γ_{\min} can be obtained by the following optimization problem:*

$$\begin{aligned} & \text{minimize } \gamma \\ & \text{subject to LMIs (21) and (22), with } \mathcal{Q} > 0, \tau > 0, \gamma > 0, \\ & \alpha > 0, \varepsilon > 0. \end{aligned}$$

Remark 11. The above designs a distributed energy-to-peak controller which is decoupled from the communication topologies. Here, we only need to select the coupling strength c to be large enough to make the controller can be used to multiple connected communication topology graphs.

4. Numerical Simulations

In this section, simulation examples are provided to illustrate the effectiveness of the theoretical results. Consider that the multiagent systems with parameter uncertainties consist of six agents as follows:

$$\begin{aligned} \dot{x}_i &= (A + \Delta A_i)x_i + Bu_i + Dw_i, & z_i &= x_i, \\ & & i &= 1, \dots, 6, \end{aligned} \tag{53}$$

where $\Delta A_i = E\Sigma_i(t)F$, $\Sigma_i^T(t)\Sigma_i(t) \leq \delta^2 I$, and

$$\begin{aligned} A &= \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, & B &= \begin{bmatrix} 0 \\ 1 \end{bmatrix}, & D &= \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, \\ E &= \begin{bmatrix} 0 \\ 1 \end{bmatrix}, & F &= [0 \ 1], & \delta &= 0.2. \end{aligned} \tag{54}$$

Hence, by applying Corollary 10 and choosing $\alpha = 0.1$ and $T = 2$, optimal solutions of LMIs (21) and (22) are calculated by

$$\begin{aligned} \mathcal{Q} &= \begin{bmatrix} 0.3887 & -0.2970 \\ -0.2970 & 0.5903 \end{bmatrix}, \\ \tau &= 3.0003, \\ \gamma_{\min} &= 0.6213. \end{aligned} \tag{55}$$

Therefore, the corresponding matrix P and the control gain matrix K are, respectively, calculated by

$$\begin{aligned} P &= \mathcal{Q}^{-1} = \begin{bmatrix} 4.1803 & 2.0134 \\ 2.0134 & 2.7523 \end{bmatrix}, \\ K &= -\frac{1}{2}B^T\mathcal{Q}^{-1} = [-1.0517 \ -1.3762]. \end{aligned} \tag{56}$$

Figure 1 shows four different communication topology graphs with six agents, a loop, that is, an edge start from one agent and end to itself, means that the agent knows its own state. An edge from agent i to agent j means that agent j can receive information from agent i . Different cases of network links are, respectively, shown in graphs \mathcal{G}_1 , \mathcal{G}_2 , and \mathcal{G}_3 , which are subgraphs of \mathcal{G} .

Hence, the corresponding weighted adjacency matrices are described by

$$\begin{aligned} \mathcal{A}_1 &= \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}, & \mathcal{A}_2 &= \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}, \\ \mathcal{A}_3 &= \begin{bmatrix} 0 & 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, & \mathcal{A} &= \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 \end{bmatrix}. \end{aligned} \tag{57}$$

Then the corresponding own-state matrices are

$$\begin{aligned} \mathcal{D}_1 &= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, & \mathcal{D}_2 &= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \\ \mathcal{D}_3 &= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, & \mathcal{D} &= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}. \end{aligned} \tag{58}$$

Then the corresponding modified Laplacian matrices are described by

$$\begin{aligned} \widehat{\mathcal{L}}_1 &= \begin{bmatrix} 3 & -1 & 0 & 0 & 0 & -1 \\ -1 & 2 & -1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & -1 & 2 & -1 \\ -1 & 0 & 0 & 0 & -1 & 2 \end{bmatrix}, \\ \widehat{\mathcal{L}}_2 &= \begin{bmatrix} 3 & -1 & 0 & 0 & 0 & -1 \\ -1 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & -1 & 0 & -1 \\ 0 & 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 \\ -1 & 0 & -1 & 0 & 0 & 2 \end{bmatrix}, \\ \widehat{\mathcal{L}}_3 &= \begin{bmatrix} 5 & -1 & 0 & -1 & -1 & -1 \\ -1 & 3 & -1 & 0 & 0 & 0 \\ 0 & -1 & 3 & -1 & 0 & 0 \\ -1 & 0 & -1 & 3 & -1 & 0 \\ -1 & 0 & 0 & -1 & 2 & 0 \\ -1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \end{aligned}$$

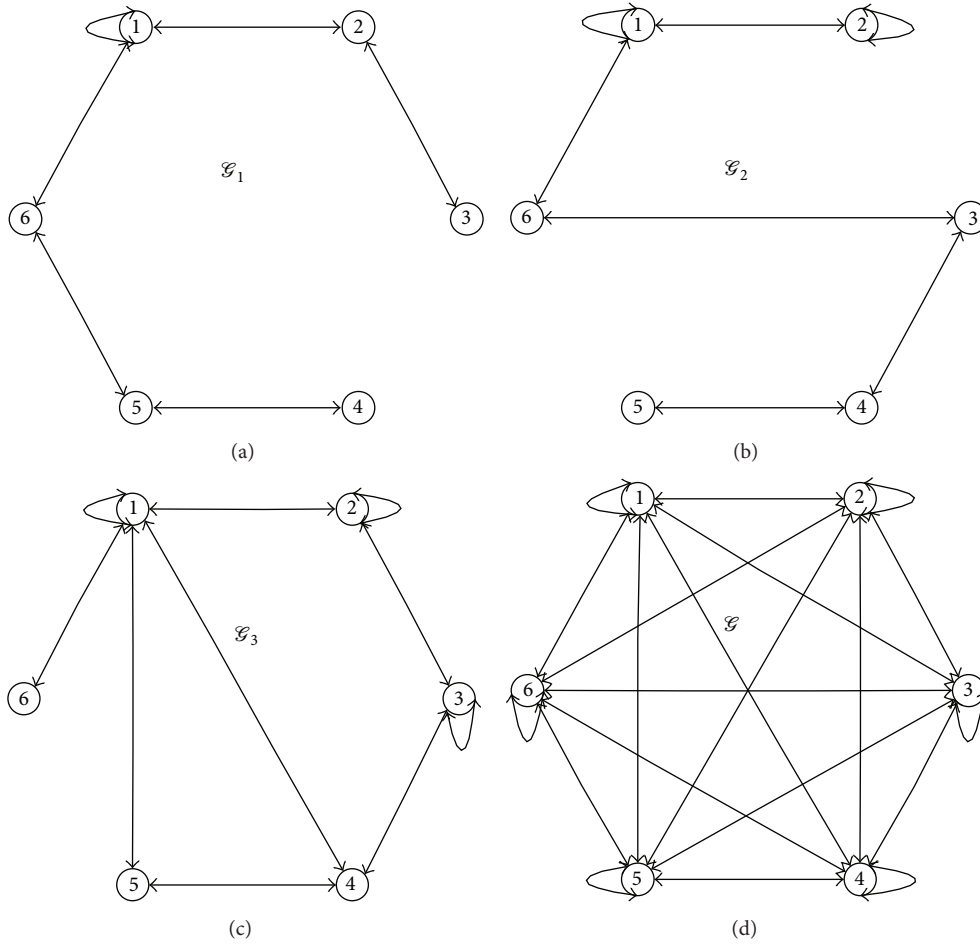


FIGURE 1: The communication topologies \mathcal{G}_1 , \mathcal{G}_2 , \mathcal{G}_3 , and \mathcal{G} .

$$\mathcal{L} = \begin{bmatrix} 6 & -1 & -1 & -1 & -1 & -1 \\ -1 & 6 & -1 & -1 & -1 & -1 \\ -1 & -1 & 6 & -1 & -1 & -1 \\ -1 & -1 & -1 & 6 & -1 & -1 \\ -1 & -1 & -1 & -1 & 6 & -1 \\ -1 & -1 & -1 & -1 & -1 & 6 \end{bmatrix}. \tag{59}$$

Hence, we can calculate the corresponding eigenvalues as follows:

$$\begin{aligned} \mathcal{G}_1: & \lambda_{11} = 0.1031, & \lambda_{12} = 0.2903, & \lambda_{13} = 1.2054, \\ & \lambda_{14} = 2.1673, & \lambda_{15} = 3.0735, & \lambda_{16} = 4.1604, \\ \mathcal{G}_2: & \lambda_{21} = 0.0913, & \lambda_{22} = 0.7302, & \lambda_{23} = 1.4413, \\ & \lambda_{24} = 2.2571, & \lambda_{25} = 3.3460, & \lambda_{26} = 4.1341, \\ \mathcal{G}_3: & \lambda_{31} = 0.3650, & \lambda_{32} = 1.0423, & \lambda_{33} = 2.0000, \\ & \lambda_{34} = 3.2526, & \lambda_{35} = 4.2366, & \lambda_{36} = 6.1034, \end{aligned}$$

$$\begin{aligned} \mathcal{G}: & \lambda_1 = 1.0000, & \lambda_2 = 7.0000, & \lambda_3 = 7.0000, \\ & \lambda_4 = 7.0000, & \lambda_5 = 7.0000, & \lambda_6 = 7.0000. \end{aligned} \tag{60}$$

The corresponding minimum eigenvalues are

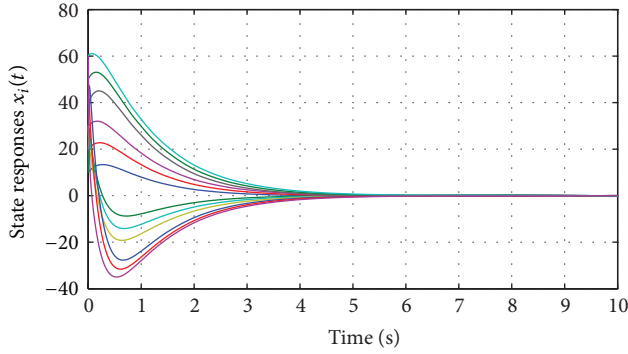
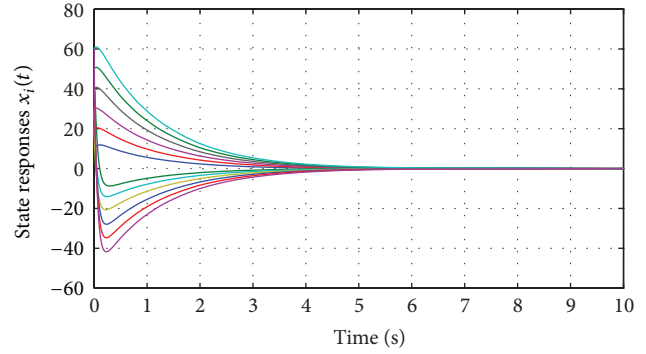
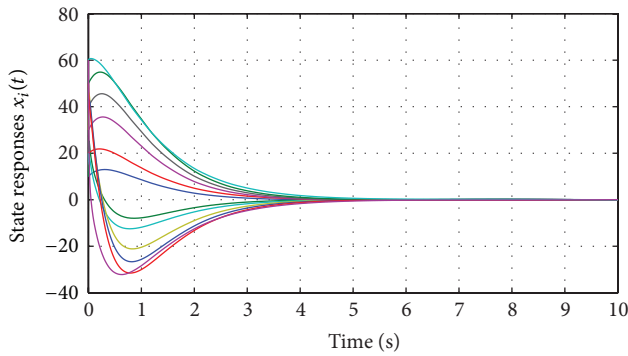
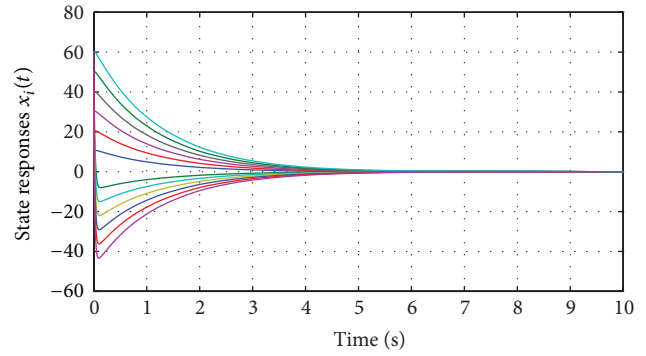
$$\begin{aligned} \lambda_{\min(\mathcal{F}_1)} &= 0.1031, & \lambda_{\min(\mathcal{F}_2)} &= 0.0913, \\ \lambda_{\min(\mathcal{F}_3)} &= 0.3650, & \lambda_{\min(\mathcal{F})} &= 1.0000. \end{aligned} \tag{61}$$

Therefore, in order to make feedback control gain cK be applied to the four different graphs, the scalar c should be chosen large enough as

$$\begin{aligned} c \geq c_{th} &= \frac{\tau}{\min\{\lambda_{\min(\mathcal{F}_1)}, \lambda_{\min(\mathcal{F}_2)}, \lambda_{\min(\mathcal{F}_3)}, \lambda_{\min(\mathcal{F})}\}} \\ &= \frac{3.0003}{0.0913} = 32.8620. \end{aligned} \tag{62}$$

Hence, one chooses the coupling strength as $c = 35$. Therefore, we can obtain the feedback control gain $cK = [-36.8102 \quad -48.1660]$.

In the next, we are going to illustrate the astringency of multiagent systems under the designed controller. Under four

FIGURE 2: The state trajectories of six agents with topology \mathcal{S}_1 .FIGURE 4: The state trajectories of six agents with topology \mathcal{S}_3 .FIGURE 3: The state trajectories of six agents with topology \mathcal{S}_2 .FIGURE 5: The state trajectories of six agents with topology \mathcal{S} .

different graphs, simulation results of six agents with parameter uncertainties will be described. The external disturbances are given by

$$w_i(t) = \begin{bmatrix} \sin(t) \\ \cos(t) \end{bmatrix}, \quad i = 1, \dots, 6. \quad (63)$$

The initial conditions of agents are defined as follows:

$$\begin{aligned} x_1(0) &= \begin{bmatrix} 10 \\ 10 \end{bmatrix}, & x_2(0) &= \begin{bmatrix} 20 \\ 20 \end{bmatrix}, & x_3(0) &= \begin{bmatrix} 30 \\ 30 \end{bmatrix}, \\ x_4(0) &= \begin{bmatrix} 40 \\ 40 \end{bmatrix}, & x_5(0) &= \begin{bmatrix} 50 \\ 50 \end{bmatrix}, & x_6(0) &= \begin{bmatrix} 60 \\ 60 \end{bmatrix}. \end{aligned} \quad (64)$$

Figures 2, 3, 4, and 5 show the consensus of agents with the designed robust energy-to-peak control protocol and their state trajectories under communication topologies \mathcal{S}_1 – \mathcal{S} , respectively. It can be seen that the state trajectories under four different graphs are almost the same which means that, although topology graphs are altered, the designed controller can still make the multiagent systems with norm bounded uncertainties achieve the consensus. In other words, the designed control protocol is an adaptive one for different cases of communication links.

5. Conclusions

In this paper, the problem of finite-time distributed L_2 – L_∞ control of uncertain multiagent systems is studied. A cooperative control protocol is described, in which the relative states of agents are used. The finite-time boundedness (FTB) and finite-time L_2 – L_∞ performance of the multiagent systems with parameter uncertainties are both analyzed. Sufficient conditions for the existence of the finite-time L_2 – L_∞ controller are derived in the form of LMIs. Finally, the simulation results show the effectiveness of the proposed approach.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of the paper.

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