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Research Article

Blow-Up Criteria for the Modified Novikov Equation

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We investigate the Cauchy problem for the modified Novikov equation. We establish blow-up criteria on the initial data to guarantee the corresponding solution blowing up in finite time.

1. Introduction

In this paper, we consider the following Cauchy problem of the modified Novikov equation:

$$\begin{aligned} &u_t - u_{xxt} + (a+b)\,u^2 u_x \\ &\quad = auu_x u_{xx} + bu^2 u_{xxx}, \quad t>0,\ x\in\mathbb{R}, \\ &u(0,x) = u_0(x), \quad x\in\mathbb{R}, \end{aligned} \tag{1}$$

where the coefficients a and b are positive constants.

In [1], Lai et al. presented the global existence of strong solutions and gave a blow-up scenario of strong solutions to the equation.

By using Green's function $G(x) = (1/2)e^{-x}$ for the operator $\Lambda = (1-\partial_x^2)^{1/2}$, (1) is equivalent to the nonlocal equation

$$\begin{aligned} &u_t + bu^2 u_x \\ &\quad = G * \left[-au^2 u_x + \frac{a-6b}{2}\left(uu_x^2\right)_x + \frac{2b-a}{2}u_x^3\right] \\ &\qquad t>0,\ x\in\mathbb{R}, \\ &u(0,x) = u_0(x), \quad x\in\mathbb{R}, \end{aligned} \tag{2}$$

where notation $*$ denotes the spatial convolution.

Letting $a = 3b$ and using the scaling translation $\tilde{u} = bu$, (1) can be reformulated into the Novikov equation

$$\tilde{u}_t - \tilde{u}_{xxt} + 4\tilde{u}^2\tilde{u}_x = 3\tilde{u}\tilde{u}_x\tilde{u}_{xx} + \tilde{u}^2 u_{xxx}, \tag{3}$$

which was derived by Novikov in a symmetry classification of nonlocal PDEs with quadratic or cubic nonlinearity [2]; subsequently, he found a scalar Lax pair for the Novikov equation (also see [3]) and proved that the Novikov equation is integrable. The equation has been investigated by many scholars. Hone and Wang gave a matrix Lax pair for the Novikov equation in [4] and showed how it was related by a reciprocal transformation to a negative flow in the Sawada-Kotera hierarchy. By using the matrix Lax pair, Hone et al. calculated the explicit formulas for multipeak on solutions of (1) in [3]. Ni and Zhou showed that the Novikov equation is well-posed in H^s, $s > 3/2$ by applying Kato's semigroup theory and the Novikov equation is locally well-posed in the Besov spaces $B^s_{2,r}$ with the critical index $s = 3/2$ and also considered the persistence properties of the solution. In [5], Jiang and Ni gave sufficient conditions on the initial data to guarantee the formulation of singularities in finite time and a global existence result was also established in [6]. It is worth pointing out recent many works have been done for the Novikov equation and the related equations, one can refer to [7–12] and the references therein.

Now, we give some elementary results and a blow-up scenario of strong solutions which will be used in this paper.

Theorem 1 (see [13]). *Given $u(x, t = 0) = u_0 \in H^s(\mathbb{R})$ with $s > 3/2$, then there exist a maximal $T = T(u_0) > 0$ and a unique solution u to* (1) *such that*

$$u = u(\cdot, u_0) \in C\left([0,T); H^s(\mathbb{R})\right) \cap C^1\left([0,T); H^{s-1}(\mathbb{R})\right). \tag{4}$$

Moreover, the solution depends continuously on the initial data; that is, the mapping $u_0 \to u(\cdot, u_0) : H^s(\mathbb{R}) \to C([0,T); H^s(\mathbb{R})) \cap C^1([0,T); H^{s-1}(\mathbb{R}))$ *is continuous.*

Theorem 2 (see [1]). *Assume* $u_0(x) \in H^s$, $s \geq 3/2$, *and let* T *be the maximal existence time of the solution* $u(x,t)$ *to* (1) *with the initial data* $u_0(x)$. *If* $a > b$, *then the corresponding solution blows up in finite time if and only if*

$$\lim_{t\uparrow T} \liminf_{x\in\mathbb{R}} (uu_x)(x,t) = -\infty. \tag{5}$$

We also need to introduce the classical particle trajectory method. Suppose $u(x,t)$ is a solution of the Novikov equation; let $q(x,t)$ be the particle line evolved by the solution u:

$$\frac{dq(x,t)}{dt} = bu^2(q(x,t),t), \qquad q(x,t=0) = x. \tag{6}$$

Then

$$q_x(x,t) = \exp\left(2\int_0^t uu_x(q,s)\,ds\right), \qquad q_x(x,0) = 1, \tag{7}$$

which is always positive before the blow-up time. Therefore, the function $q(x,t)$ is an increasing diffeomorphism of the line before blow-up.

Let $y = \Lambda^2 u = (1-\partial_x^2)u$; the following identity can be obtained:

$$y(q(x,t),t)\,q_x^2(x,t) = y_0(x)\,e^{(4b-a)\int_0^t uu_x ds}. \tag{8}$$

In fact, direct computation yields

$$\begin{aligned}
&\frac{d}{dt}\left(y(q)q_x^2\right)\\
&= y_t q_x^2 + 2yq_xq_{xt} + y_xq_tq_x^2\\
&= y_tq_x^2 + 4byuu_xq_x^2 + bu^2y_xq_x^2\\
&= \left(u_t - u_{txx} + auu_x(u-u_{xx}) + bu^2(u_x - u_{xxx})\right)q_x^2\\
&\quad - auu_xyq_x^2 + 4buu_xyq_x^2\\
&= (4b-a)\,uu_xyq_x^2.
\end{aligned} \tag{9}$$

Remark 3. From (8), it follows that if $y_0(x) = \Lambda^2u_0(x) \geq 0$ then $y(t,x) = \Lambda^2u(t,x) \geq 0$. Since $\Lambda^{-2}f = G * f$, for $f \in L^2(\mathbb{R})$, therefore, we obtain $u(x) \geq 0$. If $y_0 = (1-\partial_x^2)u_0(x) \leq 0$, the result is similar.

2. Blow-Up Criteria

In this section, we present the following blow-up criteria on the initial data to guarantee that the corresponding strong solution of (1) blowing up. Our method is partially motivated by [14].

Theorem 4. *If* $a > 3b$, *suppose that* $u_0 \in H^s(\mathbb{R})$, $s > 3/2$, *and there exists* $x_0 \in \mathbb{R}$ *such that* $u_0(x_0) \geq 0$ *and* $y_0(x_0) = (1-\partial_x^2)u_0(x_0) = 0$,

$$\begin{aligned}
&y_0(x) \geq 0\ (\not\equiv 0) \quad \textit{for } x \in (-\infty, x_0),\\
&y_0(x) \leq 0\ (\not\equiv 0) \quad \textit{for } x \in (x_0, \infty).
\end{aligned} \tag{10}$$

Then the corresponding solution $u(x,t)$ *to the modified Novikov equation* (1) *with* $u_0(x)$ *as the initial datum blows up in finite time.*

Proof. By the local well-posedness theorem and a density argument, it suffices to consider the case $s \geq 3$; without loss of generality, we take $s = 3$ for simplicity of notation. We also assume $u_0 \not\equiv 0$; otherwise, solutions are trivial.

Suppose that the solution exists globally. Due to (8) and the initial condition (10), we have

$$\begin{aligned}
&y(q(x_0,t),t) = 0,\\
&y(q(x,t),t) \geq 0\ (\not\equiv 0), \quad \text{for } x \in (-\infty, x_0),\\
&y(q(x,t),t) \leq 0\ (\not\equiv 0), \quad \text{for } x \in (x_0, \infty),
\end{aligned} \tag{11}$$

for all $t \geq 0$. Since $u(x,t) = G * y(x,t)$, $x \in \mathbb{R}$, $t > 0$, we can write $u(x,t)$ and $u_x(x,t)$ as follows:

$$\begin{aligned}
u(x,t) &= \frac{1}{2}e^{-x}\int_{-\infty}^x e^{\xi}y(\xi,t)\,d\xi + \frac{1}{2}e^{x}\int_x^{\infty} e^{-\xi}y(\xi,t)\,d\xi,\\
u_x(x,t) &= -\frac{1}{2}e^{-x}\int_{-\infty}^x e^{\xi}y(\xi,t)\,d\xi + \frac{1}{2}e^{x}\int_x^{\infty} e^{-\xi}y(\xi,t)\,d\xi.
\end{aligned} \tag{12}$$

As a result of (12)

$$(u+u_x)(x,t) = e^x\int_x^{\infty} e^{-\xi}y(\xi,t)\,d\xi, \tag{13}$$

$$(u-u_x)(x,t) = e^{-x}\int_{-\infty}^x e^{\xi}y(\xi,t)\,d\xi, \tag{14}$$

for all $t \geq 0$.

From (12) and differentiating $uu_x(q(x_0,t),t)$ with respect to t, we have

$$\begin{aligned}
&\frac{d}{dt}(2uu_x)(q(x_0,t),t)\\
&\quad= \frac{d}{dt}\left\{-\frac{1}{2}e^{-2q(x_0,t)}\left(\int_{-\infty}^{q(x_0,t)} e^{\xi}y(\xi,t)d\xi\right)^2\right.\\
&\qquad\left.+\frac{1}{2}e^{2q(x_0,t)}\left(\int_{q(x_0,t)}^{\infty} e^{-\xi}y(\xi,t)d\xi\right)^2\right\}
\end{aligned}$$

$$
\begin{aligned}
&= bu^2(q(x_0,t),t)\,e^{-2q(x_0,t)}\left(\int_{-\infty}^{q(x_0,t)} e^{\xi} y(\xi,t)\,d\xi\right)^2 \\
&\quad - e^{-2q(x_0,t)}\int_{-\infty}^{q(x_0,t)} e^{\xi} y(\xi,t)\,d\xi \int_{-\infty}^{q(x_0,t)} e^{\xi} y_t(\xi,t)\,d\xi \\
&\quad + bu^2(q(x_0,t),t)\,e^{2q(x_0,t)}\left(\int_{q(x_0,t)}^{\infty} e^{-\xi} y(\xi,t)\,d\xi\right)^2 \\
&\quad + e^{2q(x_0,t)}\int_{q(x_0,t)}^{\infty} e^{-\xi} y(\xi,t)\,d\xi \int_{q(x_0,t)}^{\infty} e^{-\xi} y_t(\xi,t)\,d\xi \\
&= bu^2(u-u_x)^2(q(x_0,t),t) \\
&\quad - (u-u_x)(q(x_0,t),t)\,e^{-q(x_0,t)}\int_{-\infty}^{q(x_0,t)} e^{\xi} y_t(\xi,t)\,d\xi \\
&\quad + bu^2(u+u_x)^2(q(x_0,t),t) \\
&\quad + (u+u_x)(q(x_0,t),t)\,e^{q(x_0,t)}\int_{q(x_0,t)}^{\infty} e^{-\xi} y_t(\xi,t)\,d\xi.
\end{aligned} \tag{15}
$$

Equation (1) can be rewritten as

$$y_t = -b\left(yu^2\right)_x + (2b-a)\,yuu_x. \tag{16}$$

Firstly, we can estimate the first term as

$$
\begin{aligned}
&e^{-q(x_0,t)}\int_{-\infty}^{q(x_0,t)} e^{\xi} y_t(\xi,t)\,d\xi \\
&= -be^{-q(x_0,t)}\int_{-\infty}^{q(x_0,t)} e^{\xi}\left(y(\xi,t)\,u^2(\xi,t)\right)_{\xi} d\xi \\
&\quad + (2b-a)\,e^{-q(x_0,t)}\int_{-\infty}^{q(x_0,t)} e^{\xi} y(\xi,t)\,u(\xi,t)\,u_{\xi}(\xi,t)\,d\xi \\
&= be^{-q(x_0,t)}\int_{-\infty}^{q(x_0,t)} e^{\xi} y(\xi,t)\,u^2(\xi,t)\,d\xi \\
&\quad + (2b-a)\,e^{-q(x_0,t)}\int_{-\infty}^{q(x_0,t)} e^{\xi} y(\xi,t)\,u(\xi,t)\,u_{\xi}(\xi,t)\,d\xi \\
&= be^{-q(x_0,t)}\int_{-\infty}^{q(x_0,t)} e^{\xi}\left(u^3 - u^2u_{xx}\right) d\xi \\
&\quad + (2b-a)\,e^{-q(x_0,t)}\int_{-\infty}^{q(x_0,t)} e^{\xi}\left(u^2u_x - uu_xu_{xx}\right) d\xi \\
&= \frac{3b-a}{3}u^3 - bu^2u_x - \frac{2b-a}{2}uu_x^2 \\
&\quad + \frac{a}{3}e^{-q(x_0,t)}\int_{-\infty}^{q(x_0,t)} e^{\xi}u^3\,d\xi \\
&\quad + \frac{6b-a}{2}e^{-q(x_0,t)}\int_{-\infty}^{q(x_0,t)} e^{\xi}uu_x^2\,d\xi \\
&\quad + \frac{2b-a}{2}e^{-q(x_0,t)}\int_{-\infty}^{q(x_0,t)} e^{\xi}u_x^3\,d\xi.
\end{aligned} \tag{17}
$$

We also apply the following inequality in [6]:

$$
\begin{aligned}
&\int_{-\infty}^{q(x_0,t)} e^{\xi}\left(2u^3 + 3uu_{\xi}^2 - u_{\xi}^3\right)(\xi,t)\,d\xi \\
&\qquad \geq e^{q(x_0,t)}u^3(q(x_0,t),t).
\end{aligned} \tag{18}
$$

So we can derive

$$
\begin{aligned}
&\frac{a}{3}e^{-q(x_0,t)}\int_{-\infty}^{q(x_0,t)} e^{\xi}u^3\,d\xi \\
&\quad + \frac{6b-a}{2}e^{-q(x_0,t)}\int_{-\infty}^{q(x_0,t)} e^{\xi}uu_x^2\,d\xi \\
&\quad + \frac{2b-a}{2}e^{-q(x_0,t)}\int_{-\infty}^{q(x_0,t)} e^{\xi}u_x^3\,d\xi \\
&= \frac{1}{6}e^{-q(x_0,t)}\int_{-\infty}^{q(x_0,t)} e^{\xi}\left[(a-3b)\left(2u^3 - 3uu_x^2 - 3u_x^3\right)\right. \\
&\qquad \left. +3b\left(2u^3 + 3uu_x^2 - 3u_x^3\right)\right] d\xi \\
&\geq \frac{a-3b}{6}u^3 + \frac{3b}{6}u^3 = \frac{a}{6}u^3.
\end{aligned} \tag{19}
$$

Putting (19) into (17), we have

$$
\begin{aligned}
&e^{-q(x_0,t)}\int_{-\infty}^{q(x_0,t)} e^{\xi} y_t(\xi,t)\,d\xi \\
&\quad \geq -bu^2(q(x_0,t),t)\,u_x(q(x_0,t),t) \\
&\qquad - \frac{2b-a}{2}u(q(x_0,t),t)\,u_x^2(q(x_0,t),t) \\
&\qquad + \frac{6b-a}{6}u^3(q(x_0,t),t).
\end{aligned} \tag{20}
$$

Similarly, we have

$$
\begin{aligned}
&e^{q(x_0,t)}\int_{q(x_0,t)}^{\infty} e^{-\xi} y_t(\xi,t)\,d\xi \\
&\quad \geq -bu^2(q(x_0,t),t)\,u_x(q(x_0,t),t) \\
&\qquad + \frac{2b-a}{2}u(q(x_0,t),t)\,u_x^2(q(x_0,t),t) \\
&\qquad + \frac{a-6b}{6}u^3(q(x_0,t),t).
\end{aligned} \tag{21}
$$

Putting (21) and (22) into (15), we obtain

$$\begin{aligned}
&\frac{d}{dt}2uu_x\left(q\left(x_0,t\right),t\right)\\
&\quad\le 2b\left(u^4+u^2u_x^2\right)\left(q\left(x_0,t\right),t\right)\\
&\qquad-\left(u-u_x\right)\left(\frac{6b-a}{6}u^3-bu^2u_x-\frac{2b-a}{2}uu_x^2\right)\\
&\qquad+\left(u+u_x\right)\left(-\frac{6b-a}{6}u^3-bu^2u_x+\frac{2b-a}{2}uu_x^2\right)\\
&\quad=\frac{a}{3}u^2\left(u^2-u_x^2\right)\left(q\left(x_0,t\right),t\right).
\end{aligned}\tag{22}$$

Here we use the facts that $(u-u_x)(q(x,t),t)\ge 0$, $x\in(-\infty,q(x_0,t))$, from (10) and (14), and $(u+u_x)(q(x,t),t)\le 0$, $x\in(q(x_0,t),\infty)$, from (10) and (13).

Claim 1. $uu_x(q(x_0,t),t)<0$ is decreasing and $u^2(q(x_0,t),t)<u_x^2(q(x_0,t),t)$ for all $t\ge 0$.

Suppose that there exists a t_0 such that $u^2(q(x_0,t),t)<u_x^2(q(x_0,t),t)$ on $[0,t_0)$ and $u^2(q(x_0,t_0),t_0)\ge u_x^2(q(x_0,t_0),t_0)$. Now, let

$$\begin{aligned}
I(t)&:=u\left(u-u_x\right)\left(q\left(x_0,t\right),t\right),\\
II(t)&:=u\left(u+u_x\right)\left(q\left(x_0,t\right),t\right).
\end{aligned}\tag{23}$$

Firstly, differentiating $I(t)$, we get

$$\begin{aligned}
\frac{dI(t)}{dt}&=-bu^2\left(q\left(x_0,t\right),t\right)\\
&\quad\times\left(e^{-q(x_0,t)}\int_{-\infty}^{q(x_0,t)}e^{\xi}y(\xi,t)d\xi\right)^2\\
&\quad+\left(e^{-q(x_0,t)}\int_{-\infty}^{q(x_0,t)}e^{\xi}y(\xi,t)\,d\xi\right)\\
&\quad\times\left(e^{-q(x_0,t)}\int_{-\infty}^{q(x_0,t)}e^{\xi}y_t(\xi,t)\,d\xi\right)\\
&\quad+\frac{1}{2}\int_{-\infty}^{q(x_0,t)}e^{\xi}y_t(\xi,t)\,d\xi\int_{q(x_0,t)}^{\infty}e^{-\xi}y(\xi,t)\,d\xi\\
&\quad+\frac{1}{2}\int_{-\infty}^{q(x_0,t)}e^{\xi}y(\xi,t)\,d\xi\int_{q(x_0,t)}^{\infty}e^{-\xi}y_t(\xi,t)\,d\xi\\
&\ge -bu^2\left(u-u_x\right)^2\left(q\left(x_0,t\right),t\right)\\
&\quad+\left(\frac{6b-a}{6}u^3-bu^2u_x-\frac{2b-a}{2}uu_x^2\right)u\left(q\left(x_0,t\right),t\right)\\
&\quad+\frac{1}{2}\left(\frac{6b-a}{6}u^3-bu^2u_x-\frac{2b-a}{2}uu_x^2\right)\\
&\quad\times\left(u-u_x\right)\left(q\left(x_0,t\right),t\right)\\
&\quad-\frac{1}{2}\left(\frac{6b-a}{6}u^3+bu^2u_x-\frac{2b-a}{2}uu_x^2\right)\\
&\quad\times\left(u-u_x\right)\left(q\left(x_0,t\right),t\right)\\
&\ge\frac{a}{6}u^2\left(u_x^2-u^2\right),\quad\text{on }[0,t_0).
\end{aligned}\tag{24}$$

Secondly, differentiating $II(t)$, we get

$$\begin{aligned}
\frac{dII(t)}{dt}&=bu^2\left(q\left(x_0,t\right),t\right)\\
&\quad\times\left(e^{q(x_0,t)}\int_{q(x_0,t)}^{\infty}e^{-\xi}y(\xi,t)d\xi\right)^2\\
&\quad+\left(e^{q(x_0,t)}\int_{q(x_0,t)}^{\infty}e^{-\xi}y(\xi,t)\,d\xi\right)\\
&\quad\times\left(e^{q(x_0,t)}\int_{q(x_0,t)}^{\infty}e^{-\xi}y_t(\xi,t)\,d\xi\right)\\
&\quad+\frac{1}{2}\int_{-\infty}^{q(x_0,t)}e^{\xi}y_t(\xi,t)\,d\xi\int_{q(x_0,t)}^{\infty}e^{-\xi}y(\xi,t)\,d\xi\\
&\quad+\frac{1}{2}\int_{-\infty}^{q(x_0,t)}e^{\xi}y(\xi,t)\,d\xi\int_{q(x_0,t)}^{\infty}e^{-\xi}y_t(\xi,t)\,d\xi\\
&\le bu^2\left(u+u_x\right)^2\left(q\left(x_0,t\right),t\right)\\
&\quad-\left(\frac{6b-a}{6}u^3+bu^2u_x-\frac{2b-a}{2}uu_x^2\right)\\
&\quad\times u_x\left(q\left(x_0,t\right),t\right)\\
&\quad+\frac{1}{2}\left(\frac{6b-a}{6}u^3-bu^2u_x-\frac{2b-a}{2}uu_x^2\right)\\
&\quad\times\left(u+u_x\right)\left(q\left(x_0,t\right),t\right)\\
&\quad-\frac{1}{2}\left(\frac{6b-a}{6}u^3+bu^2u_x-\frac{2b-a}{2}uu_x^2\right)\\
&\quad\times\left(u+u_x\right)\left(q\left(x_0,t\right),t\right)\\
&=-\frac{a}{6}u^2\left(u_x^2-u^2\right),\quad\text{on }[0,t_0).
\end{aligned}\tag{25}$$

Hence, from (24), (25), and the continuity property of ODEs, we can draw

$$u^2\left(u_x^2-u^2\right)\left(q\left(x_0,t\right),t\right)=-I(t)\,II(t)>-I(0)\,II(0)>0,\tag{26}$$

for all $t>0$. This means t_0 can be extended to infinity. This is a contradiction, so the claim is true.

Moreover, using (24) and (25) again, we have the following inequality for $u^2(u_x^2 - u^2)(q(x_0, t), t)$:

$$\begin{aligned} &\frac{d}{dt} u^2 \left(u_x^2 - u^2\right)\left(q\left(x_0, t\right), t\right) \\ &\quad \geq -\frac{a}{3} u u_x \left(q\left(x_0, t\right), t\right) u^2 \left(u_x^2 - u^2\right)\left(q\left(x_0, t\right), t\right). \end{aligned} \tag{27}$$

Putting (22) into (27) yields

$$\begin{aligned} &\frac{d}{dt} u^2 \left(u_x^2 - u^2\right)\left(q\left(x_0, t\right), t\right) \\ &\quad \geq \frac{a}{3} u^2 \left(u_x^2 - u^2\right)\left(q\left(x_0, t\right), t\right) \\ &\qquad \times \left(\frac{a}{6}\int_0^t u^2\left(u_x^2 - u^2\right)\left(q\left(x_0, s\right), s\right) ds - \frac{a}{6} u_0 u_{0x}\left(x_0\right)\right). \end{aligned} \tag{28}$$

Before completing the proof, we want the following technical lemma.

Lemma 5 (see [15]). *Suppose that Φ is a twice continuous differential satisfying*

$$\begin{gathered} \Phi''(t) \geq C_0 \Phi'(t)\Phi(t), \quad t > 0,\ C_0 > 0, \\ \Phi(0) > 0, \qquad \Phi'(0) > 0. \end{gathered} \tag{29}$$

Then $\Phi(t)$ blows up in finite time. Moreover the blow-up time T can be estimated in terms of the initial datum as

$$T \leq \max\left\{\frac{2}{C_0 \Phi(0)}, \frac{\Phi(0)}{\Phi'(0)}\right\}. \tag{30}$$

Let $\Phi(t) = \int_0^t u^2(u_x^2 - u^2)(q(x_0, s), s) ds - u_0 u_{0x}(x_0)$; then (28) is an equation of type (29) with $C_0 = a^2/18$. The proof is complete by applying Lemma 5. □

When we change the signs of $u_0(x_0)$ and $y_0(x)$ in Theorem 4, similarly, we have the following blow-up criterion.

Theorem 6. *If $a > 3b$, suppose that $u_0 \in H^s(\mathbb{R})$, $s > (3/2)$, and there exists a $x_0 \in \mathbb{R}$ such that $u_0(x_0) \leq 0$ and $y_0(x_0) = (1 - \partial_x^2)u_0(x_0) = 0$,*

$$\begin{gathered} y_0(x) \leq 0\ (\not\equiv 0) \quad \text{for } x \in \left(-\infty, x_0\right), \\ y_0(x) \geq 0\ (\not\equiv 0) \quad \text{for } x \in \left(x_0, \infty\right). \end{gathered} \tag{31}$$

Then the corresponding solution $u(x, t)$ to the modified Novikov equation (1) *with $u_0(x)$ as the initial datum blows up in finite time.*

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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