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Research Article

Some Properties of Meromorphic Solutions of Systems of Complex q-Shift Difference Equations

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In view of Nevanlinna theory, we study the properties of meromorphic solutions of systems of a class of complex difference equations. Some results obtained improve and extend the previous theorems given by Gao.

1. Introduction and Main Results

The purpose of this paper is to study some properties of meromorphic solutions of complex q-shift difference equations. The fundamental results and the standard notations of the Nevanlinna value distribution theory of meromorphic functions will be used (see [1–3]). Besides, for meromorphic function f, a meromorphic function $a(z)$ is called small function with respect to f if $T(r, a(z)) = o(T(r, f)) = S(r, f)$ for all r outside a possible exceptional set E of finite logarithmic measure $\lim_{r\to\infty}\int_{[1,r)\cap E}(dt/t) < \infty$.

In recent years, it has been a heated topic to study difference equations, difference product, and q-difference in the complex plane $\mathbb{C}$. There were articles focusing on the growth of solutions of difference equations, value distribution and uniqueness of differences analogues of Nevanlinna's theory (see [4–9]). Chiang and Feng [10] and Halburd and Korhonen [11] established a difference analogue of the logarithmic derivative lemma independently, and Barnett et al. [5] also established an analogue of the logarithmic derivative lemma on q-difference operators. By applying these theorems, a number of results on meromorphic solutions of complex difference and q-difference equations were obtained (see [12–19]).

In 2011, Korhonen [20] investigated the properties of finite-order meromorphic solution of the equation

$$H(z,\omega)P(z,\omega) = Q(z,\omega), \tag{1}$$

where $P(z,\omega) = P(z,\omega(z),\omega(z+c_1),\dots,\omega(z+c_n))$, $c_1,\dots,c_n \in \mathbb{C}$ and obtained the following result.

Theorem 1 (see [20]). *Let $\omega(z)$ be a finite-order meromorphic solution of* (1)*, where $P(z,\omega)$ is a homogeneous difference polynomial with meromorphic coefficients and $H(z,\omega)$ and $Q(z,\omega)$ are polynomials in $\omega(z)$ with meromorphic coefficients having no common factors. If* $\max\{\deg_\omega(H), \deg_\omega(Q) - \deg_\omega(P)\} > \min\{\deg_\omega(P), \mathrm{ord}_0(Q)\} - \mathrm{ord}_0(P)$, *then* $N(r,\omega) \neq S(r,\omega)$, *where* $\mathrm{ord}_0(P)$ *denotes the order of zero of $P(z,x_0,x_1,\dots,x_n)$ at $x_0 = 0$ with respect to the variable x_0.*

Let $c_j \in \mathbb{C}$ for $j = 1,\dots,n$, and let I be a finite set of multi-indexes $\lambda = (\lambda_0,\dots,\lambda_n)$. Then a difference polynomial of a meromorphic function $\omega(z)$ is defined as

$$\begin{aligned} P(z,\omega) &= P(z,\omega(z),\omega(z+c_1),\dots,\omega(z+c_n)) \\ &= \sum_{\lambda\in I} c_\lambda(z)\, w(z)^{\lambda_0} w(z+c_1)^{\lambda_1}\cdots\omega(z+c_n)^{\lambda_n}, \end{aligned} \tag{2}$$

where the coefficients $c_\lambda(z)$ are small with respect to $\omega(z)$ in the sense that $T(r,c_\lambda) = o(T(r,\omega))$ as r tends to infinity outside of an exceptional set E of finite logarithmic measure.

At the same year, Zheng and Chen [21] consider the value distribution of meromorphic solutions of zero order of a kind of q-difference equations and obtained the following result which is an extension of Theorem 1.

Theorem 2 (see [21, Theorem 1]). *Suppose that f is a nonconstant meromorphic solution of zero order of a q-difference equation of the form*

$$\sum_{\lambda\in I} c_\lambda(z) f(qz)^{i_{\lambda,1}} f(q^2z)^{i_{\lambda,2}} \cdots f(q^nz)^{i_{\lambda,n}} = \frac{P(z,f(z))}{Q(z,f(z))} = \left(a_k(z)(f(z))^k + a_{k+1}(z)(f(z))^{k+1} + \cdots + a_s(z)(f(z))^s\right) \times \left(b_0(z) + b_1(z)f(z) + \cdots + b_t(z)(f(z))^t\right)^{-1}, \quad (3)$$

where $I = \{(i_{\lambda_1}, i_{\lambda_2}, \ldots, i_{\lambda_n})\}$ is a finite index set and $i_{\lambda_1} + i_{\lambda_2} + \cdots + i_{\lambda_n} = \sigma > 0$ for all $\lambda \in I$ and $q(\neq 0,1) \in \mathbb{C}$. Moreover, suppose that $0 \le k \le s$, $a_k(z)a_s(z)b_t(z) \not\equiv 0$, the $P(z,f)$ and $Q(z,f)$ have no common factors, and that all meromorphic coefficients in (3) *are of growth of $o(T(r,f))$ on a set of logarithmic density 1. If*

$$\max\{t, s-\sigma\} > \min\{\sigma, k\}, \quad (4)$$

then

$$N(r,f) \neq o(T(r,f)) \quad (5)$$

on any set of logarithmic density 1.

Remark 3. The logarithmic density of a set F is defined by

$$\limsup_{r\to\infty} \frac{1}{\log r} \int_{[1,r]\cap F} \frac{1}{t}\,dt. \quad (6)$$

Recently, Gao [22–24] and others [25, 26] also investigated the growth and existence of meromorphic solutions of some systems of complex difference equations; one system of complex difference equation is based on (1) and obtained some interesting results.

Inspired by the idea of [21–24, 27], we will investigate the properties of meromorphic solutions of systems of a class of complex q-shift difference equations of the form

$$\begin{aligned} \Omega_1(z,w_1,w_2) &= R_1(z,w_1), \\ \Omega_2(z,w_1,w_2) &= R_2(z,w_2), \end{aligned} \quad (7)$$

where $q(\neq 0,1)$, $c_j\,(j = 1,\ldots,n) \in \mathbb{C}$, I, J are two finite sets of multi-indexes $(i_1,\ldots,i_n)$, $(j_1,\ldots,j_n)$, and $\Omega_1(z,w_1,w_2)$, $\Omega_2(z,w_1,w_2)$ are two homogeneous difference polynomials to be defined as

$$\begin{aligned} \Omega_1(z,w_1,w_2) &= \Omega_1(z, w_1(qz+c_1), w_2(qz+c_1), \ldots, w_1(q^nz+c_n), w_2(q^nz+c_n)) \\ &= \sum_{(i)} a_{(i)}(z) \prod_{k=1}^{2} (w_k(qz+c_1))^{i_{k1}} \cdots (w_k(q^nz+c_n))^{i_{kn}}, \\ \Omega_2(z,w_1,w_2) &= \Omega_2(z, w_1(qz+c_1), w_2(qz+c_1), \ldots, w_1(q^nz+c_n), w_2(q^nz+c_n)) \\ &= \sum_{(j)} b_{(j)}(z) \prod_{k=1}^{2} (w_k(qz+c_1))^{j_{k1}} \cdots (w_k(q^nz+c_n))^{j_{kn}}. \end{aligned} \quad (8)$$

The coefficients $\{a(i)\}$, $\{b(j)\}$ are small with respect to w_1, w_2 in the sense that $T(r,a_{(i)}) = o(T(r,w_l))$, $T(r,b_{(j)}) = o(T(r,w_l))$, $l = 1,2$, as r tends to infinity outside of an exceptional set E of finite logarithmic measure. The weights of $\Omega_1(z,w_1,w_2)$, $\Omega_2(z,w_1,w_2)$ are defined by

$$\begin{aligned} \sigma_{11} &= \max_{(i)} \left\{\sum_{l=1}^{n} i_{1l}\right\}, \qquad \sigma_{12} = \max_{(i)} \left\{\sum_{l=1}^{n} i_{2l}\right\}, \\ \sigma_{21} &= \max_{(j)} \left\{\sum_{l=1}^{n} j_{1l}\right\}, \qquad \sigma_{22} = \max_{(j)} \left\{\sum_{l=1}^{n} j_{2l}\right\}, \\ R_1(z,w_1) &= \frac{P_1(z,w_1)}{Q_1(z,w_1)} \\ &= \left(c^1_{k_1}(z)(w_1(z))^{k_1} + c^1_{k_1+1}(z)(w_1(z))^{k_1+1} + \cdots + c^1_{s_1}(z)(w_1(z))^{s_1}\right) \\ &\quad \times \left(d^1_0(z) + d^1_1(z)w_1(z) + \cdots, + d^1_{t_1}(z)(w_1(z))^{t_1}\right)^{-1}, \\ R_2(z,w_2) &= \frac{P_2(z,w_2)}{Q_2(z,w_2)} \\ &= \left(c^2_{k_2}(z)(w_2(z))^{k_2} + c^2_{k_2+1}(z)(w_2(z))^{k_2+1} + \cdots + c^2_{s}(z)(w_2(z))^{s_2}\right) \\ &\quad \times \left(d^2_0(z) + d^2_1(z)w_2(z) + \cdots + d^2_{t_2}(z)(w_2(z))^{t_2}\right)^{-1}. \end{aligned} \quad (9)$$

The coefficients $\{c^i_{k_i}(z)\}$, $\{d^i_{t_i}(z)\}$ are meromorphic functions and small functions,

$$\begin{aligned} S(r) = & \sum T(r, a_{(i)}) + \sum T(r, b_{(j)}) \\ & + \sum T(r, c^i_{k_i}) + \sum T(r, d^i_{t_i}). \end{aligned} \tag{10}$$

Now, we will show our main results as follows.

Theorem 4. *Let (w_1, w_2) be meromorphic solution of systems (7) satisfying $\rho = \rho(w_1, w_2) = 0$. Moreover, suppose that $0 \le k_i \le s_i$, $c^i_{k_1}(z)c^i_{s_1}(z)d^i_{t_i}(z) \not\equiv 0$, $i = 1, 2$, the $P_i(z, w_i)$ and $Q_i(z, w_i)$ are polynomials in $w_i(z)$ with meromorphic coefficients having no common factors, and that all meromorphic coefficients in (7) are of growth of $o(T(r, f))$ for all r on a set of logarithmic density 1 or outside of an exceptional set of logarithmic density 0. If*

$$\begin{aligned} \max\{t_1, s_1 - \sigma_{11}\} &> \min\{\sigma_{11}, k_1\} + \sigma_{11} + \sigma_{12}, \\ \max\{t_2, s_2 - \sigma_{22}\} &> \min\{\sigma_{22}, k_2\} + \sigma_{22} + \sigma_{21}, \end{aligned} \tag{11}$$

then $N(r, w_1) = o(T(r, w_1))$ and $N(r, w_2) = o(T(r, w_2))$ cannot hold both at the same time, for all r possibly outside of an exceptional set of logarithmic density 0, where the order of meromorphic solution (w_1, w_2) of systems (7) is defined by

$$\begin{aligned} \rho &= \rho(w_1, w_2) = \max\{\rho(w_1), \rho(w_2)\}, \\ \rho(w_i) &= \limsup_{r\to\infty} \frac{\log T(r, w_i)}{\log r}, \quad i = 1, 2. \end{aligned} \tag{12}$$

Theorem 5. *Let (w_1, w_2) be meromorphic solution of systems (7) satisfying $\rho = \rho(w_1, w_2) = 0$. Moreover, suppose that $0 \le k_i \le s_i$, $c^i_k(z)c^i_s(z)d^i_t(z) \not\equiv 0$, $i = 1, 2$, the $P_i(z, w_i)$ and $Q_i(z, w_i)$ are polynomials in $w_i(z)$ with meromorphic coefficients having no common factors, and that all meromorphic coefficients in (7) are of growth of $o(T(r, f))$ for all r on a set of logarithmic density 1 or outside of an exceptional set of logarithmic density 0, and*

$$\begin{aligned} A &= 2\sigma_{11} - \max\{s_1, t_1 + \sigma_{11}\} + \min\{\sigma_{11}, k_1\}, \\ B &= 2\sigma_{22} - \max\{s_2, t_2 + \sigma_{22}\} + \min\{\sigma_{22}, k_2\}. \end{aligned} \tag{13}$$

If

$$A < 0, \quad B < 0, \quad AB > 9\sigma_{21}\sigma_{12}, \tag{14}$$

then $m(r, w_k) = o(T(r, w_k))$, $k = 1, 2$ hold for r that runs to infinity possibly outside of an exceptional set of logarithmic density 0.

2. Some Lemmas

Lemma 6 (Valiron-Mohon'ko) ([28]). *Let $f(z)$ be a meromorphic function. Then for all irreducible rational functions in f,*

$$R(z, f(z)) = \frac{\sum_{i=0}^{m} a_i(z) f(z)^i}{\sum_{j=0}^{n} b_j(z) f(z)^j}, \tag{15}$$

with meromorphic coefficients $a_i(z)$, $b_j(z)$, the characteristic function of $R(z, f(z))$ satisfies that

$$T(r, R(z, f(z))) = dT(r, f) + O(\Psi(r)), \tag{16}$$

where $d = \max\{m, n\}$ and $\Psi(r) = \max_{i,j}\{T(r, a_i), T(r, b_j)\}$.

Lemma 7 (see [27]). *Let $f(z)$ be a nonconstant zero-order meromorphic function and $q \in \mathbb{C} \setminus \{0\}$. Then*

$$m\left(r, \frac{f(qz+\eta)}{f(z)}\right) = o(T(r, f)) = S(r, f), \tag{17}$$

on a set of logarithmic density 1 or outside of an exceptional set of logarithmic density 0.

Lemma 8 (see [29]). *Let $f(z)$ be a transcendental meromorphic function of zero order, and let q, η be two nonzero complex constants. Then*

$$\begin{aligned} T(r, f(qz+\eta)) &= T(r, f(z)) + S(r, f), \\ N(r, f(qz+\eta)) &\le N(r, f) + S(r, f), \end{aligned} \tag{18}$$

on a set of logarithmic density 1 or outside of a possibly exceptional set of logarithmic density 0.

3. The Proof of Theorem 4

From the definitions of $\Omega_i(z, w_1, w_2)$, by Lemma 7, it follows that

$$m\left(r, \frac{\Omega_1(z, w_1, w_2)}{w_1^{\sigma_{11}}}\right) \le \sigma_{12} m(r, w_2) + o(T(r, w_1)), \quad r \notin E'_1, \tag{19}$$

$$m\left(r, \frac{\Omega_2(z, w_1, w_2)}{w_2^{\sigma_{22}}}\right) \le \sigma_{21} m(r, w_1) + o(T(r, w_2)), \quad r \notin E'_2, \tag{20}$$

where E'_1, E'_2 are two sets of logarithmic density 0. By Lemma 6, we have

$$\begin{aligned} T\left(r, \frac{\Omega_1(z, w_1, w_2)}{w_1^{\sigma_{11}}}\right) &= T\left(r, \frac{P_1(z, w_1)}{Q_1(z, w_1) w_1^{\sigma_{11}}}\right) \\ &= (\max\{t_1 + \sigma_{11}, s_1\} - \min\{\sigma_{11}, k_1\}) \\ &\quad \times T(r, w_1) + o(T(r, w_1)), \\ &\qquad r \notin E'_3, \end{aligned} \tag{21}$$

$$T\left(r,\frac{\Omega_2(z,w_1,w_2)}{w_2^{\sigma_{22}}}\right)=T\left(r,\frac{P_2(z,w_2)}{Q_2(z,w_2)w_2^{\sigma_{22}}}\right)$$
$$=(\max\{t_2+\sigma_{22},s_2\}-\min\{\sigma_{22},k_2\})\times T(r,w_2)+o(T(r,w_2)),\quad r\notin E_4', \tag{22}$$

where E_3', E_4' are two sets of logarithmic density 0. Thus, from the assumptions of Theorem 4, combining (19) and (21), (20) and (22), respectively, we have

$$N\left(r,\frac{\Omega_1(z,w_1,w_2)}{w_1^{\sigma_{11}}}\right)\geq(1+\sigma_{12}+\sigma_{11})T(r,w_1)-\sigma_{12}m(r,w_2)+o(T(r,w_1)),\quad r\notin E_1=E_1'\cup E_3',$$
$$N\left(r,\frac{\Omega_2(z,w_1,w_2)}{w_2^{\sigma_{22}}}\right)\geq(1+\sigma_{21}+\sigma_{22})T(r,w_1)-\sigma_{21}m(r,w_1)+o(T(r,w_2)),\quad r\notin E_2=E_2'\cup E_4'. \tag{23}$$

Since $\rho=\rho(w_1,w_2)=0$, from Lemma 8, we have

$$N\left(r,\frac{\Omega_1(z,w_1,w_2)}{w_1^{\sigma_{11}}}\right)\leq N(r,\Omega_1(z,w_1,w_2))+\sigma_{11}N\left(r,\frac{1}{w_1}\right)$$
$$\leq\sigma_{11}N(r,w_1)+\sigma_{12}N(r,w_2)+\sigma_{11}N\left(r,\frac{1}{w_1}\right)+o(T(r,w_1))+o(T(r,w_2)),\quad r\notin E_5',$$
$$N\left(r,\frac{\Omega_2(z,w_1,w_2)}{w_2^{\sigma_{22}}}\right)\leq N(r,\Omega_2(z,w_1,w_2))+\sigma_{22}N\left(r,\frac{1}{w_2}\right)$$
$$\leq\sigma_{22}N(r,w_2)+\sigma_{21}N(r,w_1)+\sigma_{22}N\left(r,\frac{1}{w_2}\right)+o(T(r,w_1))+o(T(r,w_2)),\quad r\notin E_6', \tag{24}$$

where E_5', E_6' are the sets of logarithmic density 0.

From (23) and (24), it follows that

$$(1+\sigma_{12}+\sigma_{11})T(r,w_1)\leq\sigma_{11}N(r,w_1)+\sigma_{12}N(r,w_2)+\sigma_{11}N\left(r,\frac{1}{w_1}\right)+\sigma_{12}m(r,w_2)+o(T(r,w_1))+o(T(r,w_2))$$
$$\leq\sigma_{11}N(r,w_1)+\sigma_{12}T(r,w_2)+\sigma_{11}T(r,w_1)+o(T(r,w_1))+o(T(r,w_2)),\quad r\notin E_3=E_1\cup E_5',$$
$$(1+\sigma_{21}+\sigma_{22})T(r,w_1)\leq\sigma_{22}N(r,w_2)+\sigma_{21}N(r,w_1)+\sigma_{22}N\left(r,\frac{1}{w_2}\right)+\sigma_{21}m(r,w_1)+o(T(r,w_1))+o(T(r,w_2))$$
$$\leq\sigma_{22}N(r,w_2)+\sigma_{21}T(r,w_1)+\sigma_{22}T(r,w_2)+o(T(r,w_1))+o(T(r,w_2)),\quad r\notin E_4=E_2\cup E_6'. \tag{25}$$

Suppose now on the contrary to the assertion of Theorem 4 that $N(r,w_1)=o(T(r,w_1))$ and $N(r,w_2)=o(T(r,w_2))$, from (25); it follows that

$$(1+\sigma_{12})T(r,w_1)\leq\sigma_{12}T(r,w_2)+o(T(r,w_1))+o(T(r,w_2)),$$
$$(1+\sigma_{21})T(r,w_2)\leq\sigma_{21}T(r,w_1)+o(T(r,w_1))+o(T(r,w_2)), \tag{26}$$

that is,

$$(1+\sigma_{12}+o(1))T(r,w_1)\leq(\sigma_{12}+o(1))T(r,w_2),$$
$$(1+\sigma_{21}+o(1))T(r,w_2)\leq(\sigma_{21}+o(1))T(r,w_1). \tag{27}$$

From (27), we can get that

$$(1+\sigma_{12})(1+\sigma_{21})\leq\sigma_{12}\sigma_{21}. \tag{28}$$

From the previous inequality, we can get a contradiction.

Therefore, this completes the proof of Theorem 4.

4. The Proof of Theorem 5

Since $\rho=\rho(w_1,w_2)=0$, from the assumptions concerning the coefficients of systems (7), by Lemma 7, and from the definitions of logarithmic measure and logarithmic density, we have

$$\begin{aligned} N\left(r, \frac{\Omega_1(z,w_1,w_2)}{w_1^{\sigma_{11}}}\right) \le{}& \sigma_{11}\left[N(r,w_1)+N\left(r,\frac{1}{w_1}\right)\right] \\ &+\sigma_{12}\left[N(r,w_2)+N\left(r,\frac{1}{w_2}\right)\right] \\ &+\sigma_{12}N(r,w_2)+o(T(r,w_1)) \\ &+o(T(r,w_2)), \quad r\notin E_5, \end{aligned} \tag{29}$$

where E_5 is a set of logarithmic density 0.

From (29), we have

$$\begin{aligned} N\left(r, \frac{\Omega_1(z,w_1,w_2)}{w_1^{\sigma_{11}}}\right) \le{}& \sigma_{11}\left[N(r,w_1)+N\left(r,\frac{1}{w_1}\right)\right] \\ &+\sigma_{12}\left[2N(r,w_2)+N\left(r,\frac{1}{w_2}\right)\right] \\ &+o(T(r,w_1))+o(T(r,w_2)) \\ \le{}& \sigma_{11}\left[2T(r,w_1)-m(r,w_1)\right] \\ &+\sigma_{12}\left[3T(r,w_2)-2m(r,w_2)\right] \\ &+o(T(r,w_1))+o(T(r,w_2)), \\ & r\notin E_5. \end{aligned} \tag{30}$$

From (19) and (29), we have

$$\begin{aligned} & N\left(r, \frac{\Omega_1(z,w_1,w_2)}{w_1^{\sigma_{11}}}\right)+\sigma_{12}m(r,w_2) \\ &\ge N\left(r, \frac{\Omega_1(z,w_1,w_2)}{w_1^{\sigma_{11}}}\right)+m\left(r, \frac{\Omega_1(z,w_1,w_2)}{w_1^{\sigma_{11}}}\right) \\ &= T\left(r, \frac{P_1(z,w_1)}{Q_1(z,w_1)w_1^{\sigma_{11}}}\right) \\ &= (\max\{t_1+\sigma_{11},s_1\}-\min\{\sigma_{11},k_1\}) \\ &\quad\times T(r,w_1)+o(T(r,w_1)), \\ &\qquad r\notin F_1=E_1'\cup E_5. \end{aligned} \tag{31}$$

From the previous inequality and (30), we have for $r\notin F_1$

$$\begin{aligned} &(\max\{t_1+\sigma_{11},s_1\}-\min\{\sigma_{11},k_1\})T(r,w_1)-\sigma_{12}m(r,w_2) \\ &\quad\le \sigma_{11}\left[2T(r,w_1)-m(r,w_1)\right] \\ &\qquad+\sigma_{12}\left[3T(r,w_2)-2m(r,w_2)\right] \\ &\qquad+o(T(r,w_1))+o(T(r,w_2)). \end{aligned} \tag{32}$$

By using the same argument as in the previously mentioned, there exists a set F_2 of logarithmic density 0, for $r\notin F_2$, and we have

$$\begin{aligned} &(\max\{t_2+\sigma_{22},s_2\}-\min\{\sigma_{22},k_2\})T(r,w_2)-\sigma_{21}m(r,w_1) \\ &\quad\le \sigma_{22}\left[2T(r,w_2)-m(r,w_2)\right] \\ &\qquad+\sigma_{21}\left[3T(r,w_1)-2m(r,w_1)\right] \\ &\qquad+o(T(r,w_1))+o(T(r,w_2)). \end{aligned} \tag{33}$$

From (32) and (33), we have

$$\begin{aligned} \sigma_{11}m(r,w_1) \le{}& [2\sigma_{11}-(\max\{t_1+\sigma_{11},s_1\} \\ &-\min\{\sigma_{11},k_1\})+o(1)]T(r,w_1) \\ &+(3\sigma_{12}+o(1))T(r,w_2), \quad r\notin F_1, \\ [(\max\{t_2+\sigma_{22},s_2\}&-\min\{\sigma_{22},k_2\})-2\sigma_{22}+o(1)]T(r,w_2) \\ \le{}& (3\sigma_{21}+o(1))T(r,w_1)-\sigma_{21}m(r,w_1), \quad r\notin F_2. \end{aligned} \tag{34}$$

From (34), we have

$$\begin{aligned} &\sigma_{11}m(r,w_1) \\ &\quad\le [2\sigma_{11}-(\max\{t_1+\sigma_{11},s_1\}-\min\{\sigma_{11},k_1\}) \\ &\qquad+o(1)]T(r,w_1) \\ &\quad+((3\sigma_{12}+o(1)) \\ &\qquad\times[(3\sigma_{21}+o(1))T(r,w_1)-\sigma_{21}m(r,w_1)]) \\ &\quad\times((\max\{t_2+\sigma_{22},s_2\}-\min\{\sigma_{22},k_2\})-2\sigma_{22})^{-1}, \\ &\qquad r\notin F=F_1\cup F_2, \end{aligned} \tag{35}$$

that is,

$$\begin{aligned} &\left(\sigma_{11}-\frac{3\sigma_{12}\sigma_{21}}{B}\right)m(r,w_1) \\ &\quad\le\left[A-\frac{9\sigma_{12}\sigma_{21}+o(1)}{B}\right]T(r,w_1), \\ &\qquad r\notin F=F_1\cup F_2, \end{aligned} \tag{36}$$

where $A=2\sigma_{11}-\max\{s_1,t_1+\sigma_{11}\}+\min\{\sigma_{11},k_1\}$ and $B=2\sigma_{22}-\max\{s_2,t_2+\sigma_{22}\}+\min\{\sigma_{22},k_2\}$. From (14) and (36), we have

$$m(r,w_1)=o(T(r,w_1)) \tag{37}$$

for all r outside of F, a set of logarithmic density 0.

Similarly, we can obtain

$$m(r,w_2)=o(T(r,w_2) \tag{38}$$

for all r possibly outside of F', a set of logarithmic density 0.

Thus, this completes the proof of Theorem 5.

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