Hindawi Publishing Corporation
Abstract and Applied Analysis
Volume 2014, Article ID 250314, 9 pages
http://dx.doi.org/10.1155/2014/250314

Research Article

Fault Detection for Wireless Network Control Systems with Stochastic Uncertainties and Time Delays

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Received 2 January 2014; Accepted 21 January 2014; Published 27 February 2014

Academic Editor: Ming Liu

The fault detection problem is investigated for a class of wireless network control systems which has stochastic uncertainties in the state-space matrices, combined with time delays and nonlinear disturbance. First, the system error observer is proposed. Then, by constructing proper Lyapunov-Krasovskii functional, we acquire sufficient conditions to guarantee the stability of the fault detection observer for the discrete system, and observer gain is also derived by solving linear matrix inequalities. Finally, a simulation example shows that when a fault happens, the observer residual rises rapidly and fault can be quickly detected, which demonstrates the effectiveness of the proposed method.

1. Introduction

Over the past decades, the fault detection (FD) problem has received considerable research attention and a rich body of literature has appeared on both the theoretical research and practical construction [1–4]. Furthermore, in recent years, the fault detection based on data-driven approach has even started to appear [5, 6]. Meanwhile, the research and application of fault detection in wireless network control system has attracted more and more interest [7–12]. It is a highly interdisciplinary and knowledge integrated as hot research field, which combines sensor technology, embedded computing technology, wireless communication technology, network technology, automatic control technology and distributed information processing technology, and other areas of technological achievements. Distributed sensors are arranged in the place of interest, which is convenient for people to gather information, and measured value can be acquired through multihop technology, until they are transmitted to the user's terminal. WNCS has very broad application prospects in the military, industry, urban management, environmental monitoring, and many other important areas which have potential practical value [13–17]. However, as system is growing highly modular and complex, and system failure probability is also growing. Faults can bring disastrous damage to the entire control system. Therefore, fault detection study on WNCS is essential.

Parameter uncertainty is a kind of factor that contributes to the complexity of the system, and much effort has been made to address this problem when designing observer for state estimation or fault detection [18–29]. Reference [25] applied sliding-window strategy to minimize the worst-case quadratic cost function of estimated state variables, which lead to convert estimation problem in the form of a regularized least-squares one with uncertain data. Reference [26] addressed the problem of the system which had unknown inputs and combined time varying delay in their state variables. Linear matrix inequality formulation was used to improve computational efficiency. In addition, sufficient conditions were also derived in the reduced order system. Reference [27] investigated the effect of parameter uncertainties on genetic regulatory networks. The author presented the convergence region to guarantee the stability of uncertain genetic regulatory networks and derived all results in the case that nonlinear item is unknown. In [28], parameter uncertainties were assumed to be norm-bounded, based on the Lyapunov-Krasovskii functional and stochastic stability theory; stability criteria were obtained in terms of linear matrix inequalities. In [29], the parameter uncertainties were allowed to be norm-bounded and be entered into the state matrix; the addressed filtering problem could effectively be solved in terms of linear matrix inequalities.

It is well known that time delays play an important role in system stability analysis, which could make system unstable or lead to system crash, so many scholars devoted to this field and got fruitful result in recent years [30–36]. Reference [34] dealt with the filtering problem and designed the mean-square finite dimensional filter for incompletely measured time delay system over linear observations. Reference [35] investigated the feedback control problem in discrete time stochastic systems involving sector nonlinearities and mixed time delays. The distributed time delay in the system was first defined and then a special matrix inequality was developed to handle the distributed time delay within an algebraic framework. Reference [36] designed a full order filter for stochastic nonlinear system with time delay, sufficient conditions were derived by using an algebraic matrix inequality approach, and filter design problem was tackled based on the generalized inverse theory.

Motivated by the stated above, we aim to deal with the fault detection problem for a class of WNCSs with stochastic uncertainties and time delays. By constructing proper Lyapunov-Krasovskii functional, we can get sufficient conditions such that the system error observer is asymptotically stable in mean-square sense and acquire gain of designed observer. The main contributions of this paper are listed in the two following aspects. (1) The multiple communication delays and nonlinear disturbance are introduced for discrete-time wireless networked control systems to reflect more realistic environment. (2) The measurement delay is considered when signals are transmitted from plant to controller.

The rest of paper is organized as follows. In Section 2, the problem of WNCS is formulated and some useful lemmas are introduced. In Section 3, we present sufficient conditions to make sure of the asymptotical stability of the system error observer. Besides, the gain used in the observer can be derived by solving linear matrix inequalities. An illustrated example is given in Section 4 to demonstrate the effectiveness of proposed method. Finally, we give our conclusions in Section 5.

Notations. The notations in this paper are quite standard. $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote the n-dimensional Euclidean space and the set of $n\times m$ real matrices; the superscript "T" stands for matrix transposition; I is the matrix of appropriate dimension; $\|\cdot\|$ denotes the Euclidean norm of a vector and its induced norm of matrix; the notation $X > 0$ (resp., $X \geq 0$), for $X \in \mathbb{R}^{n\times n}$ means that the matrix X is real symmetric positive definite (resp., positive semidefinite). $\mathbb{E}\{\cdot\}$ stands for the expectation operator. What is more, we use $(*)$ to represent the entries implied by symmetry. Matrices, if not explicitly specified, are assumed to have compatible dimensions.

2. Problem Formulation

Considering a class of WNCSs as shown in Figure 1, we can know that sensor nodes of wireless network control system comprise a fixed topology in advance and gather information from plants and then transfer data to other nodes nearby. However, due to limited communication range or signal strength concern, existing signal channel may be vanished and a new one may be established. This is the main reason resulting in the uncertainties and time delays of WNCS.

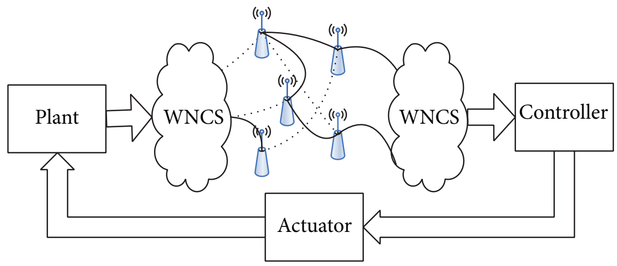

Figure 1: WNCS with stochastic uncertainties and time delays.

The system can be described by the following discrete stochastic systems:

$$x(k+1) = (A + \alpha_k \Delta A)\, x(k) + (A_d + \beta_k \Delta A_d) \times \sum_{i=1}^{N} x(k-i) + E_g g(x(k)) + E_f f(k), \qquad (1)$$
$$y(k) = Cx(k) + Dx(k - d_k),$$

where $x(k) \in \mathbb{R}^n$ is the system state vector, $y(k) \in \mathbb{R}^m$ is the measured output, ΔA and ΔA_d are internal perturbation arising from uncertain factors, $f(k) \in \mathbb{R}^l$ is the fault of the system, $g(\cdot) \in \mathbb{R}^p$ is the nonlinear disturbance, and A, A_d, E_g, E_f, C, and D are constant matrices with appropriate dimensions.

To account for the phenomena of randomly occurring uncertainties, we introduce the stochastic variables α_k and β_k, which are Bernoulli distributed white sequences governed by

$$\begin{aligned} &\text{Prob}\{\alpha_k = 1\} = \alpha, \qquad \text{Prob}\{\alpha_k = 0\} = 1 - \alpha, \\ &\text{Prob}\{\beta_k = 1\} = \beta, \qquad \text{Prob}\{\beta_k = 0\} = 1 - \beta, \end{aligned} \qquad (2)$$

where $\alpha, \beta \in [0, 1]$ are known constants.

Remark 1. For the uncertainties in system that occur in a stochastic way and α_k obeys Bernoulli distribution with a known probability, in this case, the system can be viewed as a changing system as time goes on. In addition, time delay item in system will also be variable as topology changes, we also suppose that it obeys another Bernoulli distribution β_k independent of α_k, which reflects stochastic character of the time delay in system model.

Remark 2. The random variables α_k and β_k that satisfy $\mathbb{E}\{\alpha_k\} = \alpha$, $\mathbb{E}\{\beta_k\} = \beta$, and $\mathbb{E}\{(\alpha_k - \alpha)^2\} = \alpha(1-\alpha)$, $\mathbb{E}\{(\beta_k - \beta)^2\} = \beta(1-\beta)$ are used to model the probability distribution of the randomly occurring uncertainties.

For the system shown in (1), we make the following assumption throughout the paper.

Assumption 3. ΔA and ΔA_d are time-varying matrices with appropriate dimensions and are defined as

$$[\Delta A \;\; \Delta A_d] = GD(k)\,[H \;\; H_d], \qquad (3)$$

where G, H, and H_d are known constant matrix, $D(k)$ is an unknown real time-varying matrix with Lebesgue measurable elements satisfying $D^T(k)D(k) \leq I$.

Assumption 4. $g(\cdot)$ is the nonlinear disturbance which satisfies the following Lipschitz condition:

$$\|g(x_1) - g(x_2)\| \leq \|\gamma(x_1 - x_2)\| \tag{4}$$

with γ being a known constant matrix of appropriate dimension, and $g(0) = 0$.

Assumption 5. The variable d_k denotes the output time-varying delay satisfying

$$0 \leq d_k \leq d, \tag{5}$$

where d is constant positive integers representing the upper bound on the communication delay.

In order to generate residual signal, we construct the following state observer:

$$\begin{aligned} \hat{x}(k+1) &= A\hat{x}(k) + A_d \\ &\quad \times \sum_{i=1}^{N} \hat{x}(k-i) + E_g g(\hat{x}(k)) + K[\hat{y}(k) - y(k)], \\ \hat{y}(k) &= C\hat{x}(k), \end{aligned} \tag{6}$$

where K is the gain of observer to be determined.

Define the state error $e_x(k)$ and the system residual $e_y(k)$ of the system can be written as

$$e_x(k) = x(k) - \hat{x}(k), \qquad e_y(k) = y(k) - \hat{y}(k). \tag{7}$$

If there is no fault, the system residual is close to zero. In order to detect system fault, we adopt a residual evaluation stage, including an evaluation function $J(k)$ and a threshold J_{th} of the following form:

$$J(k) = \left\{ \sum_{k=s-\zeta}^{k=s} e_y^T(k) e_y(k) \right\}^{1/2}, \qquad J_{\text{th}} = \sup_{f(k)=0} \mathbb{E}\{J(k)\}, \tag{8}$$

where ζ denotes the length of the finite evaluating time horizon. Based on (8), the occurrence of faults can be detected by comparing $J(k)$ with J_{th} according to the following rule:

$$\begin{aligned} J(k) > J_{\text{th}} &\Longrightarrow \text{with faults} \Longrightarrow \text{alarm}, \\ J(k) \leq J_{\text{th}} &\Longrightarrow \text{no faults}. \end{aligned} \tag{9}$$

In addition, from (1)–(6), we have the state error detection dynamic governed by the following system:

$$\begin{aligned} e_x(k+1) &= (A - KC) e_x(k) + (\alpha_k - \alpha) \Delta A x(k) \\ &\quad + E_g g(e_x(k)) + \alpha \Delta A x(k) \\ &\quad + (A_d + \beta \Delta A_d) \sum_{i=1}^{N} e_x(k-i) + (\beta_k - \beta) \Delta A_d \\ &\quad \times \sum_{i=1}^{N} e_x(k-i) - KDx(k-d_k), \end{aligned} \tag{10}$$

where $g(e_x(k)) = g(x(k)) - g(\hat{x}(k))$.

By augmenting $\eta(k) = [x^T(k) \;\; e_x^T(k)]^T$, we have the following augmented system:

$$\begin{aligned} \eta(k+1) &= (\overline{A} + \alpha \Delta \overline{A}) \eta(k) + (\alpha_k - \alpha) \Delta \overline{A} \eta(k) \\ &\quad + \overline{E}_g g(\eta(k)) + (\overline{A}_d + \beta \Delta \overline{A}_d) \sum_{i=1}^{N} \eta(k-i) \\ &\quad + (\beta_k - \beta) \Delta \overline{A}_d \sum_{i=1}^{N} \eta(k-i) + \overline{K} \eta(k-d_k), \end{aligned} \tag{11}$$

where

$$\begin{aligned} \overline{A} &= \begin{bmatrix} A & 0 \\ 0 & A - KC \end{bmatrix}, & \Delta \overline{A} &= \begin{bmatrix} \Delta A & 0 \\ \Delta A & 0 \end{bmatrix}, \\ \overline{E}_g &= \begin{bmatrix} E_g & 0 \\ 0 & E_g \end{bmatrix}, & \overline{A}_d &= \begin{bmatrix} A_d & 0 \\ 0 & A_d \end{bmatrix}, \\ \Delta A_d &= \begin{bmatrix} \Delta A_d & 0 \\ 0 & \Delta A_d \end{bmatrix}, & \overline{K} &= \begin{bmatrix} 0 & 0 \\ -KD & 0 \end{bmatrix}. \end{aligned} \tag{12}$$

Remark 6. Because there is a big probability of the existence of errors between theoretical and practical systems because of unexpected factors in WSNs, in order to overcome this phenomenon, it is natural to assume system uncertainties. In addition, the system uncertainties may be the random changes in environmental circumstances, for example, network-induced random failures and repairs of components and sudden environmental disturbances. Therefore, we construct the system state observer (6) without the randomly occurring uncertainties according to the original system (1).

Remark 7. Compared with [37], it is clear the system error observer (11) has only one parameter which needs to be decided. In addition, because the uncertainty item is separated from the certainty item in (11), this makes it easier to take theoretical derivation.

The purpose of our work is to design a fault detection observer for a class of WNCSs, which has stochastic uncertainties and time delays, combined with nonlinear disturbance. In the following, we will utilize the Lyapunov stability theory to proceed the proof process in terms of LMI.

Besides, some useful and important lemmas that will be used in deriving out results will be introduced as below.

Lemma 8 (Schur complement). *Given constant matrices S_1, S_2, and S_3 where $S_1 = S_1^T$ and $0 < S_2 = S_2^T$, then $S_1 + S_3^T S_2^{-1} S_3 < 0$ if and only if*

$$\begin{bmatrix} S_1 & S_3^T \\ S_3 & -S_2 \end{bmatrix} < 0 \quad \text{or} \quad \begin{bmatrix} -S_2 & S_3 \\ S_3^T & S_1 \end{bmatrix} < 0. \tag{13}$$

Lemma 9. *For any $x, y \in R^n$, $\mu > 0$, the following inequality holds:*

$$2x^T y \leq \mu x^T x + \frac{1}{\mu} y^T y. \tag{14}$$

Lemma 10. *Let $Y = Y^T$, M, N, and $D(k)$ be real matrix of proper dimensions, and $D^T(k)D(k) \le I$; then inequality $Y + MDN + (MDN)^T < 0$ holds if there exists a constant ε, which makes the following inequality holds:*

$$Y + \varepsilon NN^T + \varepsilon^{-1}M^T M < 0 \tag{15}$$

or equivalently

$$\begin{bmatrix} Y & M & \varepsilon N^T \\ * & -\varepsilon I & 0 \\ * & * & -\varepsilon I \end{bmatrix} < 0. \tag{16}$$

3. Main Results

In this part, we will construct Lyapunov-Krasovskii functional and give sufficient condition such that the system error model in (11) could be asymptotically stable in mean square.

Theorem 11. *Consider the system* (1) *and suppose that the estimator parameters K are given. The system error model* (11) *is said to be asymptotically stable in mean square, if there exists positive definite matrices $P = \operatorname{diag}\{P_{11}, P_{22}\}$, $Q > 0$, $R > 0$, and scalars $\lambda > 0$, $\varepsilon_1 > 0$, $\varepsilon_2 > 0$ satisfying the following inequality:*

$$\Gamma = \begin{bmatrix} \Gamma_{11} & 0 & 0 & \Gamma_{14} & 0 & 0 \\ * & -Q & 0 & \Gamma_{24} & 0 & 0 \\ * & * & -\dfrac{2}{(1+N)N}R & \Gamma_{34} & 0 & 0 \\ * & * & * & \Gamma_{44} & \Gamma_{45} & 0 \\ * & * & * & * & -\varepsilon_1 I & \Gamma_{56} \\ * & * & * & * & * & -\varepsilon_2 I \end{bmatrix} < 0, \tag{17}$$

where

$$\begin{gathered} \Gamma_{11} = 4\lambda M^T M - P + (1+2d)Q + NR + \varepsilon_1 \overline{H}^T \overline{H} + \varepsilon_2 \overline{H}_d^T \overline{H}_d, \\ \Gamma_{14} = \left[2\overline{A}^T P \ \ 0 \ \ 0 \ \ 0 \ \ 0\right], \\ \Gamma_{24} = \left[0 \ \ 0 \ \ 2\overline{K}^T P \ \ 0 \ \ 0\right], \\ \Gamma_{34} = \left[0 \ \ 0 \ \ 0 \ \ 2\overline{A}_d^T P \ \ 0\right], \\ \Gamma_{44} = \operatorname{diag}\{-P, -P, -P, -P, -P\}, \\ \Gamma_{45}^T = \left[\alpha \overline{P}^T G \ \ \overline{\alpha}\,\overline{P}^T G \ \ 0 \ \ 0 \ \ 0\right], \\ \Gamma_{56}^T = \left[0 \ \ 0 \ \ \beta \overline{P}^T G \ \ \overline{\beta}\,\overline{P}^T G \ \ 0\right], \\ \overline{\alpha} = \sqrt{\alpha(1-\alpha)}, \qquad \overline{\beta} = \sqrt{\beta(1-\beta)}, \\ \lambda = \lambda_{\max}\left(\overline{E}_g^T P \overline{E}_g\right), \qquad M = \operatorname{diag}\{\gamma, \gamma\}, \\ \overline{H} = [H \ \ 0], \qquad \overline{H}_d = [H_d \ \ 0], \qquad \overline{P} = \begin{bmatrix} P_{11} \\ P_{22} \end{bmatrix}. \end{gathered} \tag{18}$$

Proof. For the stability analysis of the system (11), construct the following Lyapunov-Krasovskii functional:

$$V\{\eta(k)\} = V_1\{\eta(k)\} + V_2\{\eta(k)\} + V_3\{\eta(k)\}, \tag{19}$$

where

$$\begin{aligned} V_1\{\eta(k)\} &= \eta^T(k) P \eta(k), \\ V_2\{\eta(k)\} &= \sum_{l=k-d_k}^{k-1} \eta_l^T Q \eta_l + \sum_{j=-d}^{-d} \sum_{l=k+j}^{k-1} \eta_l^T Q \eta_l, \\ V_3\{\eta(k)\} &= \sum_{i=1}^{N} \sum_{l=k-i}^{k-1} \eta_l^T R \eta_l. \end{aligned} \tag{20}$$

By calculating the difference of $V\{\eta(k)\}$ along error dynamics (11), we have

$$\begin{aligned} &\mathbb{E}\{\Delta V_1\} \\ &\quad = \mathbb{E}\left\{\eta^T(k+1) P \eta(k+1) - \eta^T(k) P \eta(k)\right\} \\ &\quad = \eta^T(k)\left(\overline{A} + \alpha\Delta\overline{A}\right)^T P \left(\overline{A} + \alpha\Delta\overline{A}\right)\eta(k) \\ &\qquad + 2\eta^T(k)\left(\overline{A} + \alpha\Delta\overline{A}\right)^T P \left(\overline{A}_d + \beta\Delta\overline{A}_d\right)\sum_{i=1}^{N}\eta(k-i) \\ &\qquad + 2\eta^T(k)\left(\overline{A} + \alpha\Delta\overline{A}\right)^T P \overline{E}_g g(\eta(k)) \\ &\qquad + 2\eta^T(k)\left(\overline{A} + \alpha\Delta\overline{A}\right)^T P \overline{K}\eta(k-d_k) \\ &\qquad + \alpha(1-\alpha)\eta^T(k)\Delta\overline{A}^T P \Delta\overline{A}\eta(k) \\ &\qquad + \sum_{i=1}^{N}\eta^T(k-i)\left(\overline{A}_d + \beta\Delta\overline{A}_d\right)^T P \left(\overline{A}_d + \beta\Delta\overline{A}_d\right) \\ &\qquad \times \sum_{i=1}^{N}\eta^T(k-i) \\ &\qquad + 2\sum_{i=1}^{N}\eta^T(k-i)\left(\overline{A}_d + \beta\Delta\overline{A}_d\right)^T P \overline{E}_g g(\eta(k)) \\ &\qquad + 2\sum_{i=1}^{N}\eta^T(k-i)\left(\overline{A}_d + \beta\Delta\overline{A}_d\right)^T P \overline{K}\eta(k-d_k) \\ &\qquad + \beta(1-\beta)\sum_{i=1}^{N}\eta^T(k-i)\Delta\overline{A}_d^T P \Delta\overline{A}_d \\ &\qquad \times \sum_{i=1}^{N}\eta(k-i) + g^T(\eta(k))\overline{E}_g^T P \overline{E}_g g(\eta(k)) \\ &\qquad + 2g^T(\eta(k))\overline{E}_g^T P \overline{K}\eta(k-d_k) \\ &\qquad + \eta^T(k-d_k)\overline{K}^T P \overline{K}\eta(k-d_k) - \eta^T(k) P \eta(k). \end{aligned} \tag{21}$$

According to Lemma 9, we have

$$\begin{aligned} &2\eta^T(k)\left(\overline{A} + \alpha\Delta\overline{A}\right)^T P \left(\overline{A}_d + \beta\Delta\overline{A}_d\right)\sum_{i=1}^{N}\eta(k-i) \\ &\quad \le \eta^T(k)\left(\overline{A} + \alpha\Delta\overline{A}\right)^T P \left(\overline{A} + \alpha\Delta\overline{A}\right)\eta(k) \\ &\qquad + \sum_{i=1}^{N}\eta^T(k-i)\left(\overline{A}_d + \beta\Delta\overline{A}_d\right)^T P \left(\overline{A}_d + \beta\Delta\overline{A}_d\right) \\ &\qquad \times \sum_{i=1}^{N}\eta(k-i), \end{aligned}$$

$$
\begin{aligned}
&2\eta^T(k)\left(\overline{A}+\alpha\Delta\overline{A}\right)^T P\overline{E}_g g(\eta(k)) \\
&\quad\le \eta^T(k)\left(\overline{A}+\alpha\Delta\overline{A}\right)^T P\left(\overline{A}+\alpha\Delta\overline{A}\right)\eta(k) \\
&\qquad + g^T(\eta(k))\,\overline{E}_g^T P\overline{E}_g g(\eta(k)), \\
&2\eta^T(k)\left(\overline{A}+\alpha\Delta\overline{A}\right)^T P\overline{K}\eta(k-d_k) \\
&\quad\le \eta^T(k)\left(\overline{A}+\alpha\Delta\overline{A}\right)^T P\left(\overline{A}+\alpha\Delta\overline{A}\right)\eta(k) \\
&\qquad + \eta^T(k-d_k)\,\overline{K}^T P\overline{K}\eta(k-d_k), \\
&2\sum_{i=1}^{N}\eta^T(k-i)\left(\overline{A}_d+\beta\Delta\overline{A}_d\right)^T P\overline{E}_g g(\eta(k)) \\
&\quad\le \sum_{i=1}^{N}\eta^T(k-i)\left(\overline{A}_d+\beta\Delta\overline{A}_d\right)^T P\left(\overline{A}_d+\beta\Delta\overline{A}_d\right) \\
&\qquad \times\sum_{i=1}^{N}\eta(k-i) \\
&\qquad + g^T(\eta(k))\,\overline{E}_g^T P\overline{E}_g g(\eta(k)), \\
&2\sum_{i=1}^{N}\eta^T(k-i)\left(\overline{A}_d+\beta\Delta\overline{A}_d\right)^T P\overline{K}\eta(k-d_k) \\
&\quad\le \sum_{i=1}^{N}\eta^T(k-i)\left(\overline{A}_d+\beta\Delta\overline{A}_d\right)^T P\left(\overline{A}_d+\beta\Delta\overline{A}_d\right) \\
&\qquad \times\sum_{i=1}^{N}\eta(k-i) \\
&\qquad + \eta^T(k-d_k)\,\overline{K}^T P\overline{K}\eta(k-d_k), \\
&2g^T(\eta(k))\,\overline{E}_g^T P\overline{K}\eta(k-d_k) \\
&\quad\le g^T(\eta(k))\,\overline{E}_g^T P\overline{E}_g g(\eta(k)) \\
&\qquad + \eta^T(k-d_k)\,\overline{K}^T P\overline{K}\eta(k-d_k).
\end{aligned}
\tag{22}
$$

Besides, according to Assumption 4, we have

$$
\begin{aligned}
&g^T(\eta(k))\,\overline{E}_g^T P\overline{E}_g g(\eta(k)) \\
&\quad\le \lambda_{\max}\left(\overline{E}_g^T P\overline{E}_g\right) g^T(\eta(k))\,g(\eta(k)) \\
&\quad\le \lambda\eta^T(k)\,M^T M\eta(k), \\
&\mathbb{E}\{\Delta V_2\} \\
&\quad= \sum_{l=k+1-d_{k+1}}^{k}\eta(l)^T Q\eta(l) - \sum_{l=k-d_k}^{k-1}\eta(l)^T Q\eta(l) \\
&\qquad + \sum_{j=-d}^{d}\left(\sum_{l=k+1+j}^{k}\eta(l)^T Q\eta(l) - \sum_{l=k+j}^{k-1}\eta(l)^T Q\eta(l)\right) \\
&\quad= \eta(k)^T Q\eta(k) + \sum_{l=k+d+1}^{k-1}\eta(k)^T Q\eta(k) \\
&\qquad + \sum_{l=k+1-d_{k+1}}^{k+d}\eta(l)^T Q\eta(l) - \eta(k-d_k)^T Q\eta(k-d_k) \\
&\qquad - \sum_{l=k+1-d_k}^{k-1}\eta(l)^T Q\eta(l) \\
&\qquad + \sum_{j=-d}^{d}\left(\sum_{l=k+1+j}^{k-1}\eta(l)^T Q\eta(l) - \sum_{l=k+j}^{k-1}\eta(l)^T Q\eta(l) + \eta(l)^T Q\eta(l)\right) \\
&\quad= (2d+1)\,\eta(k)^T Q\eta(k) - \eta(k-d_k)^T Q\eta(k-d_k) \\
&\qquad + \sum_{l=k+d+1}^{k-1}\eta(l)^T Q\eta(l) \\
&\qquad + \sum_{l=k+1-d_{k+1}}^{k+d}\eta(l)^T Q\eta(l) - \sum_{l=k+1-d_k}^{k-1}\eta(l)^T Q\eta(l) \\
&\qquad - \sum_{l=k-d}^{k+d}\eta(l)^T Q\eta(l) \\
&\quad\le (2d+1)\,\eta(k)^T Q\eta(k) - \eta(k-d_k)^T Q\eta(k-d_k), \\
&\mathbb{E}\{\Delta V_3\} \\
&\quad= \sum_{i=1}^{N}\left(\sum_{l=k+1-i}^{k}\eta(l)^T R\eta(l) - \sum_{l=k-i}^{k-1}\eta(l)^T R\eta(l)\right) \\
&\quad= \sum_{i=1}^{N}\left(\eta(k)^T R\eta(k) - \eta(k-i)^T R\eta(k-i)\right) \\
&\quad\le N\eta(k)^T R\eta(k) \\
&\qquad - \frac{2}{(1+N)\,N}\left(\sum_{i=1}^{N}\eta(k-i)\right)^T R\left(\sum_{i=1}^{N}\eta(k-i)\right).
\end{aligned}
\tag{23}
$$

The combination of (19)–(23) results in

$$
\begin{aligned}
\mathbb{E}\{\Delta V\} &= \mathbb{E}\{\Delta V_1+\Delta V_2+\Delta V_3\} \\
&= Z^T(k)\,WZ(k),
\end{aligned}
\tag{24}
$$

where

$$
\begin{aligned}
Z(k) &= \left[\eta^T(k)\ \ \eta^T(k-d_k)\ \ \sum_{i=1}^{N}\eta^T(k-i)\right]^T, \\
W &= \begin{bmatrix} W_{11} & 0 & 0 \\ 0 & W_{22} & 0 \\ 0 & 0 & W_{33} \end{bmatrix}, \\
W_{11} &= 4\left(\overline{A}+\alpha\Delta\overline{A}\right)^T P\left(\overline{A}+\alpha\Delta\overline{A}\right) + \alpha(1-\alpha)\,\Delta\overline{A}^T P\Delta\overline{A} \\
&\quad + 4\lambda M^T M - P + (1+2d)\,Q + NR,
\end{aligned}
$$

$$W_{22} = 4\overline{K}^T P\overline{K} - Q,$$
$$W_{33} = 4\left(\overline{A}_d + \beta\Delta\overline{A}_d\right)^T P\left(\overline{A}_d + \beta\Delta\overline{A}_d\right) + \beta(1-\beta)\Delta\overline{A}_d^T P\Delta\overline{A}_d - \frac{2}{(1+N)N}R. \tag{25}$$

According to Lyapunov stability theory, system dynamic (11) the stable means

$$W < 0. \tag{26}$$

According to Lemma 8, (26) is equivalent to

$$\begin{bmatrix} \widetilde{\Gamma}_{11} & 0 & 0 & 2\left(\overline{A}+\alpha\Delta\overline{A}\right)^T & \overline{\alpha}\Delta\overline{A}^T & 0 & 0 & 0 \\ * & -Q & 0 & 0 & 0 & 2\overline{K}^T & 0 & 0 \\ * & * & -\frac{2}{(1+N)N}R & 0 & 0 & 0 & 2\left(\overline{A}_d+\beta\Delta\overline{A}_d\right)^T & \overline{\beta}\Delta\overline{A}_d^T \\ * & * & * & -P^{-1} & 0 & 0 & 0 & 0 \\ * & * & * & * & -P^{-1} & 0 & 0 & 0 \\ * & * & * & * & * & -P^{-1} & 0 & 0 \\ * & * & * & * & * & * & -P^{-1} & 0 \\ * & * & * & * & * & * & * & -P^{-1} \end{bmatrix} < 0, \tag{27}$$

where

$$\widetilde{\Gamma}_{11} = 4\lambda M^T M - P + (1+2d)Q + NR. \tag{28}$$

Multiply $\mathrm{diag}\{I, I, I, P, P, P, P, P\}$ on both ends of the matrix; then

$$\begin{bmatrix} \widetilde{\Gamma}_{11} & 0 & 0 & 2\left(\overline{A}+\alpha\Delta\overline{A}\right)^T P & \overline{\alpha}\Delta\overline{A}^T P & 0 & 0 & 0 \\ * & -Q & 0 & 0 & 0 & 2\overline{K}^T P & 0 & 0 \\ * & * & -\frac{2}{(1+N)N}R & 0 & 0 & 0 & 2\left(\overline{A}_d+\beta\Delta\overline{A}_d\right)^T P & \overline{\beta}\Delta\overline{A}_d^T P \\ * & * & * & -P & 0 & 0 & 0 & 0 \\ * & * & * & * & -P & 0 & 0 & 0 \\ * & * & * & * & * & -P & 0 & 0 \\ * & * & * & * & * & * & -P & 0 \\ * & * & * & * & * & * & * & -P \end{bmatrix} < 0. \tag{29}$$

Inequality (29) can be rewritten into the following form:

$$Y + MDN + (MDN)^T < 0, \tag{30}$$

where

$$Y = \begin{bmatrix} \widetilde{\Gamma}_{11} & 0 & 0 & 2\overline{A}^T P & 0 & 0 & 0 & 0 \\ * & -Q & 0 & 0 & 0 & 2\overline{K}^T P & 0 & 0 \\ * & * & -\frac{2}{(1+N)N}R & 0 & 0 & 0 & 2\left(\overline{A}_d+\beta\Delta\overline{A}_d\right)^T P & \overline{\beta}\Delta\overline{A}_d^T P \\ * & * & * & -P & 0 & 0 & 0 & 0 \\ * & * & * & * & -P & 0 & 0 & 0 \\ * & * & * & * & * & -P & 0 & 0 \\ * & * & * & * & * & * & -P & 0 \\ * & * & * & * & * & * & * & -P \end{bmatrix} < 0,$$
$$M^T = \begin{bmatrix} 0 & 0 & 0 & \alpha G^T\overline{P} & \overline{\alpha}G^T\overline{P} & 0 & 0 & 0 \end{bmatrix},$$
$$N = \begin{bmatrix} \overline{H} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \tag{31}$$

According to Lemma 10, inequality (30) is equivalent to

$$\begin{bmatrix} \widetilde{\Gamma}_{11}+\varepsilon_1\overline{H}^T\overline{H} & 0 & 0 & 2\overline{A}^T P & 0 & 0 & 0 & 0 & 0 \\ * & -Q & 0 & 0 & 0 & 2\overline{K}^T P & 0 & 0 & 0 \\ * & * & -\frac{2}{(1+N)N}R & 0 & 0 & 0 & 2\left(\overline{A}_d+\beta\Delta\overline{A}_d\right)^T P & \overline{\beta}\Delta\overline{A}_d^T P & 0 \\ * & * & * & -P & 0 & 0 & 0 & 0 & \alpha G^T\overline{P} \\ * & * & * & * & -P & 0 & 0 & 0 & \overline{\alpha}G^T\overline{P} \\ * & * & * & * & * & -P & 0 & 0 & 0 \\ * & * & * & * & * & * & -P & 0 & 0 \\ * & * & * & * & * & * & * & -P & 0 \\ * & * & * & * & * & * & * & * & -\varepsilon_1 I \end{bmatrix} < 0. \quad (32)$$

Then applying Lemma 10 again, we can acquire inequality (17), which means $W < 0$, and the proof of Theorem 11 is complete. □

Notice that the inequality (17) in Theorem 11 demonstrates the stability of the dynamic model (11); however, the gain of fault detection observer cannot be acquired; we will try to find an accessible way to solve the problem.

Theorem 12. *The system error model* (11) *is said to be asymptotically stable in mean square, if there exists positive definite matrices* $P = \operatorname{diag}\{P_{11}, P_{22}\}$, $Q > 0$, $R > 0$, *and scalars* $\lambda > 0$, $\varepsilon_1 > 0$, $\varepsilon_2 > 0$ *satisfying the following inequality:*

$$\begin{bmatrix} \Gamma_{11} & 0 & 0 & \Omega_1 & 0 & 0 \\ * & -Q & 0 & \Omega_2 & 0 & 0 \\ * & * & -\frac{2}{(1+N)N}R & \Gamma_{34} & 0 & 0 \\ * & * & * & \Gamma_{44} & \Gamma_{45} & 0 \\ * & * & * & * & -\varepsilon_1 I & \Gamma_{56} \\ * & * & * & * & * & -\varepsilon_2 I \end{bmatrix} < 0, \quad (33)$$

where

$$\Omega_1 = \begin{bmatrix}\Omega_{11} & 0 & 0 & 0 & 0\end{bmatrix},$$
$$\Omega_2 = \begin{bmatrix}0 & 0 & \Omega_{23} & 0 & 0\end{bmatrix},$$
$$\Omega_{11} = \begin{bmatrix} 2A^T P_{11} & 0 \\ 0 & 2A^T P_{22} - 2C^T X^T \end{bmatrix}, \quad (34)$$
$$\Omega_{23} = \begin{bmatrix} 0 & -2D^T X^T \\ 0 & 0 \end{bmatrix},$$

and Γ_{11}, Γ_{34}, Γ_{44}, Γ_{45}, *and* Γ_{56} *are defined in Theorem 11; in this case, the gain of fault detection observer is*

$$K = P_{22}^{-1} X. \quad (35)$$

Proof. We set $P_{22}K = X$, so $K = P_{22}^{-1}X$, and substitute it into Theorem 11; we can get the result easily, and the proof of Theorem 12 is complete. □

Remark 13. The main results in Theorems 11–12 can be applied to a wide class of wireless network control systems that involve randomly occurring uncertainties and time delays that result typically from networked environments. The LMI conditions are established to ensure the existence of the desired estimator gain, and the explicit expression of such estimator gain is characterized in terms of the solution to a LMI that can be effectively solved.

4. Numerical Simulations

In this section, we present an illustrative example to demonstrate the effectiveness of the proposed theorems. Consider the system model (1), where

$$A = \begin{bmatrix} 0.2122 & 0.2060 \\ 0.1936 & 0.1890 \end{bmatrix}, \quad A_d = \begin{bmatrix} -0.1195 & 0.0051 \\ -0.0098 & 0.0320 \end{bmatrix},$$
$$D(k) = \begin{bmatrix} 0.6\sin(0.4k) & 0 \\ 0 & 0.6\sin(0.4k) \end{bmatrix},$$
$$G = \begin{bmatrix} 0.0515 & 0.1030 \\ 0 & 0.1185 \end{bmatrix}, \quad H = \begin{bmatrix} 0.3708 & 0.2678 \\ 0.2163 & -0.3605 \end{bmatrix},$$
$$H_d = \begin{bmatrix} 0.0947 & 0.0577 \\ 0.0499 & -0.0824 \end{bmatrix},$$
$$E_g = \begin{bmatrix} -0.0108 \\ 0.0618 \end{bmatrix}, \quad E_f = \begin{bmatrix} 0.1391 \\ 0.0978 \end{bmatrix},$$
$$C = \begin{bmatrix} 0.9888 & 0.3793 \\ 0.4496 & 0.2637 \end{bmatrix}, \quad D = \begin{bmatrix} 0.1 & -0.15 \\ 0.17 & -0.11 \end{bmatrix}. \quad (36)$$

In addition, we suppose the nonlinear function $g(x(k)) = [0.25\sin x_1(k) \;\; 0.1x_2(k)]^T$, so we have $\gamma = \operatorname{diag}\{0.25, 0.1\}$. Besides, we set $N = 2$, $\alpha = 0.1$, $\beta = 0.15$, and $d(k) = 1 + \sin(\pi k/2)$.

Parameters can be acquired based on Theorems 11 and 12; we have

$$P = \begin{bmatrix} 101.2398 & -2.8940 & 0 & 0 \\ -2.8940 & 107.1766 & 0 & 0 \\ 0 & 0 & 143.0241 & -0.9776 \\ 0 & 0 & -0.9776 & 129.4239 \end{bmatrix},$$

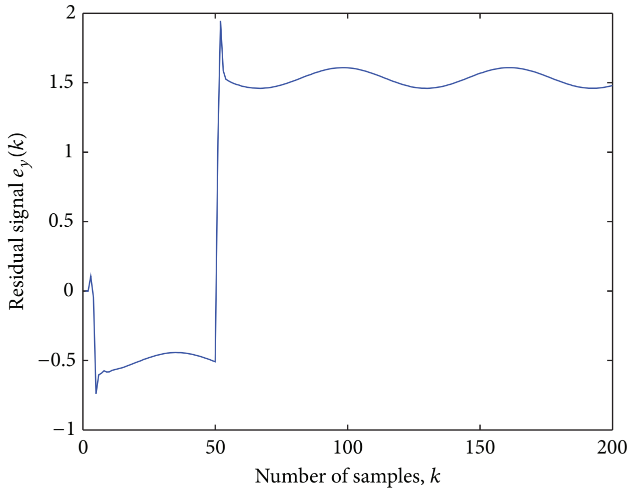

FIGURE 2: System residual signal with fault.

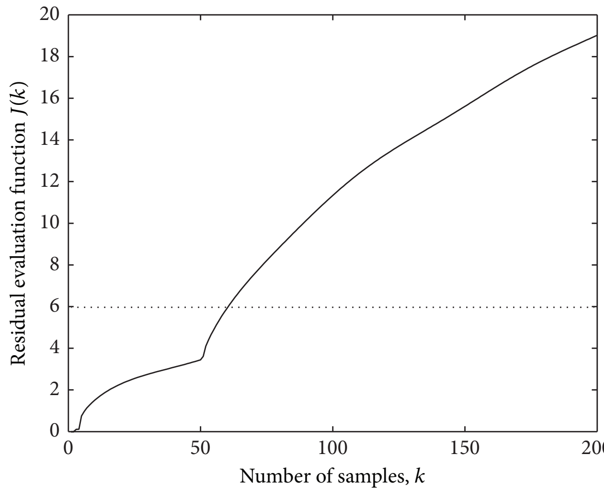

FIGURE 3: Fault detection with stochastic uncertainties and time delays.

$$Q = \begin{bmatrix} 1.8344 & 0.0332 & 23.0058 & 0.2020 \\ 0.0332 & 1.7967 & 0.2020 & 23.6888 \\ 23.0058 & 0.2020 & 186.6175 & -0.4714 \\ 0.2020 & 23.6888 & -0.4714 & 179.8521 \end{bmatrix},$$

$$R = \begin{bmatrix} 8.8777 & 0.0172 & 59.3935 & -0.0474 \\ 0.0172 & 9.1231 & -0.0474 & 52.0660 \\ 59.3935 & -0.0474 & -475.37 & -119.2 \\ -0.0474 & 52.0660 & -69.1 & -439.88 \end{bmatrix}. \quad (37)$$

So we have

$$K = \begin{bmatrix} 12.2533 & -76.2536 \\ 12.5628 & -75.1691 \end{bmatrix}. \quad (38)$$

When $k = 50$, we make $f(k) = 10$. Then the residual signal is shown in Figure 2. In addition, the threshold is selected as $J_{th} = \sup_{f(k)=0} \mathbb{E}\{\sum_{k=0}^{200} r^T(k) r(k)\}^{1/2}$, and accordingly, it can be obtained that $J_{th} = 5.9642$ in Figure 3. It can be clearly observed that the fault can be detected in 10 time steps after its occurrence and the designed observer can detect the system fault effectively when it occurs.

5. Conclusion

In this paper, we have considered the fault detection problem for a class of wireless networked control systems with randomly occurring uncertainties and time delays. A fault detection observer has been designed such that the fault detection dynamics is exponentially stable in the mean square and, at the same time, the error between the residual signal and the fault signal is made as small as possible. Sufficient conditions have been established via intensive stochastic analysis for the existence of the desired fault detection observer, and then the explicit expression of the desired observer gain has been derived by means of the feasibility of certain matrix inequality. Finally, simulation results demonstrate the effectiveness of proposed method.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

Acknowledgments

This work has been supported by the National Natural Science Foundation of China (Grant no. 61104109), the Natural Science Foundation of Jiangsu Province of China (Grant no. BK2011703), the Support of Science and Technology and Independent Innovation Foundation of Jiangsu Province of China (Grant no. BE2012178), the NUST Outstanding Scholar Supporting Program, and the Doctoral Fund of Ministry of Education of China (Grant no. 20113219110027).

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