

Research Article

Global Stability of a Host-Vector Model for Pine Wilt Disease with Nonlinear Incidence Rate

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Based on classical epidemic models, this paper considers a deterministic epidemic model for the spread of the pine wilt disease which has vector mediated transmission. The analysis of the model shows that its dynamics are completely determined by the basic reproduction number R_0 . Using a Lyapunov function and a LaSalle's invariant set theorem, we proved the global asymptotical stability of the disease-free equilibrium. We find that if $R_0 \leq 1$, the disease free equilibrium is globally asymptotically stable, and the disease will be eliminated. If $R_0 > 1$, a unique endemic equilibrium exists and is shown to be globally asymptotically stable, under certain restrictions on the parameter values, using the geometric approach method for global stability, due to Li and Muldowney and the disease persists at the endemic equilibrium state if it initially exists.

1. Introduction

Pine wilt disease (PWD) is caused by the pinewood nematode *Bursaphelenchus xylophilus* Nickle, which is vectored by the Japanese pine sawyer beetle *Monochamus alternatus*. The first epidemic of PWD was recorded in 1905 in Japan [1]. Since PWD was found in Japan, the pinewood nematode has spread to Korea, Taiwan, and China and has devastated pine forests in East Asia. Furthermore, it was also found in Portugal in 1999 [2]. The greatest losses to pine wilt have occurred in Japan. During the 20th century, the disease spread through highly susceptible Japanese black (*P. thunbergiana*) and Japanese red (*P. densiflora*) pine forests with devastating impact. Iowa, Illinois, Missouri, Kentucky, eastern Kansas, and southeastern Nebraska have experienced heavy losses of Scots pine. Thus, PWD has become the most serious threat to forest worldwide [3].

Mathematical modeling is useful in understanding the process of transmission of a disease, and determining the different factors that influence the spread of the disease. In this way, different control strategies can be developed to limit the spread of infection. Lately, some mathematical models have been formulated on pest-tree dynamics, such as PWD

transmission model which was investigated by Lee and Kim [4] and Shi and Song [5].

The incidence rate of the transmission of the disease plays an important role in the study of mathematical epidemiology. In classical epidemiological models, the incidence rate is assumed to be bilinear given by βSI , where β is the probability of transmission per contact rate, S is susceptible, and I is infective populations, respectively. However, actual data and evidence observed for many diseases show that dynamics of disease transmission are not always as simple as it is shown in these rates. In 1978, Capasso and Serio [6] introduced a saturated incidence rate $g(I)S$ in epidemic models where $g(I) = \beta I / (1 + \alpha I)$, $\beta > 0$, $\alpha > 0$. This incidence rate is important because the number of effective contacts between infected and susceptible individuals may be saturated at high infective levels in order to avoid the overcrowding effect of infective individuals.

There are many papers for mathematical models with nonlinear incidence rates [7–15]. Lee and Kim [4] introduced a model of a pine wilt disease with nonlinear incidence rate. Their model does not include an exposed class for the host population and falls within the susceptible-infected (SI) category of models. When the pine tree has been infected by

the nematode, the pine tree stopped the cessation of oleoresin exudation in 2-3 weeks. We consider the role of incubation period during disease transmission, that is, exposed pine trees E_h , the tree has been infected by the nematode but still sustains the ability for oleoresin exudation.

In this paper, we propose a mathematical model with nonlinear incidence rates to describe the host-vector interaction between pines and pine sawyers carrying nematode by means of ordinary differential equation. The vector (beetles) population is described by a system for the susceptible and infected vector and the dynamics of the host (pine trees) are described by SEI model. The ODE model shows that the dynamics are completely determined by the basic reproduction number R_0 . If $R_0 \leq 1$, the disease-free equilibrium is globally stable and the disease dies out. If $R_0 > 1$, a unique endemic equilibrium exists and is globally stable in the interior of the feasible region and the disease persists at the endemic equilibrium.

The paper is organized as follows. In Section 2, the host-vector model for pine wilt disease with nonlinear incidence rates is presented, where the dynamics of hosts and vectors are described by SEI and SI models, respectively. The stability of disease free equilibrium and the stability of endemic equilibrium are investigated in Sections 3 and 4, respectively. In Section 5, the global stability of endemic equilibrium is proved using the geometric approach method for global stability, due to Li and Muldowney [16]. Some numerical results and conclusions are presented in Section 6.

2. Model Frame Work

This model regards *Monochamus alternatus* as vector and pine tree as host, and establishes the host-vector epidemic model.

The total host population at time t , $N_h(t)$ is divided into three subclasses of susceptible pine trees at time t , $S_h(t)$; that is, the susceptible pine trees have a potential to be infected by the nematode and can exude oleoresin which acts as a physical barrier to beetle oviposition, and beetles cannot oviposit on them. Exposed pine trees $E_h(t)$ have been infected by the nematode but still sustain the ability for oleoresin exudation, and infected pine trees $I_h(t)$ have been infected by the nematode and the oleoresin exudation ability have been lost and also beetles can oviposit on it. Furthermore, we assume that the class of recovered $R_h(t)$ is negligible because every infectious pine tree dies within the year of infection or in the next year. The number of total host population is denoted by $N_h(t) = S_h(t) + E_h(t) + I_h(t)$. And then, we assume that the total vector population at time t , $N_v(t)$ is split into two subclasses the number of susceptible adult beetles $S_v(t)$ which does not carry pinewood nematode at time t and the number of infective adult beetles $I_v(t)$ which does carry pinewood nematode at time t , so that total vector population is denoted by $N_v(t) = S_v(t) + I_v(t)$. Our model excludes the immature beetles which are in the egg stage, a pupal stage, because they do not participate in the infection cycle. The parameters in the system are as follows: the parameter a_h is the constant increase rate of pine tree at time t and b_v is the constant emergence rate of adult beetles at time t (the

period of emergence). And μ_1 is the natural death rate of pine tree host and μ_2 is the natural death rate of beetles as vectors. The parameter α is denoted by the probability that infectious beetles transmit nematode by means of contact and γ is the probability of having pinewood nematode when the beetle emerges out in the $I_v(t)$. And the parameter ϕ is the average number of contact per day of the vectors adult beetles during maturation feeding period. The parameter β denotes the transfer rates between the exposed and the infectious.

In this model, the nonlinear incidence term $\alpha\phi S_h I_v / (1 + mI_v)$ denotes the rate at which the pine trees host S_h gets infected by infectious adult beetles $I_v(t)$ which do carry pinewood nematode at time t , and $\gamma I_h S_v / (1 + nI_h)$ refers to the rate at which the susceptible pine sawyers S_v have pinewood nematode when it emerges in the infected pine trees I_h and m, n determine the level at which the force of infection saturates. The incidence function forms reflect a saturating effect of diseases transmission. All parameters are assumed to be positive based on some biological reasons. Thus, a host-vector epidemic model with nonlinear incidence can be described by the following system of differential equations:

$$\begin{aligned} \frac{dS_h}{dt} &= a_h - \frac{\alpha\phi S_h I_v}{1 + mI_v} - \mu_1 S_h, \\ \frac{dE_h}{dt} &= \frac{\alpha\phi S_h I_v}{1 + mI_v} - (\beta + \mu_1) E_h, \\ \frac{dI_h}{dt} &= \beta E_h - \mu_1 I_h, \\ \frac{dS_v}{dt} &= b_v - \frac{\gamma I_h S_v}{1 + nI_h} - \mu_2 S_v, \\ \frac{dI_v}{dt} &= \frac{\gamma I_h S_v}{1 + nI_h} - \mu_2 I_v. \end{aligned} \tag{1}$$

Considering ecological signification, we restrict our attention to the dynamics of the model in $\Omega = \{(S_h, E_h, I_h, S_v, I_v) \in R_+^5 \mid S_h \geq 0, E_h \geq 0, I_h \geq 0, S_v \geq 0, I_v \geq 0\}$. We make some reasonable technical assumptions on the parameters of the model, namely, $\alpha > 0, \phi > 0, \beta > 0, \gamma > 0, a_h > 0, b_v > 0, \mu_1 > 0, \mu_2 > 0$, in Ω . The above systems for the host population and the vector are also equipped with initial conditions as follows: $S_h(0) = S_h^0, E_h(0) = E_h^0, I_h(0) = I_h^0, S_v(0) = S_v^0$, and $I_v(0) = I_v^0$.

The total host population dynamics are given by $dN_h/dt = a_h - \mu_1 N_h$.

The given initial conditions make sure that $N_h(0) \geq 0$.

The total dynamics of vector population is given by $dN_v/dt = b_v - \mu_2 N_v$. It is easily seen that both for the host population and for the vector population, the corresponding total population sizes are asymptotically constant such as $\lim_{t \rightarrow \infty} N_h(t) = a_h/\mu_1$ and $\lim_{t \rightarrow \infty} N_v(t) = b_v/\mu_2$. This implies that in our model, we assume without loss of generality that $N_h(t) = a_h/\mu_1, N_v(t) = b_v/\mu_2$ for all $t \geq 0$ provided that $S_h^0 + E_h^0 + I_h^0 = a_h/\mu_1, S_v^0 + I_v^0 = b_v/\mu_2$.

Theorem 1. Let $(S_h, E_h, I_h, S_v, I_v)$ be the solution of the system (1) with initial conditions $S_h(0) = S_h^0, E_h(0) = E_h^0, I_h(0) = I_h^0, S_v(0) = S_v^0,$ and $I_v(0) = I_v^0$ and the compact set

$$\Omega = \left\{ (S_h, E_h, I_h, S_v, I_v) \in R_+^5 \mid 0 \leq S_h + E_h + I_h \leq \frac{a_h}{\mu_1}, 0 \leq S_v + I_v \leq \frac{b_v}{\mu_2} \right\}. \tag{2}$$

Then, Ω is positively invariant and attracting under the flow described by (1).

Proof. Consider the following Lyapunov function:

$$V(t) = (V_1(t), V_2(t)) = (S_h + E_h + I_h, S_v + I_v). \tag{3}$$

Its time derivative is

$$\begin{aligned} \frac{dV}{dt} &= \left(\frac{dV_1}{dt}, \frac{dV_2}{dt} \right) \\ &= (\dot{S}_h + \dot{E}_h + \dot{I}_h, \dot{S}_v + \dot{I}_v) = (a_h - \mu_1 V_1, b_v - \mu_2 V_2). \end{aligned} \tag{4}$$

With this in mind, we can get that

$$\begin{aligned} \frac{dV_1}{dt} &= a_h - \mu_1 V_1 \leq 0, \quad \text{for } V_1 \geq \frac{a_h}{\mu_1}, \\ \frac{dV_2}{dt} &= b_v - \mu_2 V_2 \leq 0, \quad \text{for } V_2 \geq \frac{b_v}{\mu_2}. \end{aligned} \tag{5}$$

Then, it follows from (5) that $dV/dt \leq 0$ which implies that Ω is a positively invariant set. On the other hand, a standard comparison theorem [17] can be used to show that

$$0 \leq (V_1, V_2) \leq \left(\frac{a_h}{\mu_1} + V_1(0)e^{-\mu_1 t}, \frac{b_v}{\mu_2} + V_2(0)e^{-\mu_2 t} \right), \tag{6}$$

where $V_1(0)$ and $V_2(0)$ are in the initial conditions of $V_1(t)$ and $V_2(t)$, respectively.

Thus, as $t \rightarrow \infty, 0 \leq (V_1, V_2) \leq (a_h/\mu_1, b_v/\mu_2)$ and one can conclude that Ω is an attractive set. \square

The values of S_h and S_v can be determined correspondingly by $S_v = (b_v/\mu_2) - I_v, S_h = (a_h/\mu_1) - E_h - I_h$ by the results of theorem [18]. Also, we can reduce system (1) to a 3-dimensional system by eliminating S_h and S_v , respectively, in the feasible region Ω

$$\begin{aligned} \frac{dE_h}{dt} &= \frac{\alpha\phi I_v}{1 + mI_v} \left(\frac{a_h}{\mu_1} - E_h - I_h \right) - (\beta + \mu_1) E_h, \\ \frac{dI_h}{dt} &= \beta E_h - \mu_1 I_h, \\ \frac{dI_v}{dt} &= \frac{\gamma I_h}{1 + nI_h} \left(\frac{b_v}{\mu_2} - I_v \right) - \mu_2 I_v. \end{aligned} \tag{7}$$

Therefore, from now on, we will investigate the following 3-dimensional nonlinear system so that the dynamics of

system (1) and (7) are qualitatively equivalent to the dynamics of system. It is easy to verify that all of the solutions of system (7) exist and are nonnegative. The feasible region for the system (2) is

$$\Gamma = \left\{ (E_h, I_h, I_v) \in R_+^3 \mid 0 \leq E_h + I_h \leq \frac{a_h}{\mu_1}, 0 \leq I_v \leq \frac{b_v}{\mu_2}, E_h \geq 0, I_h \geq 0, I_v \geq 0 \right\}, \tag{8}$$

where R_+^3 denotes the nonnegative cone of R^3 including its lower-dimensional faces.

With respect to system (7), we have the following result.

Theorem 2. Let (E_h, I_h, I_v) be the solution of the system (7) with initial conditions $E_h(0) = E_h^0, I_h(0) = I_h^0, I_v(0) = I_v^0,$ and the closed set Γ . Then, Γ is positively invariant with respect to system (7) and attracting under the flow described by (7).

3. The Disease-Free Equilibrium and Its Stability

Direct calculations show that the system (7) always has the disease-free equilibrium point given by $E_0 = (0, 0, 0)$. The dynamics of the disease are described by the quantity $R_0 = a_h b_v \phi \alpha \beta \gamma / \mu_1^2 \mu_2^2 (\beta + \mu_1)$. R_0 is the critical threshold of model (7) that is called the basic reproduction number in the epidemic model. Using Theorem 2 in [19], at first, the following results are established.

Theorem 3. If $R_0 < 1$, the disease-free equilibrium E_0 of the model (7) is locally asymptotically stable, and is unstable if $R_0 > 1$.

Proof. We linearize the system (7) around the disease-free equilibrium E_0 . The matrix of the linearization at E_0 is given by

$$J(E_0) = \begin{pmatrix} -(\beta + \mu_1) & 0 & \frac{a_h \alpha \phi}{\mu_1} \\ \beta & -\mu_1 & 0 \\ 0 & \frac{b_v \beta}{\mu_2} & -\mu_2 \end{pmatrix}. \tag{9}$$

The characteristic equation of this matrix is given by $\det(\lambda I - J(E_0)) = 0$, where I is the 3×3 unit matrix. Expanding the determinant into a characteristic equation, we obtain the following equation, which is equivalent to

$$\lambda^3 + a_1 \lambda^2 + a_2 \lambda + a_3 = 0, \tag{10}$$

where

$$\begin{aligned} a_1 &= 2\mu_1 + \mu_2 + \beta > 0, \\ a_2 &= (\mu_1 + \mu_2)(\beta + \mu_1) + \mu_1 \mu_2 > 0, \\ a_3 &= \mu_1 \mu_2 (\beta + \mu_1) (1 - R_0) > 0. \end{aligned} \tag{11}$$

These three eigenvalues have negative real part if they satisfy the Routh-Hurwitz Criteria [20], such that $a_i > 0$ for $i = 1, 2, 3$, with $a_1 > 0$, $a_3 > 0$, and $a_1 a_2 > a_3$. If $R_0 < 1$, then

$$\begin{aligned}
 & a_1 a_2 - a_3 \\
 &= (2\mu_1 + \mu_2 + \beta) [(\mu_1 + \mu_2)(\beta + \mu_1) + \mu_1 \mu_2] \\
 &\quad - \mu_1 \mu_2 (\beta + \mu_1) (1 - R_0), \\
 &= (2\mu_1 + \mu_2 + \beta) (\mu_1 + \mu_2) (\beta + \mu_1) \\
 &\quad + \mu_1 \mu_2 (2\mu_1 + \mu_2 + \beta) - \mu_1 \mu_2 (\beta + \mu_1) (1 - R_0) \\
 &= (2\mu_1 + \mu_2 + \beta) (\mu_1 + \mu_2) (\beta + \mu_1) \\
 &\quad + \mu_1 \mu_2 (2\mu_1 + \mu_2 + \beta) - \mu_1 \mu_2 (\beta + \mu_1) \\
 &\quad + \mu_1 \mu_2 (\beta + \mu_1) R_0 \\
 &= (2\mu_1 + \mu_2 + \beta) (\mu_1 + \mu_2) (\beta + \mu_1) \\
 &\quad + \mu_1 \mu_2 (\mu_1 + \mu_2) + \mu_1 \mu_2 (\beta + \mu_1) R_0 > 0.
 \end{aligned} \tag{12}$$

According to the Routh-Hurwitz Criteria, the disease-free equilibrium E_0 of the model (7) is locally asymptotically stable. \square

Now, we study the global behavior of the disease-free equilibrium for system (7).

Theorem 4. *If $R_0 \leq 1$, the disease-free equilibrium E_0 of the model (7) is globally asymptotically stable in Γ .*

Proof. We construct the following Lyapunov function:

$$V(t) = a_1 E_h + a_2 I_h + a_3 I_v, \tag{13}$$

where

$$\begin{aligned}
 a_1 &= \frac{b_v \beta \gamma}{\mu_1 \mu_2^2 (\beta + \mu_1)}, & a_2 &= \frac{b_v \gamma}{\mu_1 \mu_2^2}, \\
 a_3 &= \frac{1}{\mu_2}.
 \end{aligned} \tag{14}$$

Its derivative along the solutions to the system (7) is

$$\begin{aligned}
 V'(t) &= a_1 E'_h + a_2 I'_h + a_3 I'_v \\
 &= a_1 \left[\frac{\alpha \phi I_v}{1 + m I_v} \left(\frac{a_h}{\mu_1} - E_h - I_h \right) - (\beta + \mu_1) E_h \right] \\
 &\quad + a_2 [\beta E_h - \mu_1 I_h] + a_3 \left[\frac{\gamma I_h}{1 + n I_h} \left(\frac{b_v}{\mu_2} - I_v \right) - \mu_2 I_v \right]
 \end{aligned}$$

$$\begin{aligned}
 &\leq a_1 \left[\alpha \phi I_v \left(\frac{a_h}{\mu_1} - E_h - I_h \right) - (\beta + \mu_1) E_h \right] \\
 &\quad + a_2 [\beta E_h - \mu_1 I_h] + a_3 \left[\gamma I_h \left(\frac{b_v}{\mu_2} - I_v \right) - \mu_2 I_v \right] \\
 &\leq \frac{b_v \beta \gamma}{\mu_1 \mu_2^2 (\beta + \mu_1)} \\
 &\quad \times \left\{ \alpha \phi I_v \left(\frac{a_h}{\mu_1} - E_h - I_h \right) - (\beta + \mu_1) E_h \right\} \\
 &\quad + \frac{b_v \gamma}{\mu_1 \mu_2^2} (\beta E_h - \mu_1 I_h) + \frac{1}{\mu_2} \left(\frac{b_v \gamma}{\mu_2} I_h - \mu_2 I_v \right) \\
 &= I_v \left\{ (R_0 - 1) - \frac{b_v \alpha \phi \beta \gamma}{\mu_1 \mu_2^2 (\beta + \mu_1)} (E_h + I_h) \right\} \leq 0.
 \end{aligned} \tag{15}$$

Thus, $V'(t)$ is negative if $R_0 \leq 1$. Furthermore, $V > 0$ along the solution of the system and is zero if and only if E_h, I_v , and I_h are zero. Also, $V' \leq 0$. If $R_0 \leq 1$, then $V' = 0$ if and only if $I_v = 0$, and in the case $R_0 = 1$, $V' = 0$ if and only if $I_v = 0$ or $E_h = I_h = 0$. Hence, the largest compact invariant set in $\{(E_h, I_h, I_v) \in \Gamma \mid V'(t) = 0\}$ when $R_0 \leq 1$, is the singleton $\{E_0\}$. By Lasalle's Invariance Principle [21], then it implies that E_0 is globally asymptotically stable in Γ . \square

4. The Endemic Equilibrium and Its Stability

Here, we study the existence and stability of the endemic equilibrium points. By straightforward computation, if $R_0 > 1$, then the host-vector model system (7) has a unique endemic equilibrium given by $E^* = (E_h^*, I_h^*, I_v^*)$ in Γ , with

$$\begin{aligned}
 E_h^* &= \frac{\mu_1}{\beta} I_h^*, \\
 I_h^* &= \frac{\mu_1 \mu_2^2 (R_0 - 1)}{\mu_1 (\mu_2 w + m \gamma b_v) + \alpha \phi \gamma b_v}, \\
 I_v^* &= \frac{\gamma b_v I_h^*}{\mu_2 (w I_h^* + \mu_2)},
 \end{aligned} \tag{16}$$

where

$$w = \gamma + n \mu_2. \tag{17}$$

In order to investigate the stability of the endemic equilibrium, the additive compound matrices approach as in [22, 23] is used. We will linearize system (7) about an endemic equilibrium E^* and get the following Jacobian matrix

$$J(E^*) = \begin{pmatrix} -\frac{\alpha\phi I_v^*}{1+mI_v^*} - (\beta + \mu_1) & -\frac{\alpha\phi I_v^*}{1+mI_v^*} & \frac{\alpha\phi((a_h/\mu_1) - E_h^* - I_h^*)}{(1+mI_v^*)^2} \\ \beta & -\mu_1 & 0 \\ 0 & \frac{\gamma((b/\mu_2) - I_v^*)}{(1+nI_h^*)^2} & -\frac{\gamma I_h^*}{1+nI_h^*} - \mu_2 \end{pmatrix}. \tag{18}$$

From the Jacobian matrix $J(E^*)$, the second additive compound matrix is given by

$$J^{[2]}(E^*) = \begin{pmatrix} -\frac{\alpha\phi I_v^*}{1+mI_v^*} - (\beta + 2\mu_1) & 0 & -\frac{\alpha\phi((a_h/\mu_1) - E_h^* - I_h^*)}{(1+mI_v^*)^2} \\ \frac{\gamma((b/\mu_2) - I_v^*)}{(1+nI_h^*)^2} & -\frac{\alpha\phi I_v^*}{1+mI_v^*} - \frac{\gamma I_h^*}{1+nI_h^*} - \beta - \mu_1 - \mu_2 & -\frac{\alpha\phi I_v^*}{1+mI_v^*} \\ 0 & \beta & -\frac{\gamma I_h^*}{1+nI_h^*} - \mu_1 - \mu_2 \end{pmatrix}. \tag{19}$$

The following lemma stated and proved in McCluskey and van den Driessche [24] is used to demonstrate the local stability of endemic equilibrium point E^* .

Lemma 5. *Let M be a 3×3 real matrix. If $\text{tr}(M)$, $\det(M)$, and $\det(M^{[2]})$ are all negative, then all eigenvalues of M have negative real part.*

Using the above Lemma, we will study the stability of the endemic equilibrium.

Theorem 6. *If $R_0 > 1$, the endemic equilibrium E^* of the model (7) is locally asymptotically stable in Γ .*

Proof. From the Jacobian matrix $J(E^*)$, we have

$$\text{tr}(M) = -\left(\frac{\alpha\phi I_v^*}{1+mI_v^*} + (\beta + \mu_1) + \mu_1 + \frac{\gamma I_h^*}{1+nI_h^*} + \mu_2\right) < 0. \tag{20}$$

Because

$$\begin{aligned} \frac{a_h}{\mu_1} - E_h^* - I_h^* &= \frac{(\beta + \mu_1)(1+mI_v^*)E_h^*}{\alpha\phi I_v^*}, \\ E_h^* &= \frac{\mu_1}{\beta} I_h^*, \end{aligned} \tag{21}$$

$$\frac{b_v}{\mu_2} - I_v^* = \frac{(1+nI_h^*)\mu_2 I_v^*}{\gamma I_h^*}.$$

From (21), it is easy to see that

$$\begin{aligned} &\frac{\alpha\phi((a_h/\mu_1) - E_h^* - I_h^*)}{(1+mI_v^*)^2} \times \frac{\gamma((b/\mu_2) - I_v^*)}{(1+nI_h^*)^2} \\ &= \frac{\mu_1\mu_2(\beta + \mu_1)}{(1+mI_v^*)(1+nI_h^*)\beta}. \end{aligned} \tag{22}$$

Thus,

$$\begin{aligned} &\det(J(E^*)) \\ &= \begin{vmatrix} -\frac{\alpha\phi I_v^*}{1+mI_v^*} - (\beta + \mu_1) & -\frac{\alpha\phi I_v^*}{1+mI_v^*} & -\frac{\alpha\phi((a_h/\mu_1) - E_h^* - I_h^*)}{(1+mI_v^*)^2} \\ \beta & -\mu_1 & 0 \\ 0 & \frac{\gamma((b/\mu_2) - I_v^*)}{(1+nI_h^*)^2} & -\frac{\gamma I_h^*}{1+nI_h^*} - \mu_2 \end{vmatrix} \\ &= -\left(\frac{\alpha\phi I_v^*}{1+mI_v^*} + \beta + \mu_1\right) \left(\mu_1\mu_2 + \frac{\mu_1\gamma I_h^*}{1+nI_h^*}\right) - \beta \left[\frac{\alpha\phi I_v^*}{1+mI_v^*} \left(\frac{\gamma I_h^*}{1+nI_h^*} + \mu_2\right) - \frac{\mu_1\mu_2(\beta + \mu_1)}{(1+mI_v^*)(1+nI_h^*)\beta}\right] \\ &= -\left(\frac{\alpha\phi I_v^*}{1+mI_v^*} + \beta + \mu_1\right) \left(\mu_1\mu_2 + \frac{\mu_1\gamma I_h^*}{1+nI_h^*}\right) - \frac{\alpha\beta\phi I_v^*}{1+mI_v^*} \left(\frac{\gamma I_h^*}{1+nI_h^*} + \mu_2\right) + \frac{\mu_1\mu_2(\beta + \mu_1)}{(1+mI_v^*)(1+nI_h^*)} \\ &= -\frac{\mu_1\gamma I_h^*}{1+nI_h^*} \left(\frac{\alpha\phi I_v^*}{1+mI_v^*} + \beta + \mu_1\right) - \mu_1\mu_2 \frac{\alpha\phi I_v^*}{1+mI_v^*} - \mu_1\mu_2(\beta + \mu_1) - \frac{\alpha\beta\phi I_v^*}{1+mI_v^*} \left(\frac{\gamma I_h^*}{1+nI_h^*} + \mu_2\right) \end{aligned}$$

$$\begin{aligned}
 & + \frac{\mu_1 \mu_2 (\beta + \mu_1)}{(1 + mI_v^*)(1 + nI_h^*)} = -\frac{\mu_1 \gamma I_h^*}{1 + nI_h^*} \left(\frac{\alpha \phi I_v^*}{1 + mI_v^*} + \beta + \mu_1 \right) \\
 & - \mu_1 \mu_2 \frac{\alpha \phi I_v^*}{1 + mI_v^*} - \frac{\alpha \phi I_v^*}{1 + mI_v^*} \left(\frac{\gamma I_h^*}{1 + nI_h^*} \right) + \mu_1 \mu_2 (\beta + \mu_1) \left(\frac{1}{(1 + mI_v^*)(1 + nI_h^*)} - 1 \right) < 0.
 \end{aligned} \tag{23}$$

Computing directly the determinant of $J^{[2]}(E^*)$, we can get

$$\begin{aligned}
 & \det(J^{[2]}(E^*)) \\
 & = \begin{vmatrix} -\frac{\alpha \phi I_v^*}{1 + mI_v^*} - (\beta + 2\mu_1) & 0 & -\frac{\alpha \phi ((a_h/\mu_1) - E_h^* - I_h^*)}{(1 + mI_v^*)^2} \\ \frac{\gamma ((b_v/\mu_2) - I_v^*)}{(1 + nI_h^*)^2} & -\frac{\alpha \phi I_v^*}{1 + mI_v^*} - \frac{\gamma I_h^*}{1 + nI_h^*} - \beta - \mu_1 - \mu_2 & -\frac{\alpha \phi I_v^*}{1 + mI_v^*} \\ 0 & \beta & -\frac{\gamma I_h^*}{1 + nI_h^*} - \mu_1 - \mu_2 \end{vmatrix} \\
 & = -\left(\frac{\alpha \phi I_v^*}{1 + mI_v^*} + \beta + 2\mu_1 \right) \left[\left(\frac{\alpha \phi I_v^*}{1 + mI_v^*} + \frac{\gamma I_h^*}{1 + nI_h^*} + \beta + \mu_1 + \mu_2 \right) \left(\frac{\gamma I_h^*}{1 + nI_h^*} + \mu_1 + \mu_2 \right) + \frac{\alpha \beta \phi I_v^*}{1 + mI_v^*} \right] \\
 & \quad - \frac{\gamma ((b_v/\mu_2) - I_v^*)}{(1 + nI_h^*)^2} \left(\frac{\alpha \beta \phi ((a_h/\mu_1) - E_h^* - I_h^*)}{(1 + mI_v^*)^2} \right) < 0.
 \end{aligned} \tag{24}$$

Hence, by lemma, the endemic equilibrium E^* of the model (7) is locally asymptotically stable in Γ . \square

5. Global Stability of the Endemic Equilibrium

We now prove the global stability of the endemic equilibrium E^* , when the reproduction number R_0 is greater than the unity. For this, first we will prove the following result.

Theorem 7. *If $R_0 > 1$, then system (7) is uniformly persistent; that is, there exists $c > 0$ (independent of initial conditions), such that $\liminf_{t \rightarrow \infty} E_h(t) \geq c$, $\liminf_{t \rightarrow \infty} I_h(t) \geq c$, $\liminf_{t \rightarrow \infty} I_v(t) \geq c$.*

Proof. Let π be a semidynamical system (7) in $(R_0^+)^3$. Let χ be a locally compact metric space, and $\Gamma_0 = \{(E_h, I_h, I_v) \in \Gamma \mid I_v = 0\}$. The set Γ_0 is a compact subset of Γ and Γ/Γ_0 is positively invariant set of system (7). Let $P : \chi \rightarrow R_0^+$ be defined by $P(E_h, I_h, I_v) = I_v$ and set $S = \{(E_h, I_h, I_v) \in \Gamma \mid P(E_h, I_h, I_v) < \rho\}$, where ρ is sufficiently small so that $R_0(1 - (\mu_2/b_v)\rho)/(1 + n\rho) > 1$. Assume that there is a solution $x \in S$ such that for each $t > 0$, we have $P(\pi(x, t)) < P(x) < \rho$. Let us consider the following:

$$L(t) = \frac{a_h \alpha \beta \phi (1 - \delta^*)}{\mu_1 \mu_2 (\beta + \mu_1)} I_v + I_h, \tag{25}$$

where $\delta^* > 0$ is a sufficiently small constant so that $R_0(1 - (\mu_2/b_v)\rho)(1 - \delta^*)/(1 + n\rho) > 1$. By a direct calculation, we have

$$\begin{aligned}
 L'(t) & \geq \mu_1 \left[\frac{a_h b_v \alpha \beta \gamma \phi (1 - \delta^*) (1 - (\mu_2/b_v)\rho)}{\mu_1^2 \mu_2^2 (\beta + \mu_1) (1 + n\rho)} - 1 \right] I_h \\
 & \quad + \frac{a_h \alpha \beta \phi \delta^*}{\mu_1} I_v.
 \end{aligned} \tag{26}$$

Let

$$\begin{aligned}
 \delta & = \min \left\{ \mu_1 \left[\frac{a_h b_v \alpha \beta \gamma \phi (1 - \delta^*) (1 - (\mu_2/b_v)\rho)}{\mu_1^2 \mu_2^2 (\beta + \mu_1) (1 + n\rho)} - 1 \right], \right. \\
 & \quad \left. \frac{\mu_2 (\beta + \mu_1) \delta^*}{1 - \delta^*} \right\} > 0.
 \end{aligned} \tag{27}$$

Thus, we have

$$L'(t) \geq \delta L(t). \tag{28}$$

The above inequality (28) implies that $L(t) \rightarrow \infty$ as $t \rightarrow \infty$. However, $L(t)$ is bounded on the set Γ . According to Theorem 1 in [25], we complete the proof of Theorem 7. \square

Here, we use the geometrical approach of Li and Muldowney to investigate the global stability of the endemic equilibrium E^* in the feasible region Ω . We have omitted the detailed introduction of this approach and we refer the interested readers to see [16]. For the applications of the Li and Muldowney approach to host-vector models (see [26, 27]). We summarize this approach as follows.

Consider a C^1 map $f : x \mapsto f(x)$ from an open set $D \subset R^n$ to R^n such that each solution $x(t, x_0)$ to the differential equation

$$x' = f(x) \tag{29}$$

is uniquely determined by the initial value $x(0, x_0)$. We have the following assumptions:

- (H_1) D is simply connected;
- (H_2) there exists a compact absorbing set $K \subset D$;
- (H_3) (29) has unique equilibrium \bar{x} in D .

Let $P : x \mapsto P(x)$ be a nonsingular $\binom{n}{2} \times \binom{n}{2}$ matrix-valued function which is C^1 in D and a vector norm $|\cdot|$ on R^N , where $N = \binom{n}{2}$.

Let μ be the Lozinskiĭ measure with respect to the $|\cdot|$. Define a quantity \bar{q}_2 as

$$\bar{q}_2 = \limsup_{t \rightarrow \infty} \sup_{x_0 \in K} \frac{1}{t} \int_0^t \mu(B(x(s, x_0))) ds, \tag{30}$$

where $B = P_f P^{-1} + P J^{[2]} P^{-1}$, the matrix P_f is obtained by replacing each entry p of P by its derivative in the direction of f , $(p_{ij})_f$, and $J^{[2]}$ is the second additive compound matrix of the Jacobian matrix J of (19). The following result has been established in Li and Muldowney [16].

Theorem 8. *Suppose that (H_1) , (H_2) , and (H_3) hold; then the unique endemic equilibrium E^* is globally stable in Ω if $\bar{q}_2 < 0$.*

We choose a suitable vector norm $|\cdot|$ in R^3 and a 3×3 matrix valued function

$$P(x) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{E_h}{I_v} & 0 \\ 0 & 0 & \frac{E_h}{I_v} \end{pmatrix}. \tag{31}$$

Obviously, P is C^1 and nonsingular in the interior of Ω . Linearizing system (2) about an endemic equilibrium E^* gives the following Jacobian matrix:

$$J = \begin{pmatrix} -\frac{\alpha\phi I_v}{1+mI_v} - (\beta + \mu_1) & -\frac{\alpha\phi I_v}{1+mI_v} & \frac{\alpha\phi}{(1+mI_v)^2} \left(\frac{a_h}{\mu_1} - E_h - I_h \right) \\ \beta & -\mu_1 & 0 \\ 0 & \frac{\gamma}{(1+nI_h)^2} \left(\frac{b_v}{\mu_2} - I_v \right) & -\frac{\gamma I_h}{1+nI_h} - \mu_2 \end{pmatrix}. \tag{32}$$

The second additive compound matrix of $J(E^*)$ is given by

$$J^{[2]} = \begin{pmatrix} -\frac{\alpha\phi I_v}{1+mI_v} - (\beta + \mu_1) - \mu_1 & 0 & -\frac{\alpha\phi}{(1+mI_v)^2} \left(\frac{a_h}{\mu_1} - E_h - I_h \right) \\ \frac{\gamma}{(1+nI_h)^2} \left(\frac{b_v}{\mu_2} - I_v \right) & -\frac{\alpha\phi I_v}{1+mI_v} - (\beta + \mu_1) - \frac{\gamma I_h}{1+nI_h} - \mu_2 & -\frac{\alpha\phi I_v}{1+mI_v} \\ 0 & \beta & -\mu_1 - \frac{\gamma I_h}{1+nI_h} - \mu_2 \end{pmatrix}. \tag{33}$$

The matrix $B = P_f P^{-1} + P J^{[2]} P^{-1}$ can be written in block form as

$$B = \begin{pmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{pmatrix}, \tag{34}$$

where

$$\begin{aligned} B_{11} &= -\frac{\alpha\phi I_v}{1+mI_v} - (\beta + \mu_1) - \mu_1, \\ B_{12} &= \left(0 - \frac{\alpha\phi}{(1+mI_v)^2} \frac{I_v}{E_h} \left(\frac{a_h}{\mu_1} - E_h - I_h \right) \right), \\ B_{21} &= \begin{pmatrix} \frac{\gamma}{(1+nI_h)^2} \left(\frac{b_v}{\mu_2} - I_v \right) \frac{E_h}{I_v} \\ 0 \end{pmatrix}, \\ B_{22} &= \begin{pmatrix} \frac{E'_h}{E_h} - \frac{I'_v}{I_v} - \frac{\alpha\phi I_v}{1+mI_v} - (\beta + \mu_1) - \frac{\gamma I_h}{1+nI_h} - \mu_2 & -\frac{\alpha\phi I_v}{1+mI_v} \\ \beta & \frac{E'_h}{E_h} - \frac{I'_v}{I_v} - \mu_1 - \frac{\gamma I_h}{1+nI_h} - \mu_2 \end{pmatrix}. \end{aligned} \tag{35}$$

Consider the norm in R^3 as $|(u, v, w)| = \max(|u|, |v| + |w|)$ where (u, v, w) denotes the vector in R^3 . The Lozinskiĭ measure with respect to this norm is defined as $\mu(B) \leq \sup(g_1, g_2)$, where

$$g_1 = \mu_1 (B_{11}) + |B_{12}|, \quad g_2 = \mu_1 (B_{22}) + |B_{21}|. \tag{36}$$

From system (2), we can write

$$\begin{aligned} \frac{E'_h}{E_h} &= \frac{\alpha\phi}{1+mI_v} \frac{I_v}{E_h} \left(\frac{a_h}{\mu_1} - E_h - I_h \right) - (\beta + \mu_1), \\ \frac{I'_h}{I_h} &= \beta \frac{E_h}{I_h} - \mu_1, \\ \frac{I'_v}{I_v} &= \frac{\gamma}{1+nI_h} \frac{I_h}{I_v} \left(\frac{b_v}{\mu_2} - I_v \right) - \mu_2. \end{aligned} \tag{37}$$

Since B_{11} is a scalar, its Lozinskiĭ measure with respect to any vector norm in R^1 will be equal to B_{11} . Thus,

$$\begin{aligned} B_{11} &= -\frac{\alpha\phi I_v}{1+mI_v} - (\beta + \mu_1) - \mu_1, \\ |B_{12}| &= \frac{\alpha\phi}{(1+mI_v)^2} \frac{I_v}{E_h} \left(\frac{a_h}{\mu_1} - E_h - I_h \right), \end{aligned} \tag{38}$$

and g_1 will become

$$\begin{aligned} g_1 &= -\frac{\alpha\phi I_v}{1+mI_v} - (\beta + \mu_1) - \mu_1 \\ &\quad + \frac{\alpha\phi}{(1+mI_v)^2} \frac{I_v}{E_h} \left(\frac{a_h}{\mu_1} - E_h - I_h \right) \\ &\leq -\frac{\alpha\phi I_v}{1+mI_v} - \mu_1 \\ &\quad + \frac{\alpha\phi}{(1+mI_v)} \frac{I_v}{E_h} \left(\frac{a_h}{\mu_1} - E_h - I_h \right) - (\beta + \mu_1) \\ &\leq \frac{E'_h}{E_h} - \frac{\alpha\phi I_v}{1+mI_v} - \mu_1. \end{aligned} \tag{39}$$

Also, $|B_{21}| = (\gamma/(1+nI_h)^2)((b_v/\mu_2) - I_v)(E_h/I_v) \cdot |B_{12}|$ and $|B_{21}|$ are the operator norms of B_{12} and B_{21} which are mapping from R^2 to R and from R to R^2 , respectively, and R^2 is endowed with the l_1 norm. $\mu_1(B_{22})$ is the Lozinskiĭ measure of 2×2 matrix B_{22} with respect to l_1 norm in R^2 . Consider the following:

$$\begin{aligned} \mu(B_{22}) &= \sup \left\{ \frac{E'_h}{E_h} - \frac{I'_v}{I_v} - \frac{\alpha\phi I_v}{1+mI_v} - \mu_1 - \frac{\gamma I_h}{1+nI_h} - \mu_2, \right. \\ &\quad \left. \frac{E'_h}{E_h} - \frac{I'_v}{I_v} + \frac{\alpha\phi I_v}{1+mI_v} - \mu_1 - \frac{\gamma I_h}{1+nI_h} - \mu_2 \right\}, \\ &= \frac{E'_h}{E_h} - \frac{I'_v}{I_v} + \frac{\alpha\phi I_v}{1+mI_v} - \mu_1 - \frac{\gamma I_h}{1+nI_h} - \mu_2. \end{aligned} \tag{40}$$

Hence,

$$\begin{aligned}
 g_2 &= \frac{E'_h}{E_h} - \frac{I'_v}{I_v} + \frac{\alpha\phi I_v}{1+mI_v} - \mu_1 - \frac{\gamma I_h}{1+nI_h} \\
 &\quad - \mu_2 + \frac{\gamma}{(1+nI_h)^2} \left(\frac{b_v}{\mu_2} - I_v \right) \frac{E_h}{I_v} \\
 &= \frac{E'_h}{E_h} - \frac{I'_v}{I_v} + \frac{\alpha\phi I_v}{1+mI_v} - \mu_1 - \frac{\gamma I_h}{1+nI_h} \\
 &\quad + \frac{\gamma}{(1+nI_h)^2} \left(\frac{b_v}{\mu_2} - I_v \right) \frac{E_h}{I_v} - \mu_2 \\
 &= \frac{E'_h}{E_h} - \frac{I'_v}{I_v} + \frac{\alpha\phi I_v}{1+mI_v} - \mu_1 - \frac{\gamma I_h}{1+nI_h} \\
 &\quad + \frac{\gamma}{(1+nI_h)^2} \left(\frac{b_v}{\mu_2} - I_v \right) \frac{E_h}{I_v} + \frac{I'_v}{I_v} \\
 &\quad - \frac{\gamma}{1+nI_h} \left(\frac{b_v}{\mu_2} - I_v \right) \frac{I_h}{I_v} \\
 &= \frac{E'_h}{E_h} + \frac{\alpha\phi I_v}{1+mI_v} - \mu_1 - \frac{\gamma I_h}{1+nI_h} \\
 &\quad + \frac{\gamma}{(1+nI_h)^2} \left(\frac{b_v}{\mu_2} - I_v \right) \frac{E_h}{I_v} - \frac{\gamma}{1+nI_h} \left(\frac{b_v}{\mu_2} - I_v \right) \frac{I_h}{I_v} \\
 &= \frac{E'_h}{E_h} + \frac{\alpha\phi I_v}{1+mI_v} - \mu_1 - \frac{\gamma I_h}{1+nI_h} + \frac{\gamma}{(1+nI_h)^2} \frac{E_h b_v}{I_v \mu_2} \\
 &\quad - \frac{\gamma}{(1+nI_h)^2} E_h - \frac{\gamma}{1+nI_h} \frac{I_h b_v}{I_v \mu_2} + \frac{\gamma}{1+nI_h} I_h \\
 &= \frac{E'_h}{E_h} + \frac{\alpha\phi I_v}{1+mI_v} - \mu_1 + \frac{\gamma}{(1+nI_h)^2} \frac{E_h b_v}{I_v \mu_2} \\
 &\quad - \frac{\gamma}{(1+nI_h)} E_h - \frac{\gamma}{1+nI_h} \frac{I_h b_v}{I_v \mu_2} \\
 &\leq \frac{E'_h}{E_h} + \frac{\alpha\phi I_v}{1+mI_v} - \mu_1 + \frac{\gamma}{(1+nI_h)^2} \frac{E_h b_v}{I_v \mu_2} \\
 &\quad - \frac{\gamma}{(1+nI_h)^2} E_h - \frac{\gamma}{(1+nI_h)^2} \frac{I_h b_v}{I_v \mu_2} \\
 &\leq \frac{E'_h}{E_h} + \frac{\alpha\phi I_v}{1+mI_v} - \mu_1 + \frac{\gamma}{(1+nI_h)^2} \frac{a_h b_v}{\mu_1 I_v \mu_2} \\
 &\quad - \frac{\gamma}{(1+nI_h)^2} E_h - \frac{\gamma}{(1+nI_h)^2} \frac{c b_v}{I_v \mu_2}.
 \end{aligned}
 \tag{41}$$

So,

$$\begin{aligned}
 g_2 &\leq \frac{E'_h}{E_h} + \frac{\alpha\phi I_v}{1+mI_v} - \mu_1 \\
 &\quad + \frac{\gamma}{(1+nI_h)^2} I_v \frac{b_v}{\mu_2} \left(\frac{a_h}{\mu_1} - c \right) - \frac{\gamma}{(1+nI_h)^2} E_h
 \end{aligned}$$

$$\begin{aligned}
 &< \frac{E'_h}{E_h} + \frac{\alpha\phi I_v}{1+mI_v} - \mu_1 \\
 &< \frac{E'_h}{E_h} + \frac{\alpha\phi I_v}{m} - \mu_1 \\
 &< \frac{E'_h}{E_h} + \frac{\alpha\phi M}{m} - \mu_1,
 \end{aligned}
 \tag{42}$$

if $\mu_1 > a_h/c$ and $M = b_v/\mu_2$.

Thus,

$$\mu(B) = \sup \{g_1, g_2\} \leq \frac{E'_h}{E_h} - \frac{\mu_1}{2},
 \tag{43}$$

if $\mu_1 > 2\alpha\phi M/m$.

Since (2) is uniformly persistent when $R_0 > 1$, so for $T > 0$ such that $t > T$ implies that $E_h(t) \geq c$, $I_h(t) \geq c$, $I_v(t) \geq c$, and $(1/t) \log I_h(t) < \mu_1/4$ for all $(S_h(0), I_h(0), I_v(0)) \in K$. Thus,

$$\frac{1}{t} \int_0^t \mu(B) dt < \frac{\log E_h(t)}{t} - \frac{\mu_1}{2} < \frac{-\mu_1}{4},
 \tag{44}$$

for all $(S_h(0), I_h(0), I_v(0)) \in K$. The condition (H_3) is satisfied provided that $\mu_1 > \max\{a_h/c, 2\alpha\phi M/m\}$. Therefore, all the conditions of Theorem 7 are satisfied. Hence, unique endemic equilibrium E^* is globally stable in Ω .

6. Discussion

We know that the basic reproduction number of the model R_0 is proportional to the total number of the host tree population available as oviposition sites for the vector beetles and the number of vector population and host infectious rates α and vector infectious rate β , respectively. The basic reproduction number R_0 does not depend on m, n definitely; numerical simulations indicate that when the disease is endemic, the steady state value of the exposed host E_h^* , infected host I_h^* decreases as m increases (see Figures 1 and 2), and the steady state value of the infective vector I_v^* decreases as n increases (see Figure 3). The numerical simulations are carried out using $S_h(0) = 300, E_h(0) = 30, I_h(0) = 20, S_v(0) = 65, I_v(0) = 20, a_h = 0.009041, b_v = 0.002691, \alpha = 0.00166, \phi = 0.2, \beta = 0.057142, \gamma = 0.00305, \mu_1 = 0.0000301, \mu_2 = 0.011764, m_1 = 0.01, m_2 = 0.03, m_3 = 0.07, m_4 = 0.09, n = 0.01, n_1 = 0.02, \text{ and } n_2 = 0.03$. Furthermore, from the expression of the basic reproduction number, we can observe that more effective control strategy seems to reduce the total number of infection and the rates of transmission and decrease the carrying capacity of the environment for vector beetles using conventional controls, such as aerial spraying of pesticide to kill pine sawyer adults, injection procedures and physical treatment (chipping and burning), or chemical treatment of wilt pines to kill their larvae.

This paper presents a host-vector model for pine wilt disease with nonlinear incidence rate. The mathematical analysis is carried out for a model for forest insect pests with pine wilt disease. The global dynamics of the model are shown

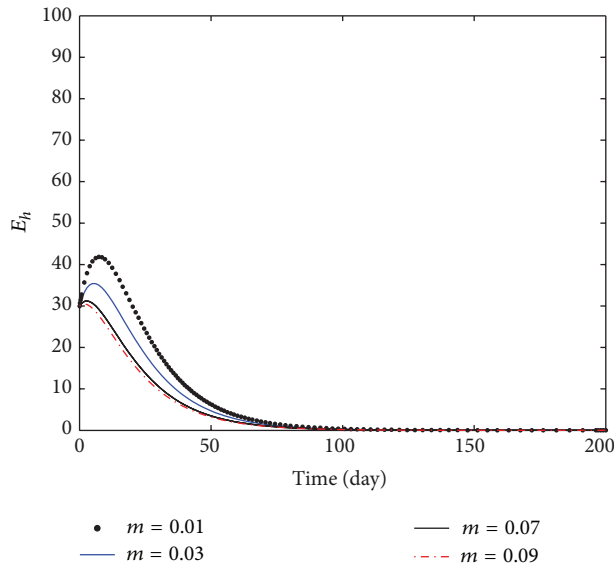


FIGURE 1: Plot of the exposed host population.

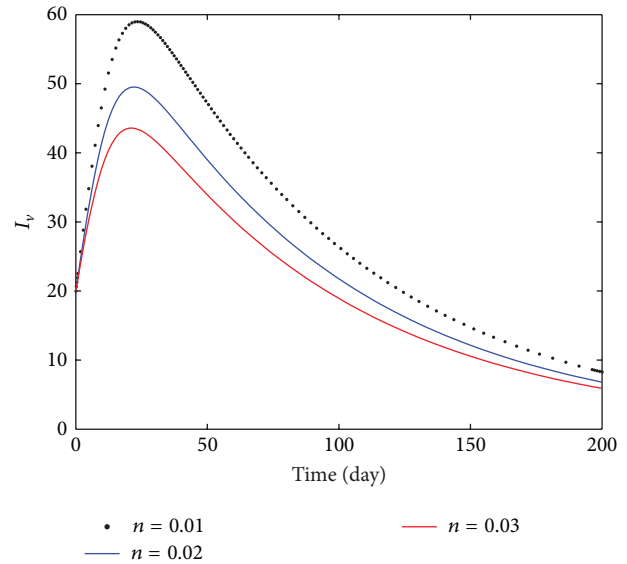


FIGURE 3: Plot of the case infected vector population.

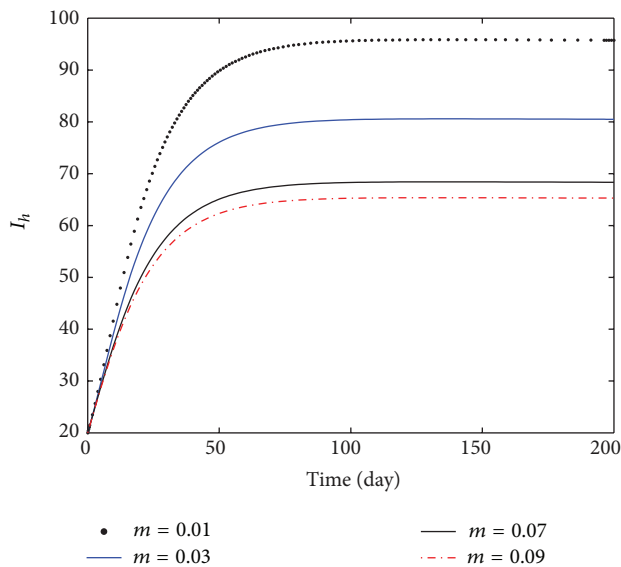


FIGURE 2: Plot of the infected host population.

to be determined by the basic reproduction number R_0 . More specifically, by constructing suitable Lyapunov function, we proved that if $R_0 \leq 1$, then disease-free equilibrium E_0 is globally asymptotically stable in Γ , and thus the disease always dies out. If $R_0 > 1$, the unique endemic equilibrium E^* exists and is globally asymptotically stable, so that the disease persists at the endemic equilibrium if it is initially present.

Conflict of Interests

The authors declared that there is no conflict of interests.

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