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Research Article

Global Behavior of a Discrete Survival Model with Several Delays

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The difference equation $y_{n+1} - y_n = -\alpha y_n + \sum_{j=1}^{m} \beta_j e^{-\gamma_j y_{n-k_j}}$ is studied and some sufficient conditions which guarantee that all solutions of the equation are oscillatory, or that the positive equilibrium of the equation is globally asymptotically stable, are obtained.

1. Introduction

The delay differential equation

$$\frac{dN(t)}{dt} = -\alpha N(t) + \beta e^{-\gamma N(t-\tau)}, \quad \text{for } t \geq 0, \tag{1}$$

was first proposed by Wazewska-Czyzewska and Lasota [1] as a model for the survival of red blood cell in an animal. Here, $N(t)$ denotes the number of red blood cells at time t, α is the probability of death of red blood cells, β and γ are positive constants which are related to the production of red blood cells, and τ is the time which is required to produce a red blood cell. The oscillation and global attractivity of (1) were studied by Győri and Ladas [2] and Li and Cheng [3], while the bifurcation and the direction of the stability were investigated by Song et al. [4]. Xu and Li [5] and Liu [6] considered its generalization with several delays and obtained sufficient conditions for the global stability of survival blood cells model with several delays and piecewise constant argument.

Research on the oscillation and global stability of the discrete analogue of (1), that is, for the equation

$$x_{n+1} - x_n = -\alpha x_n + \beta e^{-\gamma x_{n-k}}, \quad n = 0, 1, 2, \ldots, \tag{2}$$

where

$$\alpha \in (0, 1), \quad \beta, \gamma \in (0, \infty), \quad k \in \{1, 2, \ldots\}, \tag{3}$$

was proposed by Kocić and Ladas [7] as an open problem.

Kubiaczyk and Saker [8] investigated the oscillation of (2) about its positive equilibrium point $\overline{x}$, where $\overline{x}$ is the unique solution of the equation

$$\alpha \overline{x} = \beta e^{-\gamma \overline{x}}, \tag{4}$$

and showed that every solution of (2) oscillates about $\overline{x}$ if

$$\beta \gamma e^{-\gamma \overline{x}} > \left(\frac{k}{k+1}\right)^{k+1} (1-\alpha)^{k+1}. \tag{5}$$

Meng and Yan [9] investigated the global attractivity of the positive equilibrium point $\overline{x}$ and showed that $\overline{x}$ is a global attractor of all positive solutions of (2) if

$$\frac{\beta^2 \gamma^2}{\alpha^2} e^{\gamma (Q_1 + \overline{x})} < 1, \tag{6}$$

where $Q_1 = (\beta/\alpha) e^{-\beta\gamma/\alpha}$.

Zeng and Shi [10] established another condition for global attractivity of $\overline{x}$ and showed that $\overline{x}$ is a global attractor of all positive solutions of (2) if

$$\frac{\beta\gamma}{\alpha} \leq e. \tag{7}$$

Obviously, the condition (7) improves (6).

Kubiaczyk and Saker [8] also considered (2) when $k = 1$ and proved that $\overline{x}$ is a global attractor of all positive solutions of (2) provided that

$$\beta \gamma e^{-\gamma \overline{x}} < \alpha. \tag{7'}$$

Ma and Yu [11] proved that $\overline{x}$ is a global attractor of all solutions of (2) if

$$\gamma\overline{x}\left(1-(1-\alpha)^{k+1}\right)\leq 1. \tag{8}$$

By (2), we have

$$\overline{x}e^{\gamma\overline{x}}=\frac{\beta}{\alpha}. \tag{9}$$

So, if (7) holds, then we have $\gamma\overline{x}e^{\gamma\overline{x}} = \beta\gamma/\alpha \leq e$, which implies that $\gamma\overline{x}\leq 1$. Hence, (8) is satisfied. But, the converse is not true. So, the condition (8) improves (7).

In addition, we can also easily see that the conditions (7) and $(7')$ are equivalent to the condition $\gamma\overline{x}\leq 1$.

For the system with delay, many authors deemed that arbitrary finite number of discrete delays is more appropriate than the single discrete delay; see [12–14] and the references cited therein.

Stemming from the above discussion, the difference equation in the following form will be studied in this paper:

$$y_{n+1}-y_n=-\alpha y_n+\sum_{j=1}^{m}\beta_j e^{-\gamma_j y_{n-k_j}}, \tag{10}$$

where

$$\begin{gathered}\alpha\in(0,1),\quad \beta_j,\gamma_j\in(0,\infty),\quad k_j\in\{1,2,\dots\},\\ j=1,2,\dots,m;\quad \sum_{j=1}^{m}\beta_j=\beta.\end{gathered} \tag{11}$$

Besides, we denote that $k=\max_{1\leq j\leq m}\{k_j\}$, $l=\min_{1\leq j\leq m}\{k_j\}$, $\gamma^*=\min_{1\leq j\leq m}\{\gamma_j\}$.

Obviously, the case $m = 1$ is the form of (2). Besides, if $y_{-k}, y_{-k+1},\dots,y_{-1}, y_0 \in [0,\infty)$, then, the corresponding solution of (10) is positive, and (10) has a unique positive equilibrium point $\overline{y}$, which satisfies

$$\alpha\overline{y}=\sum_{j=1}^{m}\beta_j e^{-\gamma_j\overline{y}}. \tag{12}$$

The aim of this paper is to investigate the oscillation and the global asymptotic stability of (10).

2. Some Lemmas

Lemma 1 (see [7, page 6]). *Assume that $p_i\in(0,\infty)$ and $k_i\in\{0,1,\dots\}$ with $\sum_{i=1}^{m}(p_i+k_i)\neq 1$, $i=1,2,\dots,m$. Let $\{p_i(n)\}$ be sequences of positive numbers such that*

$$\liminf_{n\to\infty}p_i(n)\geqslant p_i\quad \text{for } i=1,2,\dots,m. \tag{13}$$

Suppose that the linear difference inequality

$$z_{n+1}-z_n+\sum_{i=1}^{m}p_i(n)z_{n-k_i}\leqslant 0\quad \text{for } n=0,1,\dots \tag{14}$$

has an eventually positive solution. Then, the difference equation

$$x_{n+1}-x_n+\sum_{i=1}^{m}p_i x_{n-k_i}=0 \tag{15}$$

has a positive solution.

Lemma 2 (see [7, page 5]). *Consider the linear homogeneous difference equation*

$$x_{n+k}+\sum_{i=1}^{k}q_i x_{n+k-i}=0\quad \text{for } n=0,1,\dots, \tag{16}$$

where k is a nonnegative integer and $q_i\in R$, $i=1,2,\dots,k$. Then, the following statements are equivalent:

(a) *every solution of* (16) *oscillates;*

(b) *the characteristic equation of* (16)

$$\lambda^k+\sum_{i=1}^{k}q_i\lambda^{k-i}=0 \tag{17}$$

has no positive roots.

Lemma 3 (see [7, page 12]). *Assume that $p_1,p_2,\dots,p_k\in R$ and k is a nonnegative integer. Then, $\sum_{i=1}^{k}|p_i|<1$ is a sufficient condition for the asymptotic stability of the difference equation*

$$x_{n+k}+p_1x_{n+k-1}+\cdots+p_kx_n=0\quad \text{for } n=0,1,\dots. \tag{18}$$

Lemma 4. *Assume that* (11) *holds, and $\{y_n\}$ is a solution of* (10) *with positive initial conditions $y_{-k},\dots,y_0$. Then,*

$$\limsup_{n\to\infty}y_n\leq\frac{\beta}{\alpha}. \tag{19}$$

Proof. Clearly, we have $y_n > 0$, for $n = -k,-k+1,\dots,0,1,2,\dots$. So by (10), we can find that

$$\begin{aligned}y_{n+1}&=(1-\alpha)y_n+\sum_{j=1}^{m}\beta_j e^{-\gamma_j y_{n-k_j}}\\&\leq(1-\alpha)y_n+\beta.\end{aligned} \tag{20}$$

Define a sequence $\{\omega_n\}$ by

$$\omega_{n+1}=(1-\alpha)\omega_n+\beta,\quad \omega_0=y_0. \tag{21}$$

Obviously,

$$y_n\leq\omega_n=(1-\alpha)^n\omega_0+\frac{\beta}{\alpha}\left[1-(1-\alpha)^n\right]. \tag{22}$$

So, we have

$$\limsup_{n\to\infty}y_n\leq\frac{\beta}{\alpha}. \tag{23}$$

□

Lemma 5. *Assume that* (11) *holds, and*

$$\sum_{j=1}^{m} \beta_j \gamma_j e^{-r_j \overline{y}} < \alpha. \tag{24}$$

Then, the positive equilibrium $\overline{y}$ *of* (10) *is locally asymptotically stable.*

Proof. To prove that the positive equilibrium $\overline{y}$ is locally asymptotically stable, it suffices to prove that the zero solution of the linear equation of (10) is locally asymptotically stable. The linearized equation associated with (10) about positive equilibrium $\overline{y}$ is

$$y_{n+1} = (1-\alpha)\, y_n - \sum_{j=1}^{m} \beta_j \gamma_j e^{-\gamma_j \overline{y}} y_{n-k_j}, \tag{25}$$

which satisfies

$$|1-\alpha| + \sum_{j=1}^{m} \left| \beta_j \gamma_j e^{-\gamma_j \overline{y}} \right| \le 1. \tag{26}$$

Then, by Lemma 3, the positive equilibrium solution $\overline{y}$ of (10) is locally asymptotically stable. □

Lemma 6 (see [15]). *The following system of inequalities,*

$$\mu \le e^{-\lambda} - 1, \quad \lambda \ge e^{-\mu} - 1, \tag{27}$$

with λ, μ *being real numbers, have exactly one solution* $\lambda = \mu = 0$.

3. Main Results

Theorem 7. *Assume that* (11) *holds, and*

$$\sum_{j=1}^{m} \beta_j \gamma_j e^{-\gamma_j \overline{y}} (1-\alpha)^{-l-1} > 1. \tag{28}$$

Then, every positive solution of (10) *oscillates about the positive equilibrium* $\overline{y}$.

Proof. Assume for the sake of contradiction that (10) has a positive solution $\{y_n\}$ which does not oscillate about $\overline{y}$. We assume that $y_n > \overline{y}$ eventually. If $y_n < \overline{y}$ eventually, the proof is similar and will be omitted. So, there exists an $n_0 \geqslant 0$ such that $y_n > \overline{y}$ for $n \geqslant n_0$, and consequently $y_{n-k} > \overline{y}$ for $n \geqslant n_1$, where $n_1 = n_0 + k$.

From Lemma 4, we have $\{y_n\}$ as a bounded sequence. In the following, we will claim that

$$\lim_{n\to\infty} y_n = \overline{y}. \tag{29}$$

Otherwise, let

$$\mu = \limsup_{n\to\infty} y_n. \tag{30}$$

Then, $\mu > \overline{y}$ and there exists a subsequence $\{y_{n_i}\}$ such that

$$\lim_{i\to\infty} y_{n_i+1} = \mu, \qquad y_{n_i+1} - y_{n_i} > 0 \quad \text{for } n_i \ge n_1,\ i = 1, 2, \ldots. \tag{31}$$

Equation (10) can be reformulated in the form

$$y_{n+1} = (1-\alpha)\, y_n + \sum_{j=1}^{m} \beta_j e^{-\gamma_j y_{n-k_j}}. \tag{32}$$

Then, from (31) and (32), we find that

$$\alpha y_{n_i+1} \le \sum_{j=1}^{m} \beta_j e^{-\gamma_j y_{n_i-k_j}}. \tag{33}$$

So, we obtain

$$\alpha\mu \leqslant \limsup_{i\to\infty} \sum_{j=1}^{m} \beta_j e^{-\gamma_j y_{n_i-k_j}} \le \sum_{j=1}^{m} \beta_j e^{-\gamma_j \overline{y}} = \alpha \overline{y}, \tag{34}$$

which is a contradiction. Accordingly, (29) holds.

Set

$$y_n = \overline{y} + x_n \quad \text{for } n = -k, -k+1, \ldots. \tag{35}$$

By the assumption $y_n > \overline{y}$, we have that x_n is an eventually positive solution of the difference equation

$$x_{n+1} - (1-\alpha)\, x_n + \alpha \overline{y} - \sum_{j=1}^{m} \beta_j e^{-\gamma_j (\overline{y} + x_{n-k_j})} = 0, \quad n = 0, 1, \ldots, \tag{36}$$

which can also be rewritten in the form

$$x_{n+1} - (1-\alpha)\, x_n + \sum_{j=1}^{m} p\left(n-k_j\right) x_{n-k_j} = 0, \quad n = 0, 1, \ldots, \tag{37}$$

where $p(n-k_j) = \left(\beta_j e^{-\gamma_j \overline{y}} - \beta_j e^{-\gamma_j(\overline{y} + x_{n-k_j})}\right) / \left(x_{n-k_j}\right)$, $j = 0, 1, \ldots, m$.

By some simple calculations and (29), we get

$$\lim_{n\to\infty} p\left(n-k_j\right) = \beta_j \gamma_j e^{-\gamma_j \overline{y}} > 0. \tag{38}$$

One can easily see that the hypothesis of Lemma 1 is satisfied and so the linear equation

$$x_{n+1} - (1-\alpha)\, x_n + \sum_{j=1}^{m} \beta_j \gamma_j e^{-\gamma_j \overline{y}} x_{n-k_j} = 0 \tag{39}$$

has an eventually positive solution.

Let $\{x_n\}$ be an eventually positive solution of (39); then $z_n = (1-\alpha)^{-n} x_n$ is an eventually positive solution of

$$z_{n+1} - z_n + \sum_{j=1}^{m} \beta_j \gamma_j e^{-\gamma_j \overline{y}} (1-\alpha)^{-k_j-1} z_{n-k_j} = 0, \quad n = 0, 1, \ldots. \tag{40}$$

Let

$$F(\lambda) = \lambda^{n+1} - \lambda^n + \sum_{j=1}^{m} \beta_j \gamma_j e^{-\gamma_j \overline{y}} (1-\alpha)^{-k_j-1} \lambda^{n-k_j} \tag{41}$$

be the characteristic polynomial of (40). Now, we prove that $F(\lambda) > 0$, for $\lambda > 0$.

If $\lambda \geqslant 1$, then obviously $F(\lambda) > 0$. Else if $0 < \lambda < 1$, we have

$$\begin{aligned} F(\lambda) &= \lambda^n \left(\lambda - 1 + \sum_{j=1}^{m} \beta_j \gamma_j e^{-\gamma_j \overline{y}} (1-\alpha)^{-k_j - 1} \lambda^{-k_j} \right) \\ &\geqslant \lambda^n \left(\lambda - 1 + \sum_{j=1}^{m} \beta_j \gamma_j e^{-\gamma_j \overline{y}} (1-\alpha)^{-l - 1} \lambda^{-k_j} \right) \\ &\geqslant \lambda^n \left(\lambda - 1 + \sum_{j=1}^{m} \beta_j \gamma_j e^{-\gamma_j \overline{y}} (1-\alpha)^{-l - 1} \right) \\ &\geq \lambda^{n+1} > 0. \end{aligned} \tag{42}$$

Therefore, the characteristic equation of (40)

$$\lambda^{n+1} - \lambda^n + \sum_{j=1}^{m} \beta_j \gamma_j e^{-\gamma_j \overline{y}} (1-\alpha)^{-k_j - 1} \lambda^{n-k_j} = 0 \tag{43}$$

has no positive roots.

According to Lemma 2, (40) has no nonoscillatory solution.

This is a contradiction. The proof is completed. □

Theorem 8. *Assume that* (11) *holds, and*

$$\overline{y} \left[1 - (1-\alpha)^{k+1} \right] \leq 1. \tag{44}$$

Then, the positive equilibrium $\overline{y}$ *of* (10) *is a global attractor of all positive solutions of* (10).

Proof. To prove that the positive equilibrium $\overline{y}$ is a global attractor of all positive solutions of (10), it suffices to show that (29) holds.

We will prove that (29) holds in each of the following two cases.

Case 1 ($\{y_n\}$ is nonoscillatory). Let $\{y_n\}$ be eventually positive. The case that $\{y_n\}$ is eventually positive is similar and will be omitted. So,there exists an $n_0 \geqslant 0$ such that $y_n > \overline{y}$ for $n \geqslant n_0$, and consequently $y_{n-k} > \overline{y}$ for $n \geqslant n_1$, where $n_1 = n_0 + k$.

From Lemma 4, we have $\{y_n\}$ as a bounded sequence. Assume for the sake of contradiction that (29) is not satisfied. Let

$$\mu = \limsup_{n \to \infty} y_n. \tag{45}$$

Then, $\mu > \overline{y}$ and there exists a subsequence $\{y_{n_i}\}$ such that

$$\lim_{i \to \infty} y_{n_i+1} = \mu, \qquad y_{n_i+1} - y_{n_i} > 0 \quad \text{for } n_i \geq n_1,\ i = 1, 2, \ldots. \tag{46}$$

It follows from (10) that

$$\alpha y_{n_i+1} \leq \sum_{j=1}^{m} \beta_j e^{-\gamma_j y_{n_i - k_j}}. \tag{47}$$

So, we obtain

$$\alpha \mu \leqslant \limsup_{i \to \infty} \sum_{j=1}^{m} \beta_j e^{-\gamma_j y_{n_i - k_j}} \leq \sum_{j=1}^{m} \beta_j e^{-\gamma_j \overline{y}} = \alpha \overline{y}, \tag{48}$$

which is a contradiction. Accordingly, (29) holds.

Case 2 ($\{y_n\}$ is strictly oscillatory). To show that (29) holds, it suffices to prove that $\lim_{n \to \infty} x_n = 0$ holds, when $\{x_n\}$ is a strictly oscillatory solution of (36).

To this end, let

$$\left\{ x_{p_i+1}, x_{p_i+2}, \ldots, x_{q_i} \right\} \tag{49}$$

be the ith positive semicycle of $\{x_n\}$ followed by the jth negative semicycle

$$\left\{ x_{q_i+1}, x_{q_i+2}, \ldots, x_s \right\}. \tag{50}$$

Let x_{M_i}, x_{m_i} be the extreme values in these two semicycles with the smallest possible indices M_i and m_i. Then, we claim that

$$M_i - p_i \leqslant k + 1, \qquad m_i - q_i \leqslant k + 1. \tag{51}$$

In the following, we will prove that (51) holds for positive semicycles, while for negative semicycles, the proof is similar and will be omitted. Assume for the sake of contradiction that the first inequality in (51) is not true. Then, $M_i - p_i > k + 1$ and the terms $x_{M_i - k - 1}, x_{M_i - k}, \ldots, x_{M_i - 1}$ are in a positive semicycle. Because of $x_{M_i} > x_{M_i - 1}$, (36) renders

$$\alpha x_{M_i} + \alpha \overline{y} \leq \sum_{j=1}^{m} \beta_j e^{-\gamma_j (\overline{y} + x_{M_i - k_j - 1})}. \tag{52}$$

So, we have

$$\begin{aligned} x_{M_i} &\leqslant \frac{1}{\alpha} \sum_{j=1}^{m} \beta_j e^{-\gamma_j (\overline{y} + x_{M_i - k_j - 1})} - \overline{y} \\ &= \frac{1}{\alpha} \left[\sum_{j=1}^{m} \beta_j e^{-\gamma_j (\overline{y} + x_{M_i - k_j - 1})} - \sum_{j=1}^{m} \beta_j e^{-\gamma_j \overline{y}} \right] \\ &= \frac{1}{\alpha} \left[\sum_{j=1}^{m} \beta_j e^{-\gamma_j \overline{y}} \left(e^{-\gamma_j x_{M_i - k_j - 1}} - 1 \right) \right]. \end{aligned} \tag{53}$$

So there exists at least a j s.t. $x_{M_i - k_j - 1} < 0$, which contradicts that $x_{M_i - k_j - 1}$ is in the positive semicycle. So, (51) is true. Noting that $\{y_n\}$ is bounded from Lemma 4, we can let

$$\begin{aligned} \lambda &= \liminf_{n \to \infty} x_n = \liminf_{i \to \infty} x_{m_i}, \\ \mu &= \limsup_{n \to \infty} x_n = \limsup_{i \to \infty} x_{M_i}. \end{aligned} \tag{54}$$

To prove that $\lim_{n \to \infty} x_n = 0$ holds, it is sufficient to show that $\lambda = \mu = 0$.

From (54), it follows that if $\epsilon \in (0, \lambda)$ is given, then there exists $n_2 \geqslant 0$ such that

$$\lambda - \epsilon \leqslant x_n \leqslant \mu + \epsilon \quad \text{for } n \geqslant n_2 + k. \tag{55}$$

Equation (36) can be reformulated in the form

$$x_{n+1} - (1-\alpha) x_n = -\alpha \overline{y} + \sum_{j=1}^{m} \beta_j e^{-\gamma_j (\overline{y} + x_{n-k_j})}. \tag{56}$$

Multiplying (56) by $(1-\alpha)^{-n-1}$ and then summing up from $n = p_i$ to $n = M_i - 1$ for i being sufficiently large, we get

$$\begin{aligned}(1-\alpha)^{-M_i} x_{M_i} - (1-\alpha)^{-p_i} x_{p_i} &= \sum_{n=p_i}^{M_i-1} (-\alpha\overline{y})(1-\alpha)^{-n-1} \\ &\quad + \sum_{n=p_i}^{M_i-1} \sum_{j=1}^{m} \beta_j e^{-\gamma_j(\overline{y}+x_{n-k_j})} (1-\alpha)^{-n-1}.\end{aligned} \tag{57}$$

From (55) and $x_{p_i} < 0$, we have

$$\begin{aligned}(1-\alpha)^{-M_i} x_{M_i} &\leqslant (-\alpha\overline{y}) \sum_{n=p_i}^{M_i-1} (1-\alpha)^{-n-1} \\ &\quad + \sum_{n=p_i}^{M_i-1} \sum_{j=1}^{m} \beta_j e^{-\gamma_j(\overline{y}+x_{n-k_j})} (1-\alpha)^{-n-1} \\ &\leq (-\alpha\overline{y}) \sum_{n=p_i}^{M_i-1} (1-\alpha)^{-n-1} \\ &\quad + \sum_{n=p_i}^{M_i-1} \sum_{j=1}^{m} \beta_j e^{-\gamma_j \overline{y}} e^{-\gamma^*(\lambda-\varepsilon)} (1-\alpha)^{-n-1} \\ &= (-\alpha\overline{y}) \sum_{n=p_i}^{M_i-1} (1-\alpha)^{-n-1} \\ &\quad + (\alpha\overline{y}) e^{-\gamma^*(\lambda-\varepsilon)} \sum_{n=p_i}^{M_i-1} (1-\alpha)^{-n-1} \\ &= (-\alpha\overline{y}) \frac{(1-\alpha)^{-M_i} - (1-\alpha)^{-P_i}}{\alpha} \\ &\quad + \alpha\overline{y} e^{-\gamma^*(\lambda-\varepsilon)} \frac{(1-\alpha)^{-M_i} - (1-\alpha)^{-P_i}}{\alpha} \\ &= \overline{y}\left[(1-\alpha)^{-M_i} - (1-\alpha)^{-P_i}\right]\left[e^{-\gamma^*(\lambda-\varepsilon)} - 1\right].\end{aligned} \tag{58}$$

So,

$$x_{M_i} \leqslant \overline{y}\left[e^{-\gamma^*(\lambda-\varepsilon)} - 1\right]\left[1 - (1-\alpha)^{M_i - P_i}\right]. \tag{59}$$

By using (54), ε is arbitrary and $M_i - p_i \leqslant k+1$; we get

$$\mu \leqslant \overline{y}\left[e^{-\gamma^*\lambda} - 1\right]\left[1 - (1-\alpha)^{k+1}\right]. \tag{60}$$

From the assumption of the theorem, we have

$$\mu \leqslant e^{-\gamma^*\lambda} - 1. \tag{61}$$

By the same trick as in proving (61), we can prove that

$$\lambda \geq e^{-\gamma^*\mu} - 1. \tag{62}$$

Therefore, by Lemma 6, we can get $\lambda = \mu = 0$; that is, $\lim_{n\to\infty} x_n = 0$, which implies that $\overline{y}$ is a global attractor of all positive solutions of (10).

By Lemma 3 and Theorem 8, we can get the following result. □

Theorem 9. *Suppose that* (11) *holds and that*

$$\sum_{j=1}^{m} \beta_j \gamma_j e^{-r_j \overline{y}} < \alpha, \quad \overline{y}\left[1 - (1-\alpha)^{k+1}\right] < 1. \tag{63}$$

Then, the positive equilibrium $\overline{y}$ is globally asymptotically stable.

Remark 10. From Theorem 7, it is clear that if the condition (28) holds, then the oscillation condition for $m = 1$ as established by Kubiaczyk and Saker [8] is already satisfied.

Remark 11. When $m = 1$, the condition of Theorem 8 is independent from the argument γ.

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