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Research Article

Geometrical Variable Weights Buffer GM(1,1) Model and Its Application in Forecasting of China's Energy Consumption

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In order to improve the application area and the prediction accuracy of GM(1,1) model, a novel Grey model is proposed in this paper. To remedy the defects about the applications of traditional Grey model and buffer operators in medium- and long-term forecasting, a Variable Weights Buffer Grey model is proposed. The proposed model integrates the variable weights buffer operator with the background value optimized GM(1,1) model to implement dynamic preprocessing of original data. Taking the maximum degree of Grey incidence between fitting value and actual value as objective function, then the optimal buffer factor is chosen, which can improve forecasting precision, make forecasting results embodying the internal trend of original data to the maximum extent, and improve the stability of the prediction. To verify the effectiveness of the proposed model, the energy consumption in China from 2002 to 2009 is used for the modeling to forecast the energy consumption in China from 2010 to 2020, and the forecasting results prove that the GVGM(1,1) model has remarkably improved the forecasting ability of medium- and long-term energy consumption in China.

1. Introduction

With the rapid growth of China's economy, industrialization, and urbanization, energy consumption shows a rapid growth trend in recent years [1]. Accurate energy consumption forecasting, particularly in medium- and long-term forecasts for China's energy consumption, is significant to China's energy demand planning and the sustained and healthy economic development. The changes of China's energy consumption can be regarded as a typical Grey system problem.

Grey system was introduced by Deng early in 1982 [2]; then it is developed quickly and applied extensively in the fields of forecasting science such as in industry, economy, and the stock price forecasting [3–5]. So far, it is important for the study of handling the uncertainty of raw data comprehensively to lessen error from modeling fitting by requiring a small amount of data [6, 7]. There are many scholars proposing new procedures to improve the precision of GM(1,1) model, such as Wang et al. [8], Li et al. [9], Zhao et al. [10], Lee and Tong [11], Xie and Liu [12], Chen et al. [13], Tangkuman and Yang [14], and Chen and Guo [15]. Recently, by extending the data transforming approach, Li et al. [16] proposed the EGM(1,1) to generalize the building procedure for the Grey model to grasp the data outline and information trend. For the purpose of establishing a Grey model with Grey number, Shih et al. [17] studied the Grey modification GM(1,1) model. Zhou and He [18] proposed a novel generalized GM(1,1) model to overcome the limited samples to provide better forecasting advantage for short-term problems. To overcome the disadvantages existing in the original Grey model, the GM(1,1) model, Truong and Ahn [19] investigated a novel Grey model named "Smart Adaptive Grey Model, SAGM (1,1)", and so on.

Although those improved Grey models have been successfully adopted in various fields and they have provided us with promising results, the research on accumulated generating operator of Grey model is not included. To deal with this problem, Liu and Lin [20] introduced the concept of buffer operator which was based on the principle of new information priority. By using the buffer operator in the original data preprocessing and then establishing GM(1,1) model, we can weaken the randomness and strengthen the development trend of the raw data; then we can improve the forecasting accuracy of GM(1,1) model. But during the practical forecasting applications, buffer operator still has some problems: the traditional buffer operator's effect intensity is not adjustable; we have to choose a fixed buffer operator based on our subjective judgments and qualitative analysis for data preprocessing. Besides, the effect of buffering was often too weak or too strong and had a poor adaptability [21]. In order to solve this problem, a novel Grey model based on geometrical variable weights buffer operator is proposed in this paper. Practical example shows that the proposed GVGM(1,1) model has higher performance on model prediction.

This paper is organized as follows. In Section 2, we establish the GVGM(1,1) model; in Section 3, we present an empirical analysis of energy consumption in China and compare GVGM(1,1) model with GM(1,1) and GAGM(1,1) models; in Section 4, we draw some conclusions.

2. Modeling and Methodology

In this section, we research on the proposed geometrical variable weights buffer GM(1,1) model, GVGM(1,1) model. We will introduce the establishing process of GVGM(1,1) model and the method of determining the optimal weights.

2.1. GM(1,1) Model. GM(1,1) model is one of the most frequently used Grey forecasting models. This model is a time series forecasting model, encompassing a group of differential equations adapted for parameter variance, rather than a first-order differential equation. Its difference equations have structures that vary with time rather than being general difference equations. The process of GM(1,1) model is described below.

Definition 1. Assume that $X^{(0)} = [X^{(0)}(1), X^{(0)}(2), \ldots, X^{(0)}(n)]$ is a nonnegative sequence of raw data, its 1-AGO sequence $X^{(1)}$ is $X^{(1)} = [X^{(1)}(1), X^{(1)}(2), \ldots, X^{(1)}(n)]$, where

$$X^{(1)}(k) = \sum_{i=1}^{k} X^{(0)}(i), \quad k = 1, 2, \ldots, n. \tag{1}$$

And the sequence mean generated of consecutive neighbors of $X^{(1)}$ is

$$Z^{(1)} = \left[Z^{(1)}(1), Z^{(1)}(2), \ldots, Z^{(1)}(n)\right], \tag{2}$$

where

$$Z^{(1)}(k) = 0.5X^{(1)}(k-1) + (1-\eta) X^{(1)}(k), \quad k = 1, 2, \ldots, n. \tag{3}$$

The equation

$$X^{(0)}(k) + aZ^{(1)}(k) = b \tag{4}$$

is called the basic form of the GM(1,1) model.

The equation

$$\widehat{X}^{(1)}(k+1) = \left(X^{(0)}(1) - \frac{b}{a}\right) e^{-a(k-1)} + \frac{b}{a}, \quad k = 1, 2, \ldots, n \tag{5}$$

is called the time response sequence of GM(1,1).

Then

$$\widehat{X}^{(0)}(k+1) = \left(1 - e^{a}\right)\left(X^{(0)}(1) - \frac{b}{a}\right) e^{-ak}. \tag{6}$$

From the above modeling, we know that GM(1,1) model is a special modeling approach, based on Grey exponential law resulted from accumulated generation operation, which has ideal fitting prediction effect for the data sequence with the homogeneous exponential characteristics.

2.2. Geometrical Average Weight Buffered GM(1,1) Model. Buffer operator is a core tool for the forecast of impact disturbance system in the Grey system theory [22] and it can effectively solve the problem of the unconformity between the quantitative results and the qualitative analysis in the process of modeling and forecast for the impact disturbance data sequence. Based on the principle of new information priority, the novel geometrical average buffer operator is constructed as follows.

Theorem 2. *Assume that $X = [x(1), x(2), \ldots, x(n)]$ is a nonnegative sequence of raw data, and*

$$\begin{gathered} Y = XD_1 = \left[x(1) d_1, x(2) d_1, \ldots, x(n) d_1\right], \\ y(k) = x(k) d_1 = (x(n))^{0.5} \cdot (x(k))^{0.5}, \\ x(k) > 0, \quad k = 1, 2, \ldots, n. \end{gathered} \tag{7}$$

And the sequence $XD_1 = [x(1)d_1, x(2)d_1, \ldots, x(n)d_1]$ is called geometrical average weight buffered sequence. When X is a monotonic increasing, a monotonic decreasing, or a vibrational sequence, D_1 is always a weakening buffer operator. The GM(1,1) model with geometrical average weights buffered sequence is called geometrical average weight buffer GM(1,1) model, abbreviated as GAGM(1,1) model.

2.3. Geometrical Variable Weights Buffered GM(1,1) Model

2.3.1. Geometrical Variable Weight Buffer Operator. Because the effect intensity of the novel geometrical average buffer operator is not adjustable, the original time sequence's data preprocessing is not strongly practical; the buffering effect is often too weak or too strong. So we introduce a geometrical variable weight weakening buffer operator, in order to achieve a dynamic preprocessing of the original data.

Theorem 3. *Assume that $X = [x(1), x(2), \ldots, x(n)]$ is the nonnegative sequence of raw data, and*

$$\begin{gathered} Y = XD_2 = \left[x(1) d_2, x(2) d_2, \ldots, x(n) d_2\right], \\ y(k) = x(k) d_2 = (x(n))^{\lambda} \cdot (x(k))^{1-\lambda}, \\ 0 \le \lambda \le 1, \ x(k) > 0, \ k = 1, 2, \ldots, n. \end{gathered} \tag{8}$$

When X is a monotonic increasing, a monotonic decreasing, or a vibrational sequence, where for any variable weight $0 \le \lambda \le 1$, $k = 1, 2, \ldots, n$, D_2 is always a weakening buffer operator and called geometrical variable weights weakening buffer operator.

And the sequence

$$Y = \left[(x(n))^{\lambda} \cdot (x(1))^{1-\lambda}, (x(n))^{\lambda} \cdot (x(2))^{1-\lambda}, \ldots, (x(n))^{\lambda} \cdot (x(n-1))^{1-\lambda}, x(n) \right] \tag{9}$$

is called geometrical variable weights buffered sequence.

Definition 4. Let $X = [x(1), x(2), \ldots, x(n)]$ be a nonnegative sequence of raw data, $r(k)$ is the average changing rate from $x(k)$ to $x(n)$, and $XD_2 = [x(1)d_2, x(2)d_2, \ldots, x(n)d_2]$ is the sequence which is affected by the geometrical variable weight buffer operator D_2; then

$$\delta(k) = \left| \frac{r(k) - r(k)d}{r(k)} \right|, \quad k = 1, 2, \ldots, n, \tag{10}$$

where (10) is called regulation degree of geometrical variable weight weakening buffer operator at the point k.

Definition 5. The change of regulation degree of geometric variable weight weakening buffer operator D_2 at each point is in the same direction with change of λ, and

$$\delta(k) = \frac{(x(n))^{\lambda} \cdot (x(k))^{1-\lambda} - x(k)}{x(n) - x(k)}, \quad k = 1, 2, \ldots, n, \tag{11}$$

where $\delta(k)$ reflects the effect intensity of the buffer operator to the sequence of raw data. It is obvious that by dynamically adjusting the value of λ, we can solve the problem of the buffering effect and then enhance the flexibility, controllability, and adaptability of buffer operator in the original data preprocessing.

2.3.2. Geometrical Variable Weights Buffered GM(1,1) Model. In this paper, we use the geometrical variable weight weakening buffer operator to achieve a dynamic preprocessing of the original data. Similarly, we also use a variable weight in the sequence generated of consecutive neighbors. Therefore, the model contains two variable weights. Modeling steps are as follows.

Step 1. Assume that $X^{(0)} = [x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(n)]$ is a nonnegative sequence of raw data; we use geometrical variable weight buffer operator to affect the sequence

$$Y^{(0)} = \left[y^{(0)}(1), y^{(0)}(2), \ldots, y^{(0)}(n) \right],$$
$$y^{(0)}(k) = \left(x^{(0)}(n)\right)^{\lambda} \cdot \left(x^{(0)}(k)\right)^{1-\lambda}, \tag{12}$$
$$k = 1, 2, \ldots, n.$$

Step 2. We can get its 1-AGO sequence

$$Y^{(1)} = \left[y^{(1)}(1), y^{(1)}(2), \ldots, y^{(1)}(n) \right],$$
$$y^{(1)}(k) = \sum_{i=1}^{k} y^{(0)}(i), \tag{13}$$
$$k = 1, 2, \ldots, n.$$

Step 3. The sequence variable weight generated of consecutive neighbors of $Y^{(1)}$ is

$$Z^{(1)} = \left[z^{(1)}(1), z^{(1)}(2), \ldots, z^{(1)}(n) \right],$$
$$z^{(1)}(k) = \eta y^{(1)}(k-1) + (1-\eta) y^{(1)}(k), \tag{14}$$
$$k = 1, 2, \ldots, n.$$

In (14), η is the generated variable weight, where $0 \le \eta \le 1$.

Step 4. Then the GVGM(1,1) model is as follows:

$$y^{(0)}(k) + a z^{(1)}(k) = b, \tag{15}$$

where a and b can be solved by the generalized least squares method:

$$[a, b]^T = \left(B^T B\right)^{-1} B^T Y, \tag{16}$$

where

$$B = \begin{pmatrix} -z^{(1)}(2) & 1 \\ -z^{(1)}(3) & 1 \\ \vdots & \vdots \\ -z^{(1)}(n) & 1 \end{pmatrix}, \qquad Y = \begin{pmatrix} y^{(0)}(2) \\ y^{(0)}(3) \\ \vdots \\ y^{(0)}(n) \end{pmatrix}. \tag{17}$$

Step 5. GVGM(1,1) time-response sequence is

$$\hat{y}^{(1)}(k) = \left(y^{(0)}(1) - \frac{b}{a} \right) e^{-a(k-1)} + \frac{b}{a}, \quad k = 1, 2, \ldots, n. \tag{18}$$

Step 6. By performing IAGO on $\hat{y}^{(1)}(k)$, the predicted value of $\hat{y}^{(0)}(k)$ is

$$\hat{y}^{(0)}(k) = \hat{y}^{(1)}(k) - \hat{y}^{(1)}(k-1). \tag{19}$$

And then we can get the predicted value sequence $\widehat{X}^{(0)} = [\hat{x}^{(0)}(1), \hat{x}^{(0)}(2), \ldots, \hat{x}^{(0)}(n)]$.

Here, we call the model as geometrical variable weights buffer GM(1,1) model, abbreviated as GVGM(1,1) model.

2.3.3. The Optimization of GVGM(1,1) Model. From the process of building a GVGM(1,1) model, we can see that when the raw data $X^{(0)}(k)$ is determined, the only factor to affect the accuracy of simulation and prediction is the values of λ and η. It is usually using a certain error criterion to find the optimal

λ and η. However, considering that the GVGM(1,1) model uses a weakening buffer operator, if we use that criterion, it will lead to the result of deviating from the nature of the predicting problem. In order to avoid this problem, we introduce a method to determine the optimal λ and η. It is based on the criterion of the maximum degree of Grey incidence between the simulated values and raw data. By using this method, we can maintain the internal tendency of the original sequence to the maximum extent.

Definition 6. Assume that $X = [X(1), X(2), \ldots, X(n)]$ is the nonnegative sequence of raw data and $\widehat{Y} = [\widehat{y}(1), \widehat{y}(2), \ldots, \widehat{y}(n)]$ is the sequence of predicted values based on GVGM(1,1) model; then the degree of grey incidence of X and $\widehat{Y}$ is as follows:

$$\gamma\left(X, \widehat{Y}\right) = \frac{1}{n}\sum_{i=1}^{n}\gamma\left(x(k), \widehat{y}(k)\right), \tag{20}$$

where

$$\gamma\left(x(k), \widehat{y}(k)\right) = \frac{\min_k \left|x(k) - \widehat{y}(k)\right| + 0.5\max_k \left|x(k) - \widehat{y}(k)\right|}{\left|x(k) - \widehat{y}(k)\right| + 0.5\max_k \left|x(k) - \widehat{y}(k)\right|}. \tag{21}$$

The degree of grey incidence reflects the similarity between the two sequences. The larger the value is, the better the sequence of predicted values internal tendency of the original sequence will be kept. Then we can get a more accurate result of simulation and prediction. So we use the criterion of the maximum degree of Grey incidence

$$f(\lambda, \eta) = \max \frac{1}{n}\sum_{i=1}^{n}\gamma\left(x^{(0)}(k), \widehat{y}^{(0)}(k)\right), \tag{22}$$

to determine the optimal λ and η.

In order to evaluate the forecast capability of the model, the forecasting accuracy is examined by calculating the mean relative error (MRE). It is expressed as follows:

$$\text{MRE} = \frac{1}{n}\sum_{i=1}^{n}\left|\frac{\widehat{x}^{(0)}(i) - x^{(0)}(i)}{x^{(0)}(i)}\right|, \tag{23}$$

where $\widehat{x}^{(0)}(i)$ and $x^{(0)}(i)$ are forecasting and actual values in ith, respectively, and n is the total number of predictions. Lewis [23] interprets the MRE results as a method to judge the accuracy of forecast, where more than 50% is an inaccurate forecast, 20%–50% is a reasonable forecast, 10%–20% is a good forecast, and less than 10% is a highly accurate forecast.

3. Empirical Results and Analysis

3.1. Data Analysis. The annual data on energy consumption for 1998–2012 in this paper is collected from National Bureau of Statistics of China (NBSC). Total energy consumption is measured in million ton coal equivalent (Mtce).

From Figure 1 we can see that the growth rate before 2007 is evidently faster than the second half. If the forecast is directly based on the raw data, the average prediction error will not be acceptable. In order to hold the energy consumption trends timely and accurately and make scientific and rational forecast on the energy consumption, we must process the data that develops slowly and make them meet the development trend and then forecast them on this basis.

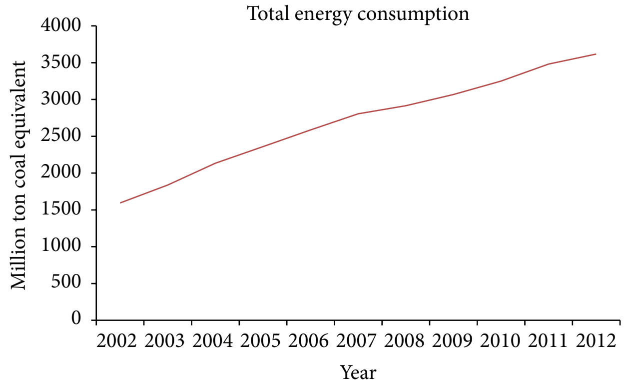

Figure 1: Time series plots of China's energy consumption 2002–2012.

3.2. Forecasting Results. In this study, the predictive capability of GVGM(1,1) model is compared with GM(1,1) and GAGM(1,1) models using eight-year (2002–2009) data as the in-sample period. This in-sample period is used to construct the models, and out-of-sample period is used to evaluate the prediction accuracy by using the MRE. We use Mathematica 8.0 software to calculate the optimal λ and η, where $\lambda = 0.2203$ and $\eta = 0.9032$. The fitting results of the three models are as shown in Tables 1 and 2.

As it is shown in Table 1, the GVGM(1,1) model has a good fitting performance, and GM(1,1) model has the best fitting performance among the three models. However, the degrees of Grey incidence of GM(1,1), GAGM(1,1), and GVGM(1,1) models are 0.628, 0.691, and 0.733, so we can see that the GVGM(1,1) model maintains the internal tendency of the original sequence to the maximum extent. According to Table 2, we can see that the GVGM(1,1) model has a strong forecasting performance because the MRE is 0.92%, which is less than GM(1,1) and GAGM(1,1) models. GVGM(1,1) model has remarkably improved the forecasting ability of Grey prediction model. Finally, this study uses the GVGM(1,1) model to forecast China's energy consumption from 2010 to 2020. The forecast results are shown in Table 3.

4. Conclusions

In this paper, it is attempted to model and forecast China's energy consumption based on GVGM(1,1) model using the historical data from 2002 to 2009; the GVGM(1,1) model displayed robust results in terms of MRE, compared with both GM(1,1) and GAGM(1,1) models. The MRE of GVGM(1,1) for out-of-sample is 0.92%. GVGM(1,1) can be used safely to predict China's energy consumption in the future. According to Table 3, China's energy consumption will reach 4033.83

Table 1: In-sample comparisons among the GVGM, GAGM, and GM models.

Time	Actual value	GVGM(1,1)		GAGM(1,1)		GM(1,1)	
		Fitting value	Relative error (%)	Fitting value	Relative error (%)	Fitting value	Relative error (%)
2002	1594.31	1841.43	15.5	2211.09	38.69	1594.31	0.00
2003	1837.92	2114.24	15.03	2450.39	33.32	1974.70	7.44
2004	2134.56	2246.20	5.23	2551.44	19.53	2135.96	0.07
2005	2359.97	2386.40	1.12	2656.67	12.57	2310.39	2.10
2006	2586.76	2535.34	1.99	2766.23	6.94	2499.06	3.39
2007	2805.08	2693.59	3.97	2880.31	2.68	2703.15	3.63
2008	2914.48	2861.71	1.81	2999.10	2.90	2923.89	0.32
2009	3066.47	3040.32	0.85	3122.79	1.84	3162.67	3.14
MRE (%)			5.69		14.81		2.51
The degree of Grey incidence		0.733		0.691		0.628	

Table 2: Out-of-sample comparisons among the GVGM, GAGM, and GM models.

Time	Actual value	GVGM(1,1)		GAGM(1,1)		GM(1,1)	
		Forecasting value	Relative error (%)	Forecasting value	Relative error (%)	Forecasting value	Relative error (%)
2010	3249.39	3230.08	0.59	3251.57	0.07	3420.94	5.28
2011	3480.02	3431.68	1.39	3385.67	2.71	3700.30	6.33
2012	3617.32	3645.87	0.79	3525.30	2.54	4002.48	10.65
MRE (%)			0.92		1.77		7.42

Table 3: Forecast of China's energy consumption from 2010 to 2020.

Year	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020
Energy consumption	3249.39	3480.02	3617.32	3707.30	3867.12	4033.83	4120.79	4260.99	4405.95	4470.48	4586.71

million tons coal equivalent in 2015. But the target of China's energy consumption is less than 4000 million tons equivalent coal by the end of 12th Five-Year Plan. So the target will be failed. The forecast results show us that China's energy consumption continues to increase sharply from 3249.39 to 4586.71 million tons coal equivalent (average annual rate of 4.1%) from 2010 to 2020. As the largest developing country in the world, China has a long way to go in order to achieve the sustainable development by improving energy efficiency and energy structures.

Conflict of Interests

The authors declare no conflict of interests.

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