

## Research Article

# The Simplest Equation Method and Its Application for Solving the Nonlinear NLSE, KGZ, GDS, DS, and GZ Equations

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A good idea of finding the exact solutions of the nonlinear evolution equations is introduced. The idea is that the exact solutions of the elliptic-like equations are derived using the simplest equation method and the modified simplest equation method, and then the exact solutions of a class of nonlinear evolution equations which can be converted to the elliptic-like equation using travelling wave reduction are obtained. For example, the perturbed nonlinear Schrödinger's equation (NLSE), the Klein-Gordon-Zakharov (KGZ) system, the generalized Davey-Stewartson (GDS) equations, the Davey-Stewartson (DS) equations, and the generalized Zakharov (GZ) equations are investigated and the exact solutions are presented using this method.

## 1. Introduction

Nonlinear phenomena exist in all areas of science and engineering, such as fluid mechanics, plasma physics, optical fibers, biology, solid state physics, chemical kinematics, and chemical physics. It is well known that many nonlinear partial differential equations (NLPDEs) are widely used to describe these complex physical phenomena. The exact solution of a differential equation gives information about the construction of complex physical phenomena. Therefore, seeking exact solutions of NLPDEs has long been one of the central themes of perpetual interest in mathematics and physics. With the development of symbolic computation packages, like Maple and Mathematica, many powerful methods for finding exact solutions have been proposed, such as the homogeneous balance method [1, 2], the auxiliary equation method [3], the sine-cosine method [4], the Jacobi elliptic function method [5], the exp-function method [6], the tanh-function method [7, 8], the Darboux transformation [9, 10], and the  $(G'/G)$ -expansion method [11, 12].

The simplest equation method is a very powerful mathematical technique for finding exact solutions of nonlinear ordinary differential equations. It has been developed by Kudryashov [13, 14] and used successfully by many authors for finding exact solutions of ODEs in mathematical physics [15–19].

In this paper, we first apply the simplest equation method and the modified simplest equation method to derive the exact solutions of the elliptic-like equation, and then the exact solutions of a class of nonlinear evolution equations which can be converted to the elliptic-like equation using travelling wave reduction are obtained.

## 2. Description of Methods

### 2.1. The Simplest Equation Method

*Step 1.* Suppose that we have a nonlinear partial differential equation (PDE) for  $u(x, t)$  in the form

$$N(u, u_t, u_x, u_{tt}, u_{xt}, u_{xx}, \dots) = 0, \quad (1)$$

where  $N$  is a polynomial in its arguments.

*Step 2.* By taking  $u(x, t) = u(\xi)$ ,  $\xi = x - ct$ , we look for traveling wave solutions of (1) and transform it to the ordinary differential equation (ODE)

$$N(u, -cu', u', c^2u'', -cu'', u'', \dots) = 0. \quad (2)$$

Step 3. Suppose the solution  $u$  of (2) can be expressed as a finite series in the form

$$u = \sum_{i=0}^n A_i (H(\xi))^i, \quad (3)$$

where  $H(\xi)$  satisfies the Bernoulli or Riccati equation,  $n$  is a positive integer that can be determined by balancing procedure, and  $A_i$  ( $i = 0, 1, 2, \dots, n$ ) are parameters to be determined.

The Bernoulli equation we consider in this paper is

$$H'(\xi) = aH(\xi) + bH^2(\xi), \quad (4)$$

where  $a$  and  $b$  are constants. Its solutions can be written as

$$H(\xi) = \frac{-aC_1}{b(C_1 + \cosh(a(\xi + \xi_0)) - \sinh(a(\xi + \xi_0)))}, \quad (5)$$

$$H(\xi) = \frac{-a(\cosh(a(\xi + \xi_0)) + \sinh(a(\xi + \xi_0)))}{b(C_2 + \cosh(a(\xi + \xi_0)) + \sinh(a(\xi + \xi_0)))},$$

where  $C_1, C_2$ , and  $\xi_0$  are constants.

For the Riccati equation

$$H'(\xi) = aH^2(\xi) + bH(\xi) + s, \quad (6)$$

where  $a, b$ , and  $s$  are constants, we will use the solutions

$$H(\xi) = -\frac{b}{2a} - \frac{\theta}{2a} \tanh\left[\frac{\theta}{2}(\xi + \xi_0)\right],$$

$$H(\xi) = -\frac{b}{2a} - \frac{\theta}{2a} \tanh\left(\frac{\theta\xi}{2}\right) + \frac{\operatorname{sech}(\theta\xi/2)}{C \cosh(\theta\xi/2) - (2a/\theta) \sinh(\theta\xi/2)}, \quad (7)$$

where  $\theta^2 = b^2 - 4as$ .

Step 4. Substituting (3) into (2) with (4) (or (6)), then the left hand side of (2) is converted into a polynomial in  $H(\xi)$ ; equating each coefficient of the polynomial to zero yields a set of algebraic equations for  $A_i, a, b$  ( $i = 0, 1, 2, \dots, n$ ). Solving the algebraic equations by symbolic computation, we can determine those parameters explicitly.

Step 5. Assuming that the constants  $A_i, a, b$  ( $i = 0, 1, 2, \dots, n$ ) can be obtained in Step 4 and substituting the results into (3), then we obtain the exact traveling wave solutions for (1).

2.2. *The Modified Simplest Equation Method.* In the modified version, one makes an ansatz for the solution  $u(\xi)$  as

$$u = \sum_{i=0}^n a_i \left( \frac{\psi'(\xi)}{\psi(\xi)} \right)^i, \quad (8)$$

where  $a_i$  ( $i = 0, 1, 2, \dots, n$ ) are arbitrary constants to be determined, such that  $a_n \neq 0$  and  $\psi(\xi)$  is an unspecified function to be determined afterward.

Substitute (8) into (2) and then we account the function  $\psi(\xi)$ . As a result of this substitution, we get a polynomial of  $\psi'(\xi)/\psi(\xi)$  and its derivatives. In this polynomial, we equate the coefficients of the same power of  $\psi^{-j}(\xi)$  to zero, where  $j \geq 0$ . This procedure yields a system of equations which can be solved to find  $a_i$  ( $i = 0, 1, 2, \dots, n$ ),  $\psi(\xi)$ , and  $\psi'(\xi)$ . Then the substitution of the values of  $a_i$  ( $i = 0, 1, 2, \dots, n$ ),  $\psi(\xi)$ , and  $\psi'(\xi)$  into (8) completes the determination of exact solutions of (1).

### 3. Solutions of the Elliptic-Like Equation

Now, let us choose the following elliptic-like equation

$$A\phi''(\xi) + B\phi(\xi) + C\phi^3(\xi) = 0, \quad (9)$$

where  $A, B$ , and  $C$  are arbitrary constants. Equation (9) is one of the most important auxiliary equations, because many nonlinear evolution equations can be converted to (9) using the travelling wave reduction.

#### 3.1. Using Simplest Equation Method

3.1.1. *Solutions of (9) Using the Bernoulli Equation as the Simplest Equation.* Considering the homogeneous balance between  $\phi''(\xi)$ , and  $\phi^3(\xi)$  we get  $n = 1$ , so the solution of (9) is the form

$$\phi(\xi) = A_0 + A_1 H(\xi). \quad (10)$$

Substituting (10) into (9) and making use of the Bernoulli equation (4) and then equating the coefficients of the functions  $H^i(\xi)$  to zero, we obtain an algebraic system of equations in terms of  $A_i$  ( $i = 0, 1$ ),  $a$ , and  $b$ . Solving this system of algebraic equations, with the aid of Maple, we obtain

$$A_0 = \pm i \sqrt{\frac{B}{C}}, \quad A_1 = \pm i \sqrt{\frac{2A}{C}}, \quad a = \sqrt{\frac{2B}{A}}, \quad (11)$$

$$A_0 = \pm i \sqrt{\frac{B}{C}}, \quad A_1 = \mp i \sqrt{\frac{2A}{C}}, \quad a = -\sqrt{\frac{2B}{A}}.$$

Therefore, using solutions (5) of (4) and ansatz (10), we obtain the following exact solution of (9):

$$\phi_1(\xi) = \pm i \sqrt{\frac{B}{C}} \left( 1 - (2C_1) \times \left( C_1 + \cosh\left(\sqrt{\frac{2B}{A}}(\xi + \xi_0)\right) - \sinh\left(\sqrt{\frac{2B}{A}}(\xi + \xi_0)\right) \right)^{-1} \right),$$

$$\begin{aligned}
 \phi_2(\xi) &= \pm i \sqrt{\frac{B}{C}} \left( 1 - (2C_1) \right. \\
 &\quad \times \left( C_1 + \cosh \left( \sqrt{\frac{2B}{A}} (\xi + \xi_0) \right) \right. \\
 &\quad \left. \left. + \sinh \left( \sqrt{\frac{2B}{A}} (\xi + \xi_0) \right) \right)^{-1} \right), \\
 \phi_3(\xi) &= \pm i \sqrt{\frac{B}{C}} \left( 1 - \left( 2 \left( \cosh \left( \sqrt{\frac{2B}{A}} (\xi + \xi_0) \right) \right. \right. \right. \\
 &\quad \left. \left. + \sinh \left( \sqrt{\frac{2B}{A}} (\xi + \xi_0) \right) \right) \right) \\
 &\quad \times \left( C_2 + \cosh \left( \sqrt{\frac{2B}{A}} (\xi + \xi_0) \right) \right. \\
 &\quad \left. \left. + \sinh \left( \sqrt{\frac{2B}{A}} (\xi + \xi_0) \right) \right)^{-1} \right), \\
 \phi_4(\xi) &= \pm i \sqrt{\frac{B}{C}} \left( 1 - \left( 2 \left( \cosh \left( \sqrt{\frac{2B}{A}} (\xi + \xi_0) \right) \right. \right. \right. \\
 &\quad \left. \left. - \sinh \left( \sqrt{\frac{2B}{A}} (\xi + \xi_0) \right) \right) \right) \\
 &\quad \times \left( C_2 + \cosh \left( \sqrt{\frac{2B}{A}} (\xi + \xi_0) \right) \right. \\
 &\quad \left. \left. - \sinh \left( \sqrt{\frac{2B}{A}} (\xi + \xi_0) \right) \right)^{-1} \right). \tag{12}
 \end{aligned}$$

3.1.2. Solutions of (9) Using Riccati Equation as the Simplest Equation. Suppose the solutions of (9) are the form

$$\phi(\xi) = B_0 + B_1 H(\xi). \tag{13}$$

Substituting (13) into (9) and making use of the Riccati equation (6) and then equating the coefficients of the functions  $H^i(\xi)$  to zero, we obtain an algebraic system of equations in terms of  $B_i$  ( $i = 0, 1$ ),  $a, b$ , and  $s$ . Solving this system of algebraic equations, with the aid of Maple, one possible set of values of  $B_i$  ( $i = 0, 1$ ),  $a, b$ , and  $s$  is

$$B_0 = \pm i \sqrt{\frac{A}{2C}} b, \quad B_1 = \pm i \sqrt{\frac{2A}{C}} a, \quad s = \frac{Ab^2 - 2B}{4Aa}. \tag{14}$$

Therefore, using solutions (7) of (6) and ansatz (13), we obtain the following exact solution of (9):

$$\begin{aligned}
 \phi_5(\xi) &= \mp i \sqrt{\frac{B}{C}} \tanh \left( \sqrt{\frac{B}{2A}} (\xi + \xi_0) \right), \\
 \phi_6(\xi) &= \mp i \sqrt{\frac{B}{2C}} \left( \left( C \sqrt{\frac{2B}{A}} \sinh \left( \sqrt{\frac{B}{2A}} \xi \right) \right. \right. \\
 &\quad \left. \left. - 2a \cosh \left( \sqrt{\frac{B}{2A}} \xi \right) \right) \right. \\
 &\quad \left. \times \left( C \sqrt{\frac{B}{A}} \cosh \left( \sqrt{\frac{B}{2A}} \xi \right) \right. \right. \\
 &\quad \left. \left. - a \sqrt{2} \sinh \left( \sqrt{\frac{B}{2A}} \xi \right) \right)^{-1} \right). \tag{15}
 \end{aligned}$$

3.2. Using Modified Simplest Equation Method. Suppose the solution of (9) is the form

$$\phi(\xi) = a_0 + a_1 \left( \frac{\psi'(\xi)}{\psi(\xi)} \right), \tag{16}$$

where  $a_0$  and  $a_1$  are constants, such that  $a_1 \neq 0$ , and  $\phi(\xi)$  is an unspecified function to be determined. It is simple to calculate that

$$\begin{aligned}
 \phi' &= a_1 \left( \frac{\psi''}{\psi} - \left( \frac{\psi'}{\psi} \right)^2 \right), \\
 \phi'' &= a_1 \left( \frac{\psi'''}{\psi} \right) - 3a_1 \left( \frac{\psi''\psi'}{\psi^2} \right) + 2a_1 \left( \frac{\psi'}{\psi} \right)^3, \tag{17} \\
 \phi^3 &= a_1^3 \left( \frac{\psi'}{\psi} \right)^3 + 3a_1^2 a_0 \left( \frac{\psi'}{\psi} \right)^2 + 3a_1 a_0^2 \left( \frac{\psi'}{\psi} \right) + a_0^3.
 \end{aligned}$$

Substituting the values of  $\phi$ ,  $\phi''$ , and  $\phi^3$  into (9) and equating the coefficients of  $\psi^0, \psi^{-1}, \psi^{-2}$ , and  $\psi^{-3}$  to zero yield

$$\psi^0 : Ca_0^3 + Ba_0 = 0, \tag{18}$$

$$\psi^{-1} : (3Ca_0^2 + B)\psi' + A\psi''' = 0, \tag{19}$$

$$\psi^{-2} : -3A\psi'' + 3Ca_0 a_1 \psi' = 0, \tag{20}$$

$$\psi^{-3} : (Ca_1^3 + 2Aa_1)(\psi')^3 = 0. \tag{21}$$

Solving (18), we obtain

$$a_0 = 0, \quad a_0 = \pm i \sqrt{\frac{B}{C}}. \tag{22}$$

And solving (21), we obtain

$$a_1 = \pm i \sqrt{\frac{2A}{C}}, \quad \text{since } a_1 \neq 0. \tag{23}$$

Case 1. When  $a_0 = 0$ , we obtain trivial solution; therefore, the case is rejected.

Case 2. When  $a_0 = \pm i\sqrt{B/C}$  and  $a_1 = \pm i\sqrt{2A/C}$ , (19) and (20) yield

$$\frac{\psi'''}{\psi''} + \sqrt{\frac{2B}{A}} = 0. \tag{24}$$

Integrating (24) with respect to  $\xi$ , we obtain

$$\psi'' = C_1 \exp\left(-\sqrt{\frac{2B}{A}}\xi\right). \tag{25}$$

Using (25), from (20), we obtain

$$\psi' = -\sqrt{\frac{A}{2B}}C_1 \exp\left(-\sqrt{\frac{2B}{A}}\xi\right). \tag{26}$$

Upon integration, we obtain

$$\psi = \frac{AC_1}{2B} \exp\left(-\sqrt{\frac{2B}{A}}\xi\right) + C_2, \tag{27}$$

where  $C_1$  and  $C_2$  are constants of integration. Therefore, the exact solution of (9) is

$$\phi_7(\xi) = \pm i\sqrt{\frac{B}{C}} \left(1 - \frac{2AC_1 \exp(-\sqrt{(2B/A)}\xi)}{AC_1 \exp(-\sqrt{(2B/A)}\xi) + 2BC_2}\right). \tag{28}$$

From (28), we obtain the exact solution of (9) which is

$$\begin{aligned} \phi_8(\xi) = \pm i\sqrt{\frac{B}{C}} \left(1 - \left(2AC_1 \left(\cosh\left(\sqrt{\frac{B}{2A}}\xi\right) - \sinh\left(\sqrt{\frac{B}{2A}}\xi\right)\right)\right) \right. \\ \left. \times \left((2BC_2 + AC_1) \cosh\left(\sqrt{\frac{B}{2A}}\xi\right) + (2BC_2 - AC_1) \sinh\left(\sqrt{\frac{B}{2A}}\xi\right)\right)^{-1}\right). \end{aligned} \tag{29}$$

We can arbitrarily choose the parameters  $C_1$  and  $C_2$ . Therefore, if we set  $C_1 = 2BC_2/A$ , (29) reduces to

$$\phi_9(\xi) = \pm i\sqrt{\frac{B}{C}} \tanh\left(\sqrt{\frac{B}{2A}}\xi\right). \tag{30}$$

Again setting  $C_1 = -(2BC_2/A)$ , (29) reduces to,

$$\phi_{10}(\xi) = \pm i\sqrt{\frac{B}{C}} \coth\left(\sqrt{\frac{B}{2A}}\xi\right). \tag{31}$$

Using hyperbolic function identities, from (30) and (31), we obtain the following periodic solutions

$$\phi_{11}(\xi) = \pm\sqrt{\frac{B}{C}} \tan\left(\sqrt{\frac{-B}{2A}}\xi\right), \tag{32}$$

$$\phi_{12}(\xi) = \pm\sqrt{\frac{B}{C}} \cot\left(\sqrt{\frac{-B}{2A}}\xi\right).$$

#### 4. Exact Solutions of Some Class of NLPDEs

4.1. The Perturbed Nonlinear Schrödinger's Equation (NLSE) in the Form [20]. Using

$$\begin{aligned} iu_t + u_{xx} + \alpha u|u|^2 \\ + i\left[\gamma_1 u_{xxx} + \gamma_2 |u|^2 u_x + \gamma_3 u(|u|^2)_x\right] = 0, \end{aligned} \tag{33}$$

where  $\gamma_1$  is the third order dispersion,  $\gamma_2$  is the nonlinear dispersion, while  $\gamma_3$  is also a version of nonlinear dispersion. We assume that (33) has exact solution in the form

$$u(x, t) = \phi(\xi) \exp(i(\lambda x - \omega t)), \quad \xi = k(x - ct), \tag{34}$$

where  $\lambda, \omega, k$ , and  $c$  are arbitrary constant to be determined. Substituting (34) into (33), removing the common factor  $\exp(i(\lambda x - \omega t))$ , we have

$$\begin{aligned} i(\gamma_1 k^3 \phi''' - k(c - 2\lambda + 3\gamma_1 \lambda^2) \phi' + k(\gamma_2 + 2\gamma_3) \phi^2 \phi') \\ + k^2(1 - 3\gamma_1 \lambda) \phi'' + (\omega - \lambda^2 + \gamma_1 \lambda^3) \phi \\ + (\alpha - \gamma_2 \lambda) \phi^3 = 0, \end{aligned} \tag{35}$$

where  $\gamma_i$  ( $i = 1, 2$ , and  $3$ ),  $\alpha$ , and  $k$  are positive constants and the prime means differentiation with respect to  $\xi$ . Then we have two equations as follows

$$\gamma_1 k^2 \phi''' - (c - 2\lambda + 3\gamma_1 \lambda^2) \phi' + (\gamma_2 + 2\gamma_3) \phi^2 \phi' = 0, \tag{36}$$

$$k^2(1 - 3\gamma_1 \lambda) \phi'' + (\omega - \lambda^2 + \gamma_1 \lambda^3) \phi + (\alpha - \gamma_2 \lambda) \phi^3 = 0. \tag{37}$$

Integrating (36) with respect to  $\xi$  once and setting the integration constant to be zero, then we have

$$\gamma_1 k^2 \phi'' + (2\lambda - c - 3\gamma_1 \lambda^2) \phi + \left(\frac{1}{3}\gamma_2 + \frac{2}{3}\gamma_3\right) \phi^3 = 0. \tag{38}$$

As (37) and (38) have the same solutions, we have the following equation:

$$\frac{\gamma_1}{1 - 3\gamma_1 \lambda} = \frac{2\lambda - c - 3\gamma_1 \lambda^2}{\omega - \lambda^2 + \gamma_1 \lambda^3} = \frac{C}{\alpha - \gamma_2 \lambda}, \tag{39}$$

where  $C = (1/3)\gamma_2 + (2/3)\gamma_3$ .

From (39), we can obtain

$$\begin{aligned} \omega = \frac{(\alpha - \gamma_2 \lambda)(2\lambda - c - 3\gamma_1 \lambda^2)}{C} + \lambda^2 - \gamma_1 \lambda^3, \\ \lambda = \frac{C - \alpha \lambda_1}{3C\gamma_1 - \gamma_1 \gamma_2}. \end{aligned} \tag{40}$$

Based on the conclusion just mentioned, we only solve (38) or (37), instead of both (37) and (38), provided that (37) and (36) are replaced by (40), respectively, we get

$$A\phi''(\xi) + B\phi(\xi) + C\phi^3(\xi) = 0. \tag{41}$$

Equation (41) is identical to (9) and  $A, B,$  and  $C$  are defined by

$$A = \gamma_1 k^2, \quad B = 2\lambda - c - 3\gamma_1 \lambda^2, \quad C = \frac{1}{3}\gamma_2 + \frac{2}{3}\gamma_3. \tag{42}$$

Then, solutions of (33) are defined as follows:

$$u(x, t) = \phi(\xi) \exp(i(\lambda x - \omega t)), \quad \xi = k(x - ct),$$

$$\omega = \frac{(\alpha - \gamma_2 \lambda)(2\lambda - c - 3\gamma_1 \lambda^2)}{C} + \lambda^2 - \gamma_1 \lambda^3, \tag{43}$$

$$\lambda = \frac{C - \alpha \lambda_1}{3C\gamma_1 - \gamma_1 \gamma_2},$$

where  $\phi(\xi)$ , appearing in these solutions, is given by relations (12), (15), and (28)–(32).  $A, B,$  and  $C$  are defined by (42).

4.2. The Klein-Gordon-Zakharov (KGZ) System [21]. Consider

$$E_{tt} - E_{xx} + E - \alpha NE - \beta |E|^2 E = 0, \tag{44}$$

$$N_{tt} - N_{xx} = \gamma(|E|^2)_{xx},$$

wherein the complex valued unknown function  $E = E(x, t)$  denotes the fast time scale component of electric field raised by electrons, and the real valued unknown function  $N = N(x, t)$  represents the deviation of ion density.  $\alpha, \beta$  and  $\gamma$  are some real parameters.

We assume that

$$E(x, t) = \phi(\xi) \exp(i(kx + \omega t)), \quad N = N(\xi), \tag{45}$$

$$\xi = \omega x + kt.$$

Substituting (45) into (44), we have

$$-\alpha N\phi - \beta\phi^3 + (k^2 - \omega^2 + 1)\phi + (k^2 - \omega^2)\phi'' = 0, \tag{46}$$

$$(k^2 - \omega^2)N'' - \gamma\omega^2(2\phi'^2 + 2\phi\phi'') = 0. \tag{47}$$

Integrating (47) with respect to  $\xi$  twice and setting the integration constant to be zero, then we have

$$N = \frac{\gamma\omega^2}{k^2 - \omega^2}\phi^2. \tag{48}$$

Substituting (48) into (46), we have

$$A\phi''(\xi) + B\phi(\xi) + C\phi^3(\xi) = 0. \tag{49}$$

Equation (49) is identical to (9) and  $A, B,$  and  $C$  are defined by

$$A = k^2 - \omega^2, \quad B = k^2 - \omega^2 + 1, \tag{50}$$

$$C = -\left(\beta + \frac{\alpha\gamma\omega^2}{k^2 - \omega^2}\right).$$

Then, solutions of the Klein-Gordon-Zakharov (KGZ) system are defined as follows:

$$E(x, t) = \phi(\xi) \exp(i(kx + \omega t)), \quad N = \frac{\gamma\omega^2}{k^2 - \omega^2}\phi^2, \tag{51}$$

$$\xi = \omega x + kt,$$

where  $\phi(\xi)$  appearing in these solutions is given by relations (12), (15), and (28)–(32).  $A, B,$  and  $C$  are defined by (50).

4.3. A Class of Nonlinear Partial Differential Equations (NPDEs). We consider a class of NLPDEs with constant coefficients [22]

$$iu_t + \mu(u_{xx} + D_1 u_{yy}) + E_1 |u|^2 u + A_1 un = 0, \tag{52}$$

$$D_2 n_{tt} + (n_{xx} - E_2 u_{yy}) + A_2 (|u|^2)_{xx} = 0,$$

where  $\mu, D_i, E_i,$  and  $A_i$  ( $i = 1, 2$ ) are real constants and  $\mu \neq 0, D_1 \neq 0, A_1 \neq 0,$  and  $A_2 \neq 0$ . Equations (52) are a class of physically important equations. In fact, if one takes

$$\mu = \frac{1}{2}\kappa^2, \quad D_1 = 2\mu, \quad E_1 = \alpha, \quad A_1 = -1, \tag{53}$$

$$D_2 = 0, \quad E_2 = D_1, \quad A_2 = -2\alpha, \quad \kappa^2 = \pm 1,$$

then (52) represent the Davey-Stewartson (DS) equations [23]

$$iu_t + \frac{1}{2}\kappa^2(u_{xx} + \kappa^2 u_{yy}) + \alpha |u|^2 u - un = 0, \tag{54}$$

$$n_{xx} - \kappa^2 n_{yy} - 2\alpha (|u|^2)_{xx} = 0.$$

If one takes

$$n = n(x, t), \quad \text{that is, } n_y = 0, \quad \mu = 1, \tag{55}$$

$$D_1 = 0, \quad E_1 = -2\lambda, \tag{55}$$

$$E_2 = -1, \quad A_2 = -1, \quad A_1 = 2,$$

then (52) become generalized Zakharov (GZ) equations [24]

$$iu_t + u_{xx} - 2\lambda |u|^2 u + 2un = 0, \tag{56}$$

$$n_{tt} - n_{xx} + (|u|^2)_{xx} = 0.$$

Since  $u$  is a complex function, we assume that

$$u(x, t) = \phi(\xi) \exp(i(kx + ly - \Omega t)), \tag{57}$$

$$n = n(\xi) \quad \xi = px + qy - \omega t,$$

where both  $\phi(\xi)$  and  $n(\xi)$  are real functions and  $k, l, p, q, \Omega$ , and  $\omega$  are constants to be determined later. Substituting (57) into (52), we have the following ODE for  $\phi(\xi)$  and  $n(\xi)$ :

$$E_1\phi^3 + (\Omega - \mu(k^2 + D_1l^2) + A_1n)\phi + \mu(D_1q^2 + p^2)\phi'' + i(2\mu(pk + D_1ql) - \omega)\phi' = 0, \quad (58)$$

$$(D_2\omega^2 + p^2 - E_2q^2)n'' + A_2p^2(2\phi'^2 + 2\phi\phi'') = 0. \quad (59)$$

If we set

$$\omega = 2\mu(pk + D_1ql), \quad (60)$$

then (58) reduces to

$$E_1\phi^3 + (\Omega - \mu(k^2 + D_1l^2) + A_1n)\phi + \mu(D_1q^2 + p^2)\phi'' = 0. \quad (61)$$

Integrating (59) twice to  $\xi$ , we get

$$n = \frac{D}{D_2\omega^2 + p^2 - E_2q^2} - \frac{A_2p^2}{D_2\omega^2 + p^2 - E_2q^2}\phi^2, \quad (62)$$

where  $D$  is the integrating constant and Substituting (62) into (61) yields

$$A\phi'' + B\phi + C\phi^3 = 0. \quad (63)$$

Equation (63) is identical to (9) and  $A, B$ , and  $C$  are defined by

$$A = \mu(D_1q^2 + p^2), \quad (64)$$

$$B = \Omega - \mu(k^2 + D_1l^2) + \frac{A_1D}{D_2\omega^2 + p^2 - E_2q^2},$$

$$C = E_1 - \frac{A_1A_2p^2}{D_2\omega^2 + p^2 - E_2q^2}.$$

Then, solutions of (52) are defined as follows:

$$u(x, y, t) = \phi(\xi) \exp(i(kx + ly - \Omega t)),$$

$$\xi = px + qy - \omega t, \quad \omega = 2\mu(pk + D_1ql),$$

$$n(x, y, t) = \frac{D}{D_2\omega^2 + p^2 - E_2q^2} - \frac{A_2p^2}{D_2\omega^2 + p^2 - E_2q^2}\phi^2, \quad (65)$$

where  $\phi(\xi)$  appearing in these solutions is given by relations (12), (15), and (28)–(32) and  $A, B$ , and  $C$  are defined by (64).

We may obtain from (54) that

$$\omega = \kappa^2(pk + \kappa^2ql),$$

$$u(x, y, t) = \phi(\xi) \exp(i(kx + ly - \Omega t)), \quad (66)$$

$$n(x, y, t) = \frac{D}{\omega^2 - \kappa^2q^2} + \frac{2\alpha p^2}{\omega^2 - \kappa^2q^2}\phi^2,$$

where  $D$  is the integrating constant and then (54) reduce to

$$A\phi'' + B\phi + C\phi^3 = 0. \quad (67)$$

This equation coincides also with (9), where  $A, B$ , and  $C$  are defined as follows:

$$A = \frac{1}{2}\kappa^2(p^2 + \kappa^2q^2),$$

$$B = \Omega - \frac{1}{2}\kappa^2(k^2 + \kappa^2l^2) - \frac{D}{\omega^2 - \kappa^2q^2}, \quad (68)$$

$$C = \alpha - \frac{2\alpha p^2}{\omega^2 - \kappa^2q^2},$$

where  $\xi = px + qy - \omega t$ ,  $\phi(\xi)$  appearing in these solutions is given by relations (12), (15), and (28)–(32) and  $A, B$ , and  $C$  are defined by (68).

We may obtain from (56) that

$$\omega = 2pk,$$

$$u(x, t) = \phi(\xi) \exp(i(kx - \Omega t)), \quad (69)$$

$$n(x, t) = \frac{D}{p^2 - \omega^2} + \frac{p^2}{p^2 - \omega^2}\phi^2,$$

where  $D$  is the integrating constant and then (56) reduce to

$$A\phi'' + B\phi + C\phi^3 = 0. \quad (70)$$

This equation coincides also with (9), where  $A, B$ , and  $C$  are defined as follows:

$$A = p^2, \quad B = \Omega - k^2 + \frac{2D}{p^2 - \omega^2}, \quad (71)$$

$$C = 2\left(\frac{p^2}{p^2 - \omega^2} - \lambda\right),$$

where  $\xi = px - \omega t$ ,  $\phi(\xi)$  appearing in these solutions is given by relations (12), (15), and (28)–(32) and  $A, B$ , and  $C$  are defined by (71).

## 5. Conclusions

The simplest equation method is a very powerful mathematical technique for finding exact solutions of nonlinear ordinary differential equations, and the elliptic-like equation is one of the most important auxiliary equations because many nonlinear evolution equations, such as the perturbed nonlinear Schrödinger's equation, the Klein-Gordon-Zakharov system, the generalized Davey-Stewartson equations, the Davey-Stewartson equations, the generalized Zakharov equations, the Hamilton amplitude equation, the generalized Hirota-Satsuma coupled KdV system, and the generalized ZK-BBM equation, can be converted to this equation using the travelling wave reduction.

In this paper, we apply the simplest equation method and the modified simplest equation method to derive the exact solutions of the elliptic-like equation. The exact solutions of

the perturbed nonlinear Schrödinger's equation, the Klein-Gordon-Zakharov system, the generalized Davey-Stewartson equations, the Davey-Stewartson equations, and the generalized Zakharov equations are derived. Comparing the currently proposed method with other methods, such as the  $(G'/G)$ -expansion method, the various extended hyperbolic methods, and the exp-function method, we might conclude that some exact solutions that we obtained can be investigated using these methods with the aid of the symbolic computation software, such as Matlab, Mathematica, and Maple to facilitate the complicated algebraic computations. But, by means of the simplest equation method and the modified simplest equation method the exact solutions to these equations have been gained in this paper without using the symbolic computation software since the computations are simple. This study shows that the simplest equation method and the modified simplest equation method are much more simple than the other methods and can be applied to many other nonlinear evolution equations.

### Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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