Hindawi Publishing Corporation
Abstract and Applied Analysis
Volume 2013, Article ID 891840, 2 pages
http://dx.doi.org/10.1155/2013/891840

Letter to the Editor

Comment on "Perturbation Analysis of the Nonlinear Matrix Equation $X - \sum_{i=1}^{m} A_i^* X^{pi} A_i = Q$"

Maher Berzig[1] and Erdal Karapınar[2]

[1] *Tunis College of Sciences and Techniques, Tunis University, 5 Avenue Taha Hussein, P.O. Box 56, Bab Manara, Tunis, Tunisia*
[2] *Department of Mathematics, Atilim University, Incek, 06836 Ankara, Turkey*

Correspondence should be addressed to Erdal Karapınar; erdalkarapinar@yahoo.com

Received 7 August 2013; Accepted 4 September 2013

Academic Editor: Ahmed El-Sayed

We show that the perturbation estimate for the matrix equation $X - \sum_{i=1}^{m} A_i^* X^{pi} A_i = Q$ due to J. Li, is wrong. Our discussion is supported by a counterexample.

1. Introduction and Preliminaries

The following definitions and the notations are the same as in [1]. We denote by $\mathscr{C}^{n\times n}$ the set of $n \times n$ complex matrices, by $\|\cdot\|$ the spectral norm, and by $\lambda_{\min}(M)$ the minimal eigenvalues of M.

Consider the matrix equation

$$X - \sum_{i=1}^{m} A_i^* X^{p_i} A_i = Q, \tag{1}$$

where $A_i \in \mathscr{C}^{n\times n}$ for $1 \le i \le m$. The existence and uniqueness of its positive definite solution X is proved in [2]. Next, consider the perturbed equation

$$\widetilde{X} - \sum_{i=1}^{m} \widetilde{A}_i^* \widetilde{X}^{p_i} \widetilde{A}_i = \widetilde{Q}, \tag{2}$$

where $0 < p_i < 1$ and $\widetilde{A}_i$ and $\widetilde{Q}$ are small perturbations of A_i and Q, respectively. We assume that X and $\widetilde{X}$ are solutions of (1) and (2), respectively. Let

$$\Delta X = \widetilde{X} - X, \qquad \Delta Q = \widetilde{Q} - Q, \qquad \Delta A_i = \widetilde{A}_i - A_i. \tag{3}$$

In [3, 4], some comments on perturbation estimates for particular cases of (1) and (2) have been furnished. In this note, we focus on the following recent result obtained by J. Li.

Theorem 1 (see [1, Theorem 5]). *Let*

$$\begin{gathered}\beta = \lambda_{\min}(Q) + \sum_{i=1}^{m} \lambda_{\min}(A_i^* A_i)\, \lambda_{\min}^{p_i}(Q), \\ b = \beta + \|\Delta Q\| - \sum_{i=1}^{m} p_i \beta^{p_i} \|A_i\|^2, \\ s = \sum_{i=1}^{m} \beta^{p_i} \|\Delta A_i\| \left(2\|A_i\| + \|\Delta A_i\|\right).\end{gathered} \tag{4}$$

If

$$\begin{gathered}0 < b < 2(\beta - s) \\ b^2 - 4(\beta - s)(s + \|\Delta Q\|) \ge 0,\end{gathered} \tag{5}$$

then

$$\frac{\|\widetilde{X} - X\|}{\|X\|} \le \rho \sum_{i=1}^{m} \|\Delta A_i\| + \omega \|\Delta Q\|, \tag{6}$$

where

$$\rho = \frac{2s}{\sum_{i=1}^{m} \|\Delta A_i\| \left(b + \sqrt{b^2 - 4(\beta - s)(s + \|\Delta Q\|)}\right)},$$
$$\omega = \frac{2}{b + \sqrt{b^2 - 4(\beta - s)(s + \|\Delta Q\|)}}. \tag{7}$$

2. Counterexample

The following counterexample shows that the perturbation estimates in Theorem 1 are not true in general. Consider

$$q = \frac{3}{4}, \quad m = 1, \quad A = \frac{1}{2},$$
$$\widetilde{A} = A + \frac{1}{10}, \quad X = 1, \quad \widetilde{X} = X + \frac{1}{100}. \tag{8}$$

Now, we compute Q and $\widetilde{Q}$ by using

$$Q = X - A^* X^q A, \qquad \widetilde{Q} = \widetilde{X} - \widetilde{A}^* \widetilde{X}^q \widetilde{A}, \tag{9}$$

so we get

$$Q = 0.75, \qquad \widetilde{Q} \approx 0.64730. \tag{10}$$

Finally, using (8)–(10), we obtain that the hypothesis of Theorem 1 is satisfied, that is,

$$0 < b \approx 0.66815 < 2(\beta - s) \approx 1.69102,$$
$$b^2 - 4(\beta - s)(s + \|\Delta Q\|) \approx 0.43535 \geq 0, \tag{11}$$

whereas

$$\frac{\|\widetilde{X} - X\|}{\|X\|} \approx 0.01000 \nleq \rho \sum_{i=1}^{m} \|\Delta A_i\| + \omega \|\Delta Q\| \approx 0.00491. \tag{12}$$

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

Authors' Contribution

All the authors contributed equally to this work and significantly in writing this paper. All the authors read and approved the final paper.

References

[1] J. Li, "Perturbation analysis of the nonlinear matrix equation $X - \sum_{i=1}^{m} A_i^* X^{p_i} A_i = Q$," *Abstract and Applied Analysis*, vol. 2013, Article ID 979832, 11 pages, 2013.

[2] X. Duan, A. Liao, and B. Tang, "On the nonlinear matrix equation $X - \sum_{i=1}^{m} A_i^* X^{\delta i} A_i = Q$," *Linear Algebra and its Applications*, vol. 429, no. 1, pp. 110–121, 2008.

[3] M. Berzig, "Comment to: perturbation estimates for the nonlinear matrix equation $X - A^* X^q A = Q (0 < q < 1)$ by G. Jia and D. Gao," *Journal of Applied Mathematics and Computing*, vol. 41, no. 1-2, pp. 501–503, 2013.

[4] X. Duan and M. Berzig, "A note concerning Gao's and Zhang's perturbation results of the matrix equation $X - A^* X^q A = I$," *Mathematica Numerica Sinica*, vol. 34, no. 4, pp. 447–447, 2013 (Chinese).