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Research Article

Regularization Method for the Approximate Split Equality Problem in Infinite-Dimensional Hilbert Spaces

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We studied the approximate split equality problem (ASEP) in the framework of infinite-dimensional Hilbert spaces. Let H_1, H_2, and H_3 be infinite-dimensional real Hilbert spaces, let $C \subset H_1$ and $Q \subset H_2$ be two nonempty closed convex sets, and let $A : H_1 \to H_3$ and $B : H_2 \to H_3$ be two bounded linear operators. The ASEP in infinite-dimensional Hilbert spaces is to minimize the function $f(x, y) = (1/2)\|Ax - By\|_2^2$ over $x \in C$ and $y \in Q$. Recently, Moudafi and Byrne had proposed several algorithms for solving the split equality problem and proved their convergence. Note that their algorithms have only weak convergence in infinite-dimensional Hilbert spaces. In this paper, we used the regularization method to establish a single-step iterative for solving the ASEP in infinite-dimensional Hilbert spaces and showed that the sequence generated by such algorithm strongly converges to the minimum-norm solution of the ASEP. Note that, by taking $B = I$ in the ASEP, we recover the approximate split feasibility problem (ASFP).

1. Introduction

Let $C \subseteq R^N$ and $Q \subseteq R^M$ be closed, nonempty convex sets, and let A and B be J by N and J by M real matrices, respectively. The split equality problem (SEP) in finite-dimensional Hilbert spaces is to find $x \in C$ and $y \in Q$ such that $Ax = By$; the approximate split equality problem (ASEP) in finite-dimensional Hilbert spaces is to minimize the function $f(x, y) = (1/2)\|Ax - By\|_2^2$ over $x \in C$ and $y \in Q$. When $J = M$ and $B = I$, the SEP reduces to the well-known split feasibility problem (SFP) and the ASEP becomes the approximate split feasibility problem (ASFP). For information on the split feasibility problem, please see [1–9].

In this paper, we work in the framework of infinite-dimensional Hilbert spaces. Let H_1, H_2, and H_3 be infinite-dimensional real Hilbert spaces, let $C \subset H_1$ and $Q \subset H_2$ be two nonempty closed convex sets, and let $A : H_1 \to H_3$ and $B : H_2 \to H_3$ be two bounded linear operators. The ASEP in infinite-dimensional Hilbert spaces is

$$\text{to minimize the function } f(x, y) = \frac{1}{2}\|Ax - By\|_2^2 \tag{1}$$

over $x \in C$ and $y \in Q$.

Very recently, for solving the SEP, Moudafi introduced the following alternating CQ-algorithms (ACQA) in [10]:

$$\begin{aligned} x_{k+1} &= P_C\left(x_k - \gamma_k A^*\left(Ax_k - By_k\right)\right), \\ y_{k+1} &= P_Q\left(y_k + \gamma_k B^*\left(Ax_{k+1} - By_k\right)\right). \end{aligned} \tag{2}$$

Then, he proved the weak convergence of the sequence $\{x_k, y_k\}$ to a solution of the SEP provided that the solution set $\Gamma := \{x \in C, y \in Q; Ax = By\}$ is nonempty and some conditions on the sequence of positive parameters (γ_k) are satisfied.

The ACQA involves two projections P_C and P_Q and, hence, might be hard to be implemented in the case where one of them fails to have a closed-form expression. So, Moudafi proposed the following relaxed CQ-algorithm (RACQA) in [11]:

$$\begin{aligned} x_{k+1} &= P_{C_k}\left(x_k - \gamma A^*\left(Ax_k - By_k\right)\right), \\ y_{k+1} &= P_{Q_k}\left(y_k + \beta B^*\left(Ax_{k+1} - By_k\right)\right), \end{aligned} \tag{3}$$

where C_k, Q_k were defined in [11], and then he proved the weak convergence of the sequence $\{x_k, y_k\}$ to a solution of the SEP.

In [12], Byrne considered and studied the algorithms to solve the approximate split equality problem (ASEP), which can be regarded as containing the consistent case and the inconsistent case of the SEP. There, he proposed a simultaneous iterative algorithm (SSEA) as follows:

$$x^{k+1} = P_C\left(x^k - \gamma_k A^T\left(Ax^k - By^k\right)\right),$$
$$y^{k+1} = P_Q\left(y^k + \gamma_k B^T\left(Ax^k - By^k\right)\right), \quad (4)$$

where $\epsilon \leq \gamma_k \leq (2/\rho(G^T G)) - \epsilon$. Then, he proposed the relaxed SSEA (RSSEA) and the perturbed version of the SSEA (PSSEA) for solving the ASEP, and he proved their convergence. Furthermore, he used these algorithms to solve the approximate split feasibility problem (ASFP), which is a special case of the ASEP. Note that he used the projected Landweber algorithm as a tool in that article.

Note that the algorithms proposed by Moudafi and Byrne have only weak convergence in infinite-dimensional Hilbert spaces. In this paper, we use the regularization method to establish a single-step iterative to solve the ASEP in infinite-dimensional Hilbert spaces, and we will prove its strong convergence.

2. Preliminaries

Let H be a real Hilbert space with inner product $\langle\cdot,\cdot\rangle$ and norm $\|\cdot\|$, respectively, and let K be a nonempty closed convex subset of H. Recall that the projection from H onto K, denoted as P_K, is defined in such a way that, for each $x \in H$, $P_K x$ is the unique point in K with the property

$$\|x - P_K x\| = \min\{\|x - y\| : y \in K\}. \quad (5)$$

The following important properties of projections are useful to our study.

Proposition 1. *Given that $x \in H$ and $z \in K$;*

(a) *$z = P_K x$ if and only if $\langle x - z, y - z\rangle \leq 0$, for all $y \in K$;*

(b) *$\langle P_K u - P_K v, u - v\rangle \geq \|P_K u - P_K v\|^2$, for all $u, v \in H$.*

Definition 2. A mapping $T : H \to H$ is said to be

(a) nonexpansive if

$$\|Tx - Ty\| \leq \|x - y\|, \quad \forall x, y \in H; \quad (6)$$

(b) firmly nonexpansive if $2T - I$ is nonexpansive, or equivalently,

$$\langle Tx - Ty, x - y\rangle \geq \|Tx - Ty\|^2, \quad \forall x, y \in H; \quad (7)$$

alternatively, T is firmly nonexpansive if and only if T can be expressed as

$$T = \left(\frac{1}{2}\right)(I + S), \quad (8)$$

where $S : H \to H$ is nonexpansive. It is well known that projections are (firmly) nonexpansive.

Definition 3. Let T be a nonlinear operator whose domain is $D(T) \subseteq H$ and whose range is $R(T) \subseteq H$.

(a) T is said to be monotone if

$$\langle Tx - Ty, x - y\rangle \geq 0, \quad \forall x, y \in D(T). \quad (9)$$

(b) Given a number $\beta > 0$, T is said to be β-strongly monotone if

$$\langle Tx - Ty, x - y\rangle \geq \beta\|x - y\|^2, \quad \forall x, y \in D(T). \quad (10)$$

(c) Given a number $L > 0$, T is said to be L-Lipschitz if

$$\|Tx - Ty\| \leq L\|x - y\|, \quad \forall x, y \in D(T). \quad (11)$$

Lemma 4 (see [13]). *Assume that a_n is a sequence of nonnegative real numbers such that*

$$a_{n+1} \leq (1 - \gamma_n) a_n + \gamma_n \delta_n, \quad n \geq 0, \quad (12)$$

where γ_n, δ_n are sequences of real numbers such that

(i) *$\gamma_n \subset (0, 1)$ and $\sum_{n=0}^{\infty} \gamma_n = \infty$;*

(ii) *either $\limsup_{n\to\infty} \delta_n \leq 0$ or $\sum_{n=1}^{\infty} \gamma_n |\delta_n| < \infty$.*

Then, $\lim_{n\to\infty} a_n = 0$.

Next, we will state and prove our main result in this paper.

3. Regularization Method for the ASEP

Let $S = C \times Q$. Define

$$G = [A \;\; -B], \quad \omega = \begin{bmatrix} x \\ y \end{bmatrix}. \quad (13)$$

The ASEP can now be reformulated as finding $\omega \in S$ with minimizing the function $\|G\omega\|$ over $\omega \in S$. Therefore, solving the ASEP (1) is equivalent to solving the following minimization problem (14).

The minimization problem

$$\min_{\omega\in S} f(\omega) = \frac{1}{2}\|G\omega\|^2 \quad (14)$$

is generally ill-posed. We consider the Tikhonov regularization (for more details about Tikhonov approximation, please see [8, 14] and the references therein)

$$\min_{\omega\in S} f_\varepsilon(\omega) = \frac{1}{2}\|G\omega\|^2 + \frac{1}{2}\varepsilon\|\omega\|^2, \quad (15)$$

where $\varepsilon > 0$ is the regularization parameter. The regularization minimization (15) has a unique solution which is denoted by ω_ε. Assume that the minimization (14) is consistent, and let $\omega_{\min}$ be its minimum-norm solution; namely, $\omega_{\min} \in \Gamma$ (Γ is the solution set of the minimization (14)) has the property

$$\|\omega_{\min}\| = \min\{\|\overline{\omega}\| : \overline{\omega} \in \Gamma\}. \quad (16)$$

The following result is easily proved.

Proposition 5. *If the minimization* (14) *is consistent, then the strong* $\lim_{\varepsilon\to 0}\omega_\varepsilon$ *exists and is the minimum-norm solution of the minimization* (14).

Proof. For any $\overline{\omega}\in\Gamma$, we have

$$f(\overline{\omega})+\frac{\varepsilon}{2}\|\omega_\varepsilon\|^2\le f(\omega_\varepsilon)+\frac{\varepsilon}{2}\|\omega_\varepsilon\|^2 = f_\varepsilon(\omega_\varepsilon)\le f_\varepsilon(\overline{\omega}) = f(\overline{\omega})+\frac{\varepsilon}{2}\|\overline{\omega}\|^2. \quad (17)$$

It follows that, for all $\varepsilon>0$ and $\overline{\omega}\in\Gamma$,

$$\|\omega_\varepsilon\|\le\|\overline{\omega}\|. \quad (18)$$

Therefore, ω_ε is bounded. Assume that $\varepsilon_j\to 0$ is such that $\omega_{\varepsilon_j}\rightharpoonup\omega^*$. Then, the weak lower semicontinuity of f implies that, for any $\omega\in S$,

$$\begin{aligned}f(\omega^*)&\le\liminf_{j\to\infty}f(\omega_{\varepsilon_j})\\&\le\liminf_{j\to\infty}f_{\varepsilon_j}(\omega_{\varepsilon_j})\\&\le\liminf_{j\to\infty}f_{\varepsilon_j}(\omega)\\&=\liminf_{j\to\infty}\left[f(\omega)+\frac{\varepsilon_j}{2}\|\omega\|^2\right]\\&=f(\omega).\end{aligned} \quad (19)$$

This means that $\omega^*\in\Gamma$. Since the norm is weak lower semicontinuous, we get from (18) that $\|\omega^*\|\le\|\overline{\omega}\|$ for all $\overline{\omega}\in\Gamma$; hence, $\omega^*=\omega_{\min}$. This is sufficient to ensure that $\omega_\varepsilon\rightharpoonup\omega_{\min}$. To obtain the strong convergence, noting that (18) holds for $\omega_{\min}$, we compute

$$\|\omega_\varepsilon-\omega_{\min}\|^2=\|\omega_\varepsilon\|^2-2\langle\omega_\varepsilon,\omega_{\min}\rangle+\|\omega_{\min}\|^2\le 2\left(\|\omega_{\min}\|^2-\langle\omega_\varepsilon,\omega_{\min}\rangle\right). \quad (20)$$

Since $\omega_\varepsilon\rightharpoonup\omega_{\min}$, we get $\omega_\varepsilon\to\omega_{\min}$ in norm. So, we complete the proof. □

Next we will state that $\omega_{\min}$ can be obtained by two steps. First, observing that the gradient

$$\nabla f_\varepsilon=\nabla f+\varepsilon I=G^TG+\varepsilon I \quad (21)$$

is $(\varepsilon+\|G\|^2)$-Lipschitz and ε-strongly monotone, the mapping $P_S(I-\gamma\nabla f_\varepsilon)$ is a contraction with coefficient

$$\sqrt{1-\gamma\left(2\varepsilon-\gamma\left(\|G\|^2+\varepsilon\right)^2\right)}\left(\le\sqrt{1-\varepsilon\gamma}\le 1-\frac{1}{2}\varepsilon\gamma\right), \quad (22)$$

where

$$0<\gamma\le\frac{\varepsilon}{\left(\|G\|^2+\varepsilon\right)^2}. \quad (23)$$

Indeed, observe that

$$\begin{aligned}&\|P_S(I-\gamma\nabla f_\varepsilon)(x)-P_S(I-\gamma\nabla f_\varepsilon)(y)\|^2\\&\le\|(I-\gamma\nabla f_\varepsilon)(x)-(I-\gamma\nabla f_\varepsilon)(y)\|^2\\&=\|x-y\|^2-2\gamma\langle\nabla f_\varepsilon(x)-\nabla f_\varepsilon(y),x-y\rangle\\&\quad+\gamma^2\|\nabla f_\varepsilon(x)-\nabla f_\varepsilon(y)\|^2\\&\le\left(1-2\gamma\varepsilon+\gamma^2\left(\varepsilon+\|G\|^2\right)^2\right)\|x-y\|^2\\&=\left[1-\gamma\left(2\varepsilon-\gamma\left(\varepsilon+\|G\|^2\right)^2\right)\right]\|x-y\|^2\\&\le(1-\varepsilon\gamma)\|x-y\|^2.\end{aligned} \quad (24)$$

Note that ω_ε is a fixed point of the mapping $P_S(I-\gamma\nabla f_\varepsilon)$ for any $\gamma>0$ satisfying (23) and can be obtained through the limit as $n\to\infty$ of the sequence of Picard iterates as follows:

$$\omega_{n+1}^\varepsilon=P_S(I-\gamma\nabla f_\varepsilon)\omega_n^\varepsilon. \quad (25)$$

Secondly, letting $\varepsilon\to 0$ yields $\omega_\varepsilon\to\omega_{\min}$ in norm. It is interesting to know whether these two steps can be combined to get $\omega_{\min}$ in a single step. The following result shows that for suitable choices of γ and ε, the minimum-norm solution $\omega_{\min}$ can be obtained by a single step, motivated by Xu [8].

Theorem 6. *Assume that the minimization problem* (14) *is consistent. Define a sequence* ω_n *by the iterative algorithm*

$$\omega_{n+1}=P_S\left(I-\gamma_n\nabla f_{\varepsilon_n}\right)\omega_n=P_S\left((1-\varepsilon_n\gamma_n)\omega_n-\gamma_nG^TG\omega_n\right), \quad (26)$$

where ε_n *and* γ_n *satisfy the following conditions:*

(i) $0<\gamma_n\le\varepsilon_n/(\|G\|^2+\varepsilon_n)^2$ *for all (large enough) n;*

(ii) $\varepsilon_n\to 0$ *and* $\gamma_n\to 0$*;*

(iii) $\sum_{n=1}^\infty\varepsilon_n\gamma_n=\infty$*;*

(iv) $(|\gamma_{n+1}-\gamma_n|+\gamma_n|\varepsilon_{n+1}-\varepsilon_n|)/(\varepsilon_{n+1}\gamma_{n+1})^2\to 0$.

Then, ω_n *converges in norm to the minimum-norm solution of the minimization problem* (14).

Proof. Note that for any γ satisfying (23), ω_ε is a fixed point of the mapping $P_S(I-\gamma\nabla f_\varepsilon)$. For each n, let z_n be the unique fixed point of the contraction

$$T_n:=P_S\left(I-\gamma_n\nabla f_{\varepsilon_n}\right). \quad (27)$$

Then, $z_n=\omega_{\varepsilon_n}$, and so

$$z_n\longrightarrow\omega_{\min}\ \text{in norm}. \quad (28)$$

Thus, to prove the theorem, it suffices to prove that

$$\|\omega_{n+1}-z_n\|\longrightarrow 0. \quad (29)$$

Noting that T_n has contraction coefficient of $(1 - (1/2)\varepsilon_n\gamma_n)$, we have

$$\begin{aligned}\|\omega_{n+1} - z_n\| &= \|T_n\omega_n - T_n z_n\| \\ &\le \left(1 - \frac{1}{2}\varepsilon_n\gamma_n\right)\|\omega_n - z_n\| \\ &\le \left(1 - \frac{1}{2}\varepsilon_n\gamma_n\right)\|\omega_n - z_{n-1}\| + \|z_n - z_{n-1}\|.\end{aligned} \tag{30}$$

We now estimate

$$\begin{aligned}\|z_n - z_{n-1}\| &= \|T_n z_n - T_{n-1}z_{n-1}\| \\ &\le \|T_n z_n - T_n z_{n-1}\| + \|T_n z_{n-1} - T_{n-1}z_{n-1}\| \\ &\le \left(1 - \frac{1}{2}\varepsilon_n\gamma_n\right)\|z_n - z_{n-1}\| \\ &\quad + \|T_n z_{n-1} - T_{n-1}z_{n-1}\|.\end{aligned} \tag{31}$$

This implies that

$$\|z_n - z_{n-1}\| \le \frac{2}{\varepsilon_n\gamma_n}\|T_n z_{n-1} - T_{n-1}z_{n-1}\|. \tag{32}$$

However, since z_n is bounded, we have, for an appropriate constant $M > 0$,

$$\begin{aligned}&\|T_n z_{n-1} - T_{n-1}z_{n-1}\| \\ &\quad = \left\|P_S\left(I - \gamma_n\nabla f_{\varepsilon_n}\right)z_{n-1} - P_S\left(I - \gamma_{n-1}\nabla f_{\varepsilon_{n-1}}\right)z_{n-1}\right\| \\ &\quad \le \left\|\left(I - \gamma_n\nabla f_{\varepsilon_n}\right)z_{n-1} - \left(I - \gamma_{n-1}\nabla f_{\varepsilon_{n-1}}\right)z_{n-1}\right\| \\ &\quad = \left\|\gamma_n\nabla f_{\varepsilon_n}(z_{n-1}) - \gamma_{n-1}\nabla f_{\varepsilon_{n-1}}(z_{n-1})\right\| \\ &\quad = \left\|(\gamma_n - \gamma_{n-1})\nabla f_{\varepsilon_n}(z_{n-1}) + \gamma_{n-1}\left(\nabla f_{\varepsilon_n}(z_{n-1}) - \nabla f_{\varepsilon_{n-1}}(z_{n-1})\right)\right\| \\ &\quad \le |\gamma_n - \gamma_{n-1}|\,\|\nabla f(z_{n-1}) + \varepsilon_n z_{n-1}\| \\ &\qquad + \gamma_{n-1}|\varepsilon_n - \varepsilon_{n-1}|\,\|z_{n-1}\| \\ &\quad \le \left(|\gamma_n - \gamma_{n-1}| + \gamma_{n-1}|\varepsilon_n - \varepsilon_{n-1}|\right)M.\end{aligned} \tag{33}$$

Combining (30), (32), and (33), we obtain

$$\|\omega_{n+1} - z_n\| \le \left(1 - \frac{1}{2}\varepsilon_n\gamma_n\right)\|\omega_n - z_{n-1}\| + \left(\frac{1}{2}\varepsilon_n\gamma_n\right)\beta_n, \tag{34}$$

where

$$\beta_n = \frac{4M\left(|\gamma_n - \gamma_{n-1}| + \gamma_{n-1}|\varepsilon_n - \varepsilon_{n-1}|\right)}{(\varepsilon_n\gamma_n)^2} \longrightarrow 0. \tag{35}$$

Now applying Lemma 4 to (34) and using the conditions (ii)–(iv), we conclude that $\|\omega_{n+1} - z_n\| \to 0$; therefore, $\omega_n \to \omega_{\min}$ in norm. □

Remark 7. Note that $\varepsilon_n = n^{-\delta}$ and $\gamma_n = n^{-\sigma}$ with $0 < \delta < \sigma < 1$ and $\sigma + 2\delta < 1$ satisfy the conditions (i)–(iv).

Remark 8. We can express the algorithm (26) in terms of x and y, and we get

$$\begin{aligned}x_{n+1} &= P_C\left((1 - \varepsilon_n\gamma_n)x_n - \gamma_n A^T(Ax_n - By_n)\right), \\ y_{n+1} &= P_Q\left((1 - \varepsilon_n\gamma_n)y_n + \gamma_n B^T(Ax_n - By_n)\right).\end{aligned} \tag{36}$$

And we can obtain that the whole sequence (x_n, y_n) generated by the algorithm (36) strongly converges to the minimum-norm solution of the ASEP (1) provided that the ASEP (1) is consistent and ε_n and γ_n satisfy the conditions (i)–(iv).

Remark 9. Now, we apply the algorithm (36) to solve the ASFP. Let $B = I$; the iteration in (36) becomes

$$\begin{aligned}x_{n+1} &= P_C\left((1 - \varepsilon_n\gamma_n)x_n - \gamma_n A^T(Ax_n - y_n)\right), \\ y_{n+1} &= P_Q\left((1 - \varepsilon_n\gamma_n)y_n + \gamma_n(Ax_n - y_n)\right).\end{aligned} \tag{37}$$

This algorithm is different from the algorithms that have been proposed to solve the ASFP, but it does solve the ASFP.

In this paper, we considered the ASEP in infinite-dimensional Hilbert spaces, which has broad applicability in modeling significant real-world problems. Then, we used the regularization method to propose a single-step iterative and showed that the sequence generated by such an algorithm strongly converges to the minimum-norm solution of the ASEP (1). We also gave an algorithm for solving the ASFP in Remark 9.

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