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Research Article

Inverse Problem Models of Oil-Water Two-Phase Flow Based on Buckley-Leverett Theory

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Based on Buckley-Leverett theory, one inverse problem model of the oil-water relative permeability was modeled and proved when the oil-water relative permeability equations obey the exponential form expression, and under the condition of the formation permeability that natural logarithm distribution always obey normal distribution, the other inverse problem model on the formation permeability was proved. These inverse problem models have been assumed in up-scaling cases to achieve the equations by minimization of objective function different between calculation water cut and real water cut, which can provide a reference for researching oil-water two-phase flow theory and reservoir numerical simulation technology.

1. Introduction

Reservoir numerical simulation technology is a growing new discipline with the emergence and development of the computer technology and computational mathematics, which has achieved rapid development and wide application all over the world, for example, the study of reservoir numerical simulation based on formation parameters [1], the well models and impacts [2], a numerical simulator for low-permeability reservoirs [3], and so on. However, the majority of methods have a great amount of calculation. It will waste a lot of time and energy when we research the oil-water relative permeability equations or the formation permeability distribution. Therefore, according to the inverse problem modeling based on oil-water two-phase flow, we propose a method to obtain the relevant results for our research. It is important to define the formation permeability distribution and oil-water relative permeability equations, which provide necessary information for oil reservoir evaluation, for example, large-scaling evaluation, which could be applied in reservoir numerical simulation during evaluation.

In order to get the answers which will be able to apply in large-scaling cases and solve the corresponding problems in reservoir scaling, inverse problem models have to be assumed to achieve the equations by minimization of objective function differently between real water cut and calculation water cut. The theoretical grid model is shown in Figure 1.

If we know the distribution $\{K_1, K_2, \ldots, K_n\}$, then the inverse problem model will be as shown in (1).

The first question: The optimization distribution $\{a_w, b_w, a_o, b_o\}$ and the relative permeability equations $K_{rw}(S_w)$, $K_{ro}(S_w)$.

The inverse problem mathematical model on oil-water relative permeability is as follows:

objective function:

$$E = \min \sum_{t=1}^{nt} \left(f_w(t) - f_w^{(\text{history})}(t) \right)^2,$$

where nt is time steps;

initial condition:

$$S_{w\,j}^{\,i(0)} = S_{Iw\,j}, \qquad Q_j^0 = 0$$

$$0 \le Kro_j^i < 1, \qquad 0 \le Krw_j^i < 1$$

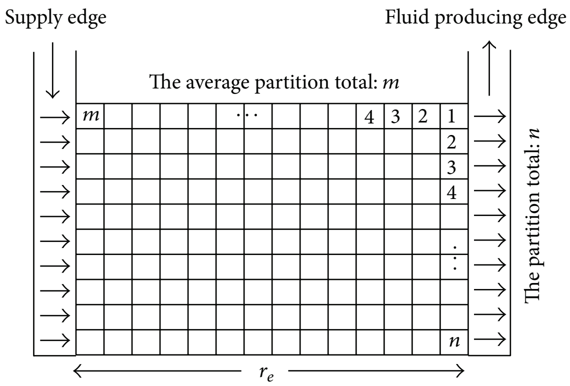

Figure 1: Reservoir profile model.

$$K_{rw}(S_w) = a_w\left(\frac{S_w - S_{wc}}{1 - S_{wc} - S_{or}}\right)^{b_w}$$

$$K_{ro}(S_w) = a_o\left(\frac{1 - S_{or} - S_w}{1 - S_{wc} - S_{or}}\right)^{b_o}$$

$$\text{if } r_e > r_{wf\,j}^{\;i},\ \text{then } S_{w\,j}^{\;i(t)} = S_{Iw\,j}$$

$$\text{if } r_e \le r_{wf\,j}^{\;i},\ \text{then } S_{w\,j}^{\;i(t)} = f_w^{-1\prime}(S_w)$$

$$f_w = f_w(S_w);$$

mathematical model:

$$Kro_j^i = Kro\left(S_{w\,j}^{\;i(t)}\right), \qquad Krw_j^i = Krw\left(S_{w\,j}^{\;i(t)}\right)$$

$$T_j^i = \frac{2\pi h_j K_j}{\ln\left(r/r_j^i\right)}\left(\frac{K_{ro\,j}^{\;i}}{\mu_o} + \frac{K_{rw\,j}^{\;i}}{\mu_w}\right)$$

$$T_j = \frac{1}{\sum_{i=1}^{m}\left(1/T_j^i\right)}, \qquad T = \sum_{j=1}^{n} T_j, \qquad Q_j^t = \frac{T_j}{T}\cdot Q_t$$

$$r_e^2 - r_j^{i\,2} = \frac{f_w^t\left(S_{w\,j}^{\;i}\right)' Q_j^t}{\phi_j\cdot\pi h_j}, \qquad r_{wf\,j}^{\;2} = r_e^2 - \frac{f_w^t\left(S_{wf\,j}^{\;i}\right)' Q_j^t}{\phi_j\cdot\pi h_j}$$

$$Q_{wj}^t = Q_j^t\cdot f_w^t\left(S_{w\,j}^1\right), \qquad Q_{oj}^t = Q_j^t\cdot\left(1 - f_w^t\left(S_{w\,j}^1\right)\right)$$

$$Q_w^t = \sum_{j=1}^{n} Q_{wj}^t, \qquad Q_o^t = \sum_{j=1}^{n} Q_{oj}^t, \qquad f_w(t) = \frac{Q_w^t}{Q_w^t + Q_o^t}. \tag{1}$$

If we know the equations $K_{rw}(S_w)$, $K_{ro}(S_w)$, another inverse problem model will be obtained as shown in (2).

The second question: an optimization distribution problem $\{K_1, K_2, \ldots, K_n\}$ and the standard deviation σ.

The inverse problem mathematical model of the formation permeability is as follows:

objective function:

$$E = \min\sum_{t=1}^{nt}\left(f_w(t) - f_w^{(\text{history})}(t)\right)^2,$$

where nt is time steps;

initial condition:

$$S_{w\,j}^{\;i(0)} = S_{wc\,j}, \qquad Q_j^0 = 0$$

$$f\left(K_j\right) = \frac{1}{\sqrt{2\pi}\cdot\sigma} e^{-(\ln K_j - \mu)^2/2\sigma^2}$$

$$\mu = \mathrm{Ln}\overline{K}$$

$$\text{if } r_e > r_{wf\,j}^{\;i},\ \text{then } S_{w\,j}^{\;i(t)} = S_{wc\,j}$$

$$\text{if } r_e \le r_{wf\,j}^{\;i},\ \text{then } S_{w\,j}^{\;i(t)} = f_w^{-1\prime}(S_w)$$

$$f_w = f_w(S_w)$$

mathematical model:

$$Kro_j^i = Kro\left(S_{w\,j}^{\;i(t)}\right), \qquad Krw_j^i = Krw\left(S_{w\,j}^{\;i(t)}\right)$$

$$T_j^i = \frac{2\pi h_j K_j}{\ln\left(r/r_j^i\right)}\left(\frac{K_{ro\,j}^{\;i}}{\mu_o} + \frac{K_{rw\,j}^{\;i}}{\mu_w}\right)$$

$$T_j = \frac{1}{\sum_{i=1}^{m}\left(1/T_j^i\right)}, \qquad T = \sum_{j=1}^{n} T_j, \qquad Q_j^t = \frac{T_j}{T}\cdot Q_t$$

$$r_e^2 - r_j^{i\,2} = \frac{f_w^t\left(S_{w\,j}^{\;i}\right)' Q_j^t}{\phi_j\cdot\pi h_j}, \qquad r_{wf\,j}^{\;2} = r_e^2 - \frac{f_w^t\left(S_{wf\,j}^{\;i}\right)' Q_j^t}{\phi_j\cdot\pi h_j}$$

$$Q_{wj}^t = Q_j^t\cdot f_w^t\left(S_{w\,j}^1\right), \qquad Q_{oj}^t = Q_j^t\cdot\left(1 - f_w^t\left(S_{w\,j}^1\right)\right)$$

$$Q_w^t = \sum_{j=1}^{n} Q_{wj}^t, \qquad Q_o^t = \sum_{j=1}^{n} Q_{oj}^t, \qquad f_w(t) = \frac{Q_w^t}{Q_w^t + Q_o^t}. \tag{2}$$

Here, $f_w^{(\text{history})}(t)$ is history water cut and $f_w(t)$ is calculation water cut; the first water cut is a known amount from production and another is calculated by our inverse problem models. Water cut means water production rate is a key parameter in reservoir engineering, which can represent water production capacity in reservoir development. If water cut is too high, it may bring negative effects to reservoir production. Therefore, the history matching for water production rate plays an important role in reservoir dynamic analysis and numerical simulation [4–6].

And thus $i = 1, 2, 3, \ldots, m$, $j = 1, 2, 3, \ldots, n$, m is the average partition total, n is the partition total, T is grid conductivity, h_j is each longitudinal layer stratum

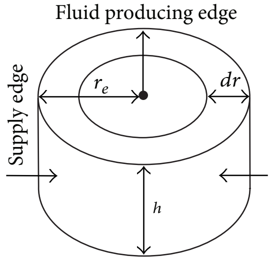

Figure 2: Radial flow unit model.

thickness, K_j is each longitudinal layer permeability, ϕ_j is each longitudinal layer porosity, S_{wj}^i is grid water saturation at different time, S_{wc} is the initial water saturation, E is the objective function, r_e is reservoir radius, $\overline{K}$ is average permeability, K_{ro} is oil relative permeability, K_{rw} is water relative permeability, Q_t is the total fluid production, Q_w^t is the total water yield of fluid producing edge at different time, Q_o^t is the total oil production of fluid producing edge at different time, Q_j^t is the total fluid production of each longitudinal layer at different time, S_{wf} is the frontier saturation, r_{wf} is the corresponding displacement of frontier saturation.

The inverse models (1) and (2) contain Buckley-Leverett theory of two-phase flow [7] and establish the oil-water two-phase plane radial flow function: $r_e^2 - r^2 = f_w' \cdot Q^t/\phi \cdot \pi \cdot h$ without considering these factors of gravity and capillary pressure; in addition, that shows the movement rules of isosaturation surface. As said above, most researchers always rely on forward problems [8] of core sampling experiment and mathematical statistics for an answer, As a result, there is always a large error between the calculation water cut and the history water cut in reservoir water cut analysis. In this paper, according to historical water cut and calculation water cut to establish the objective function, $E = \min \sum_{t=1}^{nt} (f_w(t) - f_w^{(\text{history})}(t))^2$. By the end an inverse problem model on the optimal solution distribution $\{a_w, b_w, a_o, b_o\}$ and oil-water relative permeability equations $K_{rw}(S_w)$, $K_{ro}(S_w)$ based on the Buckley-Leveret theory of two-phase flow can be realized, and another inverse problem model was modeled under the condition of the formation permeability logarithmic function are always obey normal distribution, Finally, it can provide a key information for reservoir numerical simulation studies.

2. Mathematical Model of Oil-Water Two-Phase Plane Radial Flow

Theorem 1. *Without considering these factors of gravity and capillary pressure, through the Buckley-Leveret theory of two-phase flow one established the oil-water two-phase plane radial flow mathematical model:*

$$r_e^2 - r^2 = \frac{f_w' \cdot Q^t}{\phi \cdot \pi h}. \tag{3}$$

Proof. We suppose the liquid flow rule is a plane radial flow from reservoir limit to well and choose a volume element in the vertical direction of streamline, as shown in Figure 2.

According to the seepage principle [9], we can get a flow equation of the volume element:

$$q_w = q \cdot f_w. \tag{4}$$

In the dt time, the flow volume of the volume element is

$$Q_{\text{out}} = dq_w \cdot dt. \tag{5}$$

We can derive from (4) and (5) that

$$Q_{\text{out}} = q \cdot df_w \cdot dt. \tag{6}$$

Meanwhile, the inflow volume of the volume element is

$$Q_{\text{in}} = \phi \cdot 2\pi h \cdot rdr \cdot ds_w. \tag{7}$$

In the dt time, relying on the Buckley-Leveret theory of two-phase flow, $Q_{\text{in}} = Q_{\text{out}}$. from (6) and (7), we have that $q \cdot df_w \cdot dt = \phi \cdot 2\pi h \cdot rdr \cdot dSw$ as follows:

$$q \cdot \frac{df_w}{dSw} \cdot dt = \phi \cdot 2\pi h \cdot rdr. \tag{8}$$

From $f_w = f_w(S_w)$, we can get that $f_w' = df_w/dS_w$.

Then deriving from (8) that $q \cdot df_w(Sw) \cdot dt = \phi \cdot 2\pi h \cdot rdr$, after infinitesimal calculus, we obtain

$$f_w' \int_0^t q \cdot dt = \phi \cdot 2\pi h \cdot \int_r^{r_e} rdr. \tag{9}$$

If $Q^t = \int_0^t q \cdot dt$, then (9) turns to $f_w' \cdot Q^t = \phi \cdot \pi h \cdot (r_e^2 - r^2)$. We obtain

$$r_e^2 - r^2 = \frac{f_w' \cdot Q^t}{\phi \cdot \pi h}. \tag{10}$$

From the reservoir profile grid model, the plane radial flow model of the grids in the longitudinal each layer can shows that

$$r_e^2 - {r_j^i}^2 = \frac{f_w^t\left(S_{wj}^{\ i}\right)' Q_j^t}{\phi_j \cdot \pi h_j}. \tag{11}$$

□

Theorem 2. *A special saturation definition about "S_{wf}": from the water cut function curve $f_w = f_w(S_w)$, selecting the point of the irreducible water saturation as a fixed point and joining any other point on the curve, and constructing function $k(S_{wf}) = (f_w(S_{wf}) - f_w(S_{w1}))/(S_{wf} - S_{w1})$, if $\mathrm{Max}\{k(S_{wf})\}$, then one can call "$S_{wf}$" frontier saturation. Now the corresponding displacement of frontier saturation r_{wf} is*

$$r_{wf\,j}^{\ 2} = r_e^2 - \frac{f_w^t\left(S_{wf\,j}^{\ i}\right)' Q_j^t}{\phi_j \cdot \pi h_j}. \tag{12}$$

3. The Inverse Problem Model on Oil-Water Relative Permeability

3.1. The Oil-Water Relative Permeability Equations. In the reservoir engineering, the different lithological character has the different corresponding oil-water relative permeability curve equations [10], but the most commonly form is used as a kind of the exponential form expression, as follows:

$$K_{rw}(S_w) = a_w\left(\frac{S_w - S_{wc}}{1 - S_{wc} - S_{or}}\right)^{b_w},$$
$$K_{ro}(S_w) = a_o\left(\frac{1 - S_{or} - S_w}{1 - S_{wc} - S_{or}}\right)^{b_o}. \quad (13)$$

In reservoir profile grid model, the oil-water relative permeability curve equations of each grid can be showed:

$$K_{rw}\left(S_{wj}^i\right) = a_w\left(\frac{S_{wj}^i - S_{wc}}{1 - S_{wc} - S_{or}}\right)^{b_w},$$
$$K_{ro}\left(S_{wj}^i\right) = a_o\left(\frac{1 - S_{or} - S_{wj}^i}{1 - S_{wc} - S_{or}}\right)^{b_o}. \quad (14)$$

In the equations, we can know different $\{a_w, b_w, a_o, b_o\}$ corresponding to its own $K_{rw}(S_w)$, $K_{ro}(S_w)$.

3.2. The Inverse Problem Mathematical Model of Oil-Water Relative Permeability Equations

Theorem 3. *If the oil-water relative permeability equations $K_{rw}(S_w)$, $K_{ro}(S_w)$ always obey (13) and one can calculate the water cut $f_w(t)$ based on the model of the Theorem 1, if the objective function $E = \min\sum_{t=1}^{nt}(f_w(t) - f_w^{(history)}(t))^2$ can be satisfied, then the inverse problem model (1) will have the optimal solution of the distribution $\{a_w, b_w, a_o, b_o\}$ and oil-water relative permeability equations $K_{rw}(S_w)$, $K_{ro}(S_w)$.*

Proof. Relying on oil-water relative permeability equations (14) and the conductivity definition of numerical reservoir simulation [11], we get the following:

$$\text{initial value:} \quad a_w^0,\ b_w^0,\ a_o^0,\ b_o^0,$$
$$K_{rw}(S_w) = a_w^0\left(\frac{S_w - S_{wc}}{1 - S_{wc} - S_{or}}\right)^{b_w^0}$$
$$K_{ro}(S_w) = a_o^0\left(\frac{1 - S_{or} - S_w}{1 - S_{wc} - S_{or}}\right)^{b_o^0}$$
$$T_j^i = \frac{2\pi h_j K_j}{\ln\left(r/r_j^i\right)}\left(\frac{K_{ro}{}_j^i}{\mu_o} + \frac{K_{rw}{}_j^i}{\mu_w}\right) \quad (15)$$
$$T_j = \frac{1}{\sum_{i=1}^m\left(1/T_j^i\right)}, \qquad T = \sum_{j=1}^n T_j,$$
$$Q_j^t = \frac{T_j}{T}\cdot Q_t.$$

We can derive from (12) and (15): r_{wf}.

If $r_e > r_{wf}$, then $S_{wj}^{i(t)} = S_{wcj}$. If $r_e \le r_{wf}$ solved inverse function of (11) and calculated water saturation of the fluid producing edge, then $S_{wj}^{i(t)} = {f_w^{-1}}'(S_w)$, and so forth $i = 1$.

If we obtain the value of the water saturation [12], relying on the function $f_w(S_w) \sim S_w$, we can calculate the value of the longitudinal each layer water cut: $f_w^t(S_{wj}^1)$ and the constructing mathematic model as follows

$$Q_{wj}^t = Q_j^t\cdot f_w^t\left(S_{wj}^1\right),$$
$$Q_{oj}^t = Q_j^t\cdot\left(1 - f_w^t\left(S_{wj}^1\right)\right),$$
$$Q_w^t = \sum_{j=1}^n Q_{wj}^t, \qquad Q_o^t = \sum_{j=1}^n Q_{oj}^t, \quad (16)$$
$$f_w(t) = \frac{Q_w^t}{Q_w^t + Q_o^t}.$$

From (16) we can calculate the total water cut of the fluid producing edge $f_w(t)$ and rely on the objective function: $E = \min\sum_{t=1}^{nt}(f_w(t) - f_w^{(\text{history})}(t))^2$ with numerical optimization calculation in the inverse problem model; an optimization problem of the distribution $\{a_w, b_w, a_o, b_o\}$ will be obtained, so the $K_{rw}(S_w)$ and $K_{ro}(S_w)$ can be formed. The proof of Theorem 3 is completed. □

4. The Inverse Problem Model on the Formation Permeability Logarithmic Function Always Obey Normal Distribution

Theorem 4. *If the formation permeability logarithmic function distribution $\{\operatorname{Ln} K_1, \operatorname{Ln} K_2, \ldots, \operatorname{Ln} K_n\}$ always obey normal distribution $f(K) = (1/\sqrt{2\pi}\cdot\sigma)e^{-(\ln K-\mu)^2/2\sigma^2}$, $\mu = \operatorname{Ln}\overline{K}$, meanwhile, one calculates the value of the $f_w(t)$ based on the model of Theorem 1, which can be adapted to the objective function $E = \min\sum_{t=1}^{nt}(f_w(t) - f_w^{(history)}(t))^2$; then the inverse problem model (2) will have the optimal distribution $\{\operatorname{Ln} K_1, \operatorname{Ln} K_2, \ldots, \operatorname{Ln} K_n\}$ and $\{K_1, K_2, \ldots, K_n\}$.*

Proof. According to 3σ principle: for normal distribution curve, if $P(\mu - 3\sigma - a < X \le \mu + 3\sigma + a) \to 1$, then there are a left end point value and a right end point value on the curve; the X value of the left end point is $\ln K_{\min}$, or $\mu - 3\sigma - a$. The X value of the right end point is $\ln K_{\max}$, or $\mu + 3\sigma + a$. And thus $a > 0$ and $\mu = \ln\overline{K} = 0.5\cdot(\ln K_{\min} + \ln K_{\max})$.

According to the area superposition principle of normal distribution curve, Select $x = \ln\overline{K}$ as the starting point, step size Δx and make a subdivision for probability curve; if the area summation of these formed closed figures can infinitely approach 1, then we can obtain the $\ln K_{\min}$ value of the left end point and the $\ln K_{\max}$ value of the right end point, and $\ln K_{\max} = 2\cdot\ln\overline{K} - \ln K_{\min}$.

According to the area superposition principle of normal distribution curve and giving a initial value σ of normal distribution, then $X_0 = \ln K_{\min}$, and $X_n = \ln K_{\max}$. $F(X_j)$ is a cumulative distribution function of the normal distribution [13]. If we use equal step size ΔX make a subdivision for probability curve, then the area summation of every closed figures $S_j = F(X_j) - F(X_{j-1})$, each longitudinal layer stratum thickness $h_j = h \cdot S_j$, and $X_j = \Delta X \cdot j + X_0$, $K_j = e^{X_j}$; if we use equal area ΔS make a subdivision for probability curve, then the area summation of every closed figures $S_j = F(X_j) - F(X_{j-1})$, $S_j = \Delta S = 1/n$, $h_j = h \cdot S_j$, and $X_j = F^{-1}(S_j)$, $K_j = e^{X_j}$. And thus $j = 1, 2, 3, \ldots, n$ (n is the total number of vertical stratification).

Relying on the distribution $\{K_1, K_2, \ldots, K_n\}$, oil-water relative permeability functions: $kro = kro(S_w)$, $krw = krw(S_w)$ [14], the conductivity definition of numerical reservoir simulation, we get the following equations:

$$kro_j^i = kro\left(S_{wj}^{i(t)}\right), \qquad krw_j^i = krw\left(S_{wj}^{i(t)}\right)$$
$$T_j^i = \frac{2\pi h_j K_j}{\ln\left(r/r_j^i\right)}\left(\frac{K_{roj}^{\ i}}{\mu_o} + \frac{K_{rwj}^{\ i}}{\mu_w}\right)$$
$$T_j = \frac{1}{\sum_{i=1}^{m}\left(1/T_j^i\right)}, \qquad T = \sum_{j=1}^{n} T_j \tag{17}$$
$$Q_j^t = \frac{T_j}{T} \cdot Q_t.$$

We can derive from (12) and (17) that $r_{wf\,j}$.

If $r_e \le r_{wf\,j}$, solving inverse function of (11) and calculating water saturation of the fluid producing edge, then $S_{wj}^{i(t)} = F^{-1}(r_j^i, \phi_j, h_j, Q_j^t)$, and thus $i = 1$. If $r_e > r_{wf\,j}$, then $S_{wj}^{i(t)} = S_{wc\,j}$, and thus $i = 1$.

If we obtain the value of the water saturation (S_w), relying on the function $f_w(S_w) \sim S_w$, we can calculate the value of the longitudinal each layer water cut: $f_w^t(S_{wj}^1)$; then mathematic model can be constructed as follows:

$$Q_{wj}^t = Q_j^t \cdot f_w^t\left(S_{wj}^1\right), \qquad Q_{oj}^t = Q_j^t \cdot \left(1 - f_w^t\left(S_{wj}^1\right)\right)$$
$$Q_w^t = \sum_{j=1}^{n} Q_{wj}^t, \qquad Q_o^t = \sum_{j=1}^{n} Q_{oj}^t \tag{18}$$
$$f_w(t) = \frac{Q_w^t}{\left(Q_w^t + Q_o^t\right)}.$$

Equation (18) can calculate the total water cut of the fluid producing edge: $f_w(t)$ and relies on the objective function: $E = \min \sum_{t=1}^{nt}(f_w(t) - f_w^{(\text{history})}(t))^2$. With numerical optimization calculation in the inverse problem model, the standard deviation σ and an optimization problem of the distribution $\{\text{Ln}K_1, \text{Ln}K_2, \ldots, \text{Ln}K_n\}$ will be obtained, so the distribution $\{K_1, K_2, \ldots, K_n\}$ will be given. The proof of Theorem 4 is completed. □

5. Discussions and Conclusions

The different distribution $\{a_w, b_w, a_o, b_o\}$ corresponds to a group of oil-water relative permeability equations $K_{rw}(S_w)$ and $K_{ro}(S_w)$. Combining with the objective function $E = \min \sum_{t=1}^{nt}(f_w(t) - f_w^{(\text{history})}(t))^2$ through the optimization solution method, it will finally bring a set of the optimum distribution $\{a_w, b_w, a_o, b_o\}$ and the oil-water relative permeability equations $K_{rw}(S_w)$ and $K_{ro}(S_w)$.

According to the above inverse problem mathematical model, based on the definition of the normal distribution, different (μ_i, σ_i^2) corresponds to its own normal distribution curve with the certain expectation μ_i. If σ_i goes up, the volatility of its normal distribution will be stronger according to the definition of standard deviation, which will finally lead to the large differential permeability distribution, namely, the strong heterogeneity. With the certain expectation μ_i, each different value of σ_i corresponds to a group of original values of (μ_i, σ_i^2), which will yield a group of values of $\{K_1, K_2, \ldots K_{n-1}, K_n\}$, namely, the value of permeability of each single formation. And it will also work out liquid production, water production, and integrated water cut of the whole liquid outlet of each single formation. Combining with the objective function $E = \min \sum_{t=1}^{nt}(f_w(t) - f_w^{(\text{history})}(t))^2$, it will finally bring a set of optimum normal distributions of $X \sim (\mu_i, \sigma_i^2)$ (Etc. $X = \ln K$) through the optimization solution method. Analysis of the changes of water driving place, oil production, and water production can be done through the related mathematical model.

Finally, the idea of constructing inverse problem models, according to the historical dynamic production data, can be realized, which attaches great importance to formation heterogeneity, observation of water flooding front position, and prediction of dynamic producing performance.

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