Hindawi Publishing Corporation
Abstract and Applied Analysis
Volume 2014, Article ID 240784, 4 pages
http://dx.doi.org/10.1155/2014/240784

Letter to the Editor

Comment on "New Exact Solutions to the KdV-Burgers-Kuramoto Equation with the Exp-Function Method"

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Received 4 December 2013; Accepted 21 January 2014; Published 24 February 2014

Academic Editor: Ahmed El-Sayed

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We point out in this paper that the claims made by Kim et al. in the commented paper are incorrect and no new exact solution was obtained.

1. Introduction

In [1], Kim and Chun had investigated exact solutions to the following KdV-Burgers-Kuramoto equation:

$$u_t + \nu u u_x + \mu u u_{xxx} + \alpha u_{xx} + \gamma u_{xxxx} = 0. \tag{1}$$

They took traveling wave transformation $u = u(\eta)$ with $\eta = kx + \omega t$ into account and transformed (1) into an ordinary differential equation:

$$\omega u' + k\nu u u' + k^2 \alpha u'' + k^3 \mu u''' + k^4 \gamma u'''' = 0. \tag{2}$$

After implementing the Exp-function method [2] based on the truncated Painlevé, they had constructed the following four new generalized solitary wave solutions to (1).

Case 1. Consider the following:

$$u(x,t) = \frac{15k}{76\gamma\nu} \frac{c_3 Y^6 + c_2 Y^5 + c_1 Y^4 + c_0 Y^3 + c_{-1} Y^2 + c_{-2} Y + c_{-3}}{(b_0 + b_1 Y)^6}, \tag{3}$$

where

$$\begin{gathered} c_{-3} = \left(\alpha^2 - 16\mu\gamma\right) b_1^6, \qquad c_{-2} = \left(6\alpha^2 - 96\mu\gamma\right) b_1^5 b_0, \\ c_{-1} = \left(15\alpha^2 - 240\mu\gamma\right) b_1^4 b_0^2, \qquad c_0 = \left(20\alpha^2 - 320\mu\gamma\right) b_1^3 b_0^3, \\ c_1 = \left(15\alpha^2 - 240\mu\gamma\right) b_1^2 b_0^4, \qquad c_2 = \left(6\alpha^2 - 96\mu\gamma\right) b_1 b_0^5, \\ c_3 = \left(\alpha^2 - 16\mu\gamma\right) b_0^6, \end{gathered} \tag{4}$$

$Y \triangleq \exp(\eta)$, as in the following, is an introduced variable, and k, ω, b_0, and b_1 are arbitrary constants.

Other Three Cases. Consider

$$u(x,t) = K \cdot \frac{c_3 Y^6 + c_2 Y^5 + c_1 Y^4 + c_0 Y^3 + c_{-1} Y^2 + c_{-2} Y + c_{-3}}{(b_0 + b_{-1} Y)^6}, \tag{5}$$

where

$$c_{-3} = \left(\alpha^2 - 16\mu\gamma\right) b_0^6, \qquad c_{-2} = \left(6\alpha^2 - 96\mu\gamma\right) b_0^5 b_{-1},$$
$$c_{-1} = \left(15\alpha^2 - 240\mu\gamma\right) b_0^4 b_{-1}^2, \qquad c_0 = \left(20\alpha^2 - 320\mu\gamma\right) b_0^3 b_{-1}^3,$$
$$c_1 = \left(15\alpha^2 - 240\mu\gamma\right) b_0^2 b_{-1}^4, \qquad c_2 = \left(6\alpha^2 - 96\mu\gamma\right) b_0 b_{-1}^5,$$
$$c_3 = \left(\alpha^2 - 16\mu\gamma\right) b_{-1}^6, \tag{6}$$

and b_0, b_{-1} are arbitrary constants.

Case 2. $K = -15k/76\gamma\nu$, k is an arbitrary constant, and ω subjects to

$$\omega = \frac{k^2\left(15\alpha^2 - 76\alpha k\gamma - 76\gamma^2k^2 - 316\mu\gamma\right)}{76\gamma}. \tag{7}$$

Case 3. $K = -15k/76\gamma\nu$, and k and ω subject to

$$k = \pm\frac{\sqrt{-\mu\gamma}}{\gamma}, \qquad \omega = \frac{\mu\left(240\mu\gamma \pm 76\alpha\sqrt{-\mu\gamma} - 15\alpha^2\right)}{76\gamma^2}. \tag{8}$$

Case 4. $K = -15k/2888\gamma\nu$, and k and ω subject to

$$k = \frac{-19\alpha\gamma \pm \sqrt{437\alpha^2\gamma^2 - 1216\gamma^3\mu}}{152\gamma^2},$$
$$\omega = \frac{20848\mu\alpha^2\gamma + 39936\mu^2\gamma^2 + 707\alpha^4}{-184832\gamma^3} - \frac{\left(1168\gamma\alpha\mu - 41\alpha^3\right)\sqrt{437\alpha^2 - 1216\mu\gamma}}{184832\gamma^3}. \tag{9}$$

The authors claimed that the above four solutions could not be directly constructed form the Exp-function method.

They also claimed that they had obtained a new solitary wave solution in the case where $p = q = 2$ and $d = q = 2$, which was given by

$$u(x,t) = \frac{a_2\exp(2\eta) + a_2b_1\exp(\eta) + a_2b_0 + a_2b_{-1}\exp(-\eta)}{\exp(2\eta) + b_1\exp(\eta) + b_0 + b_{-1}\exp(-\eta)}. \tag{10}$$

2. Comment and Analysis

In this section, we will analyze the claims by Kim and Chun in [1].

2.1. Comment 1. It is not difficult to rewrite (3) and (5) in the following form:

$$u(x,t) = \frac{A_6Y^6 + A_5Y^5 + A_4Y^4 + A_3Y^3 + A_2Y^2 + A_1Y + A_0}{B_6Y^6 + B_5Y^5 + B_4Y^4 + B_3Y^3 + B_2Y^2 + B_1Y + B_0}, \tag{11}$$

where A_i and B_i are certain constants.

Therefore, we have

$$\begin{aligned} u(x,t) &= \left(A_6Y^6 + A_5Y^5 + A_4Y^4 + A_3Y^3 + A_2Y^2 + A_1Y + A_0\right) \\ &\quad\times\left(B_6Y^6 + B_5Y^5 + B_4Y^4 + B_3Y^3 + B_2Y^2 + B_1Y + B_0\right)^{-1} \\ &= (A_6\exp(6\eta) + A_5\exp(5\eta) + A_4\exp(4\eta) \\ &\quad + A_3\exp(3\eta) + A_2\exp(2\eta) + A_1\exp(\eta) + A_0) \\ &\quad\times(B_6\exp(6\eta) + B_5\exp(5\eta) + B_4\exp(4\eta) \\ &\quad + B_3\exp(3\eta) + B_2\exp(2\eta) + B_1\exp(\eta) + B_0)^{-1} \\ &= (A_6\exp(5\eta) + A_5\exp(4\eta) + A_4\exp(3\eta) \\ &\quad + A_3\exp(2\eta) + A_2\exp(\eta) + A_1 + A_0\exp(-\eta)) \\ &\quad\times(B_6\exp(5\eta) + B_5\exp(4\eta) + B_4\exp(3\eta) \\ &\quad + B_3\exp(2\eta) + B_2\exp(\eta) + B_1 + B_0\exp(-\eta))^{-1} \\ &= (A_6\exp(4\eta) + A_5\exp(3\eta) + A_4\exp(2\eta) \\ &\quad + A_3\exp(\eta) + A_2 + A_1\exp(-\eta) + A_0\exp(-2\eta)) \\ &\quad\times(B_6\exp(4\eta) + B_5\exp(3\eta) + B_4\exp(2\eta) \\ &\quad + B_3\exp(\eta) + B_2 + B_1\exp(-\eta) + B_0\exp(-2\eta))^{-1} \\ &= (A_6\exp(3\eta) + A_5\exp(2\eta) + A_4\exp(\eta) + A_3 \\ &\quad + A_2\exp(-\eta) + A_1\exp(-2\eta) + A_0\exp(-3\eta)) \\ &\quad\times(B_6\exp(3\eta) + B_5\exp(2\eta) + B_4\exp(\eta) + B_3 \\ &\quad + B_2\exp(-\eta) + B_1\exp(-2\eta) + B_0\exp(-3\eta))^{-1}. \end{aligned} \tag{12}$$

According to the Exp-function method [2], we can reobtain the solution (11) by assuming that the solution of (2) can be expressed in the form

$$u(\eta) = \frac{M_{-d}\exp(-d\eta) + \cdots + M_c\exp(c\eta)}{N_{-q}\exp(-q\eta) + \cdots + N_p\exp(p\eta)}, \tag{13}$$

where (i) $p = c = 5$ and $d = q = 1$, (ii) $p = c = 4$ and $d = q = 2$, (iii) $p = c = d = q = 3$, respectively. It is worth to mention that the fact that the three cases of (i), (ii), and (iii) are equivalent has been emphasized in [3–5].

Thus the claim in the commented paper that "new generalized solitary wave solutions are constructed for the KdV-Burgers-Kuramoto equation, which cannot be directly constructed from the Exp-function method" is not true.

2.2. Comment 2. In this section, we show that the solutions in the mentioned four cases are incorrect. Here, we should point out that it is difficult for us to solve original algebra system appearing in [1] and therefore we verify the four cases in an ad hoc way.

2.2.1. Solution Analysis. At the beginning, substituting (4) into (3), we find

$$u(x,t) = \frac{15k\left(\alpha^2 - 16\mu\gamma\right)}{76\gamma\nu} \cdot \frac{\left(b_0 Y + b_1\right)^6}{\left(b_0 + b_1 Y\right)^6}, \tag{14}$$

and substituting (6) into (5), we find

$$u(x,t) = K\left(\alpha^2 - 16\mu\gamma\right) \cdot \frac{\left(b_0 + b_{-1} Y\right)^6}{\left(b_0 Y + b_{-1}\right)^6}. \tag{15}$$

So we assume that the solution of (2) can be expressed in the form

$$u(\eta) = L \cdot \frac{\left(k_0 + k_1 Y\right)^6}{\left(k_1 + k_0 Y\right)^6}, \tag{16}$$

where L, k_0, and k_1 are constants to be determined.

We emphasize that (16) needs only three undetermined parameters. Unlike (5), there are ten undetermined parameters. Hence assumption (16) can reduce the computation burden.

Substituting (16) into (2) and setting the coefficients of all powers of Y^i to zero yield a system of algebraic equations for L, k_0, k_1, k, and ω. Solving these algebraic equations, we can determine certain solutions to (2). Obviously, these solutions cover (14) and (15). However, it is to our surprise that with the aid of Maple we determine none of nontrivial solutions after solving the above system equations. Hence further verification should be made.

2.2.2. Solution Check. In what follows, after careful numerical inspection, we show that the four cases are incorrect.

Firstly, we check the solution in Case 1, namely, (3) and (4) (or (14)) with arbitrary constants b_0, b_1, k, and ω. We observe that the nontrivial solution (3) with (4) is independent of ω, which is impossible. Indeed, given $\phi(\eta)$ is a nontrivial solution of Case 1, we rewrite (2) as

$$-\omega u' = k\nu u u' + k^2\alpha u'' + k^3\mu u''' + k^4\gamma u''''. \tag{17}$$

Substituting $\phi(\eta)$ into (17), we have

$$\begin{aligned} -\omega\phi'(\eta) = {} & k\nu\phi(\eta)\phi'(\eta) + k^2\alpha\phi''(\eta) + k^3\mu\phi'''(\eta) \\ & + k^4\gamma\phi''''(\eta). \end{aligned} \tag{18}$$

Fixing b_0, b_1, and k and leaving ω free, we can find that the right-hand side of (18) is determined, while the left-hand side is not. This is a contradiction.

Secondly, we take Case 2, namely, (5) with (6) and (7), into account. Setting $b_0 = k = 1$, $b_{-1} = 2$, $\gamma = \mu = 1/4$, $\alpha = 1/2$, and $\nu = 45/19$ for simplicity, we have

$$\omega = -\frac{121}{76}, \qquad u = \frac{1}{4}\left(\frac{1+2Y}{2+Y}\right)^6 = \frac{1}{4}\left(\frac{1+2\exp(\eta)}{2+\exp(\eta)}\right)^6. \tag{19}$$

Substituting above values into the left-hand side of (2), we obtain

$$\begin{aligned} & \omega u' + k\nu u u' + k^2\alpha u'' + k^3\mu u''' + k^4\gamma u'''' \\ & \quad = \frac{27\exp(\eta)\left(1+2\exp(\eta)\right)^2}{152\left(2+\exp(\eta)\right)^{13}} \cdot F(\eta), \end{aligned} \tag{20}$$

where

$$\begin{aligned} F(\eta) = {} & -945 + 23246\exp(\eta) + 233632\exp(2\eta) \\ & + 562784\exp(3\eta) + 290396\exp(4\eta) \\ & - 108644\exp(5\eta) - 77299\exp(6\eta) \\ & + 34514\exp(7\eta) + 32332\exp(8\eta) + 7256\exp(9\eta). \end{aligned} \tag{21}$$

And by taking $\eta = 0$, we have

$$\frac{27\exp(\eta)\left(1+2\exp(\eta)\right)^2}{152\left(2+\exp(\eta)\right)^{13}} \cdot F(\eta)\Big|_{\eta=0} = 1. \tag{22}$$

Since the right-hand side of (20) is not zero for all value of η, we conclude that the solution in Case 2 is not admitted by the original ordinary differential equation (2) and KdV-Burgers-Kuramoto equation (1).

Case 3 and Case 4 can be checked in a way similar to Case 2; here we omit the details.

At the end of this section, we should point out that (10) can be exactly simplified to the constant a_2 as follows:

$$\begin{aligned} & u(x,t) \\ & \quad = \frac{a_2\exp(2\eta) + a_2 b_1\exp(\eta) + a_2 b_0 + a_2 b_{-1}\exp(-\eta)}{\exp(2\eta) + b_1\exp(\eta) + b_0 + b_{-1}\exp(-\eta)} = a_2, \end{aligned} \tag{23}$$

which is trivial.

So, we conclude that not any new exact solution was obtained.

3. Conclusion

In this paper, we emphasize that the paper [1] contains some errors. We have to point out that similar mistakes had been analyzed in some published papers (see, e.g., [6, 7]). We hope that the results will help people have a good understanding of the work made by Kim et al.

Conflict of Interests

The author declares that there is no conflict of interests regarding the publication of this paper.

Acknowledgment

This work is supported by NSF of China (11201427).

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