Hindawi Publishing Corporation
Abstract and Applied Analysis
Volume 2013, Article ID 645262, 9 pages
http://dx.doi.org/10.1155/2013/645262

Research Article

Exponential Stability and Periodicity of Fuzzy Delayed Reaction-Diffusion Cellular Neural Networks with Impulsive Effect

Guowei Yang,[1] Yonggui Kao,[2] and Changhong Wang[2]

[1] *College of Information Engineering, Nanchang Hangkong University, Nanchang 330063, China*
[2] *Space Control and Inertial Technology Research Center, Harbin Institute of Technology, Harbin, Heilongjiang 150001, China*

Correspondence should be addressed to Changhong Wang; chwang@hit.edu.cn

Received 3 September 2012; Accepted 4 January 2013

Academic Editor: Tingwen Huang

This paper considers dynamical behaviors of a class of fuzzy impulsive reaction-diffusion delayed cellular neural networks (FIRDDCNNs) with time-varying periodic self-inhibitions, interconnection weights, and inputs. By using delay differential inequality, M-matrix theory, and analytic methods, some new sufficient conditions ensuring global exponential stability of the periodic FIRDDCNN model with Neumann boundary conditions are established, and the exponential convergence rate index is estimated. The differentiability of the time-varying delays is not needed. An example is presented to demonstrate the efficiency and effectiveness of the obtained results.

1. Introduction

The fuzzy cellular neural networks (FCNNs) model, which combines fuzzy logic with the structure of traditional neural networks (CNNs) [1–3], has been proposed by Yang et al. [4, 5]. Unlike previous CNNs structures, the FCNNs model has fuzzy logic between its template and input and/or output besides the "sum of product" operation. Studies have shown that the FCNNs model is a very useful paradigm for image processing and pattern recognition [6–8]. These applications heavily depend on not only the dynamical analysis of equilibrium points but also on that of the periodic oscillatory solutions. In fact, the human brain is naturally in periodic oscillatory [9], and the dynamical analysis of periodic oscillatory solutions is very important in learning theory [10, 11], because learning usually requires repetition. Moreover, an equilibrium point can be viewed as a special periodic solution of neural networks with arbitrary period. Stability analysis problems for FCNNs with and without delays have recently been probed; see [12–22] and the references therein. Yuan et al. [13] have investigated stability of FCNNs by linear matrix inequality approach, and several criteria have been provided for checking the periodic solutions for FCNNs with time-varying delays. Huang [14] has probed exponential stability of fuzzy cellular neural networks with distributed delay, without considering reaction-diffusion effects.

Strictly speaking, reaction-diffusion effects cannot be neglected in both biological and man-made neural networks [19–32], especially when electrons are moving in noneven electromagnetic field. In [19], stability is considered for FCNNs with diffusion terms and time-varying delay. Wang and Lu [20] have probed global exponential stability of FCNNs with delays and reaction-diffusion terms. Song and Wang [21] have studied dynamical behaviors of fuzzy reaction-diffusion periodic cellular neural networks with variable coefficients and delays without considering pulsing effects. Wang et al. [22] have discussed exponential stability of impulsive stochastic fuzzy reaction-diffusion Cohen-Grossberg neural networks with mixed delays. Zhao and Mao [30] have investigated boundedness and stability of nonautonomous cellular neural networks with reaction-diffusion terms. Zhao and Wang [31] have considered existence of periodic oscillatory solution of reaction-diffusion neural networks with delays without fuzzy logic and impulsive effect.

As we all know, many practical systems in physics, biology, engineering, and information science undergo abrupt changes at certain moments of time because of impulsive inputs [33]. Impulsive differential equations and impulsive neural networks have been received much interest in recent years; see, for example, [34–42] and the references therein. Yang and Xu [36] have investigated existence and exponential stability of periodic solution for impulsive delay differential equations and applications. Li and Lu [38] have discussed global exponential stability and existence of periodic solution of Hopfield-type neural networks with impulses without reaction-diffusion. To the best of our knowledge, few authors have probed the existence and exponential stability of the periodic solutions for the FIRDDCNN model with variable coefficients, and time-varying delays. As a result of the simultaneous presence of fuzziness, pulsing effects, reaction-diffusion phenomena, periodicity, variable coefficients and delays, the dynamical behaviors of this kind of model become much more complex and have not been properly addressed, which still remain important and challenging.

Motivated by the above discussion, we will establish some sufficient conditions for the existence and exponential stability of periodic solutions of this kind of FIRDDCNN model, applying delay differential inequality, M-matrix theory, and analytic methods. An example is employed to demonstrate the usefulness of the obtained results.

Notations. Throughout this paper, $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote, respectively, the n-dimensional Euclidean space and the set of all $n\times m$ real matrices. The superscript "T" denotes matrix transposition and the notation $X \geq Y$ (resp., $X > Y$), where X and Y are symmetric matrices, means that $X - Y$ is positive semidefinite (resp., positive definite). $\Omega = \{x = (x_1, \ldots, x_m)^{\mathrm{T}}, |x_i| < \mu\}$ is a bounded compact set in space $\mathbb{R}^m$ with smooth boundary $\partial\Omega$ and measure $\operatorname{mes}\Omega > 0$; Neumann boundary condition $\partial u_i/\partial n = 0$ is the outer normal to $\partial\Omega$; $L^2(\Omega)$ is the space of real functions Ω which are L^2 for the Lebesgue measure. It is a Banach space with the norm $\|u(t,x)\|_2 = (\sum_{i=1}^{n}\|u_i(t,x)\|_2^2)^{1/2}$, where $u(t,x) = (u_1(t,x),\ldots,u_n(t,x))^{\mathrm{T}}$, $\|u_i(t,x)\|_2 = (\int_\Omega |u_i(t,x)|^2 dx)^{1/2}$, $|u(t,x)| = (|u_1(t,x)|,\ldots,|u_n(t,x)|)^{\mathrm{T}}$. For function $g(x)$ with positive period ω, we denote $\overline{g} = \max_{t\in[0,\omega]} g(t)$, $\underline{g} = \min_{t\in[0,\omega]} g(t)$. Sometimes, the arguments of a function or a matrix will be omitted in the analysis when no confusion can arise.

2. Preliminaries

Consider the impulsive fuzzy reaction-diffusion delayed cellular neural networks (FIRDDCNN) model:

$$
\begin{aligned}
\frac{\partial u_i(t,x)}{\partial t} &= \sum_{l=1}^{m}\frac{\partial}{\partial x_l}\left(D_i\frac{\partial u_i(t,x)}{\partial x_l}\right)\\
&\quad - c_i(t)\,u_i(t,x) + \sum_{j=1}^{n} a_{ij}(t)\,f_j\left(u_j(t,x)\right)\\
&\quad + \sum_{j=1}^{n} b_{ij}(t)\,v_j(t) + J_i(t)\\
&\quad + \bigwedge_{j=1}^{n}\alpha_{ij}(t)\,g_j\left(u_j\left(t-\tau_j(t),x\right)\right)\\
&\quad + \bigvee_{j=1}^{n}\beta_{ij}(t)\,g_j\left(u_j\left(t-\tau_j(t),x\right)\right)\\
&\quad + \bigwedge_{j=1}^{n}T_{ij}(t)\,v_j(t) + \bigvee_{j=1}^{n}H_{ij}(t)\,v_j(t),\\
&\qquad\qquad t\neq t_k,\ x\in\Omega,\\
u_i(t_k^+,x) - u_i(t_k^-,x) &= I_{ik}\left(u_i(t_k^-,x)\right),\\
&\qquad\qquad t = t_k,\ k\in\mathbb{Z}_+,\ x\in\Omega,\\
\frac{\partial u_i(t,x)}{\partial n} &= 0,\quad t\geq t_0,\ x\in\partial\Omega,\\
u_i(t_0+s,x) &= \psi_i(s,x),\quad -\tau_j\leq s\leq 0,\ x\in\Omega,
\end{aligned}
\tag{1}
$$

where $n \geq 2$ is the number of neurons in the network and $u_i(t,x)$ corresponds to the state of the ith neuron at time t and in space x; $D = \operatorname{diag}(D_1, D_2, \ldots, D_n)$ is the diffusion-matrix and $D_i \geq 0$; $\Delta = \sum_{k=1}^{m}(\partial^2/\partial x_k^2)$ is the Laplace operator; $f_j(u_j(t,x))$ denotes the activation function of the jth unit and $v_j(t)$ the activation function of the jth unit; $J_i(t)$ is an input at time t; $c_i(t) > 0$ represents the rate with which the ith unit will reset its potential to the resting state in isolation when disconnected from the networks and external inputs at time t; $a_{ij}(t)$ and $b_{ij}(t)$ are elements of feedback template and feed forward template at time t, respectively. Moreover, in model (1), $\alpha_{ij}(t)$, $\beta_{ij}(t)$, $T_{ij}(t)$, and $H_{ij}(t)$ are elements of fuzzy feedback MIN template, fuzzy feedback MAX template, fuzzy feed forward MIN template, and fuzzy feed forward MAX template at time t, respectively; the symbols "$\bigwedge$" and "$\bigvee$" denote the fuzzy AND and fuzzy OR operation, respectively; time-varying delay $\tau_j(t)$ is the transmission delay along the axon of the jth unit and satisfies $0 \leq \tau_j(t) \leq \tau_j$ (τ_j is a constant); the initial condition $\phi_i(s,x)$ is bounded and continuous on $[-\tau, 0]\times\Omega$, where $\tau = \max_{1\leq j\leq n}\tau_j$. The fixed moments t_k satisfy $0 = t_0 < t_1 < t_2\ldots$, $\lim_{k\to+\infty}t_k = +\infty$, $k \in \mathbb{N}$. $u_i(t_k^+,x)$ and $u_i(t_k^-,x)$ denote the right-hand and left-hand limits at t_k, respectively. We always assume $u_i(t_k^+,x) = u_i(t_k,x)$, for all $k \in N$. The initial value functions $\psi(s,x)$ belong to $\mathrm{PC}_\Omega([-\tau,0]\times\Omega; R^n)$. $\mathrm{PC}_\Omega(J\times\Omega, L^2(\Omega)) = \{\psi : J\times\Omega \to L^2(\Omega) \mid$ for every $t \in J, \psi(t,x) \in L^2(\Omega)$; for any fixed $x\in\Omega, \psi(t,x)$ is continuous for all but at most countable points $s \in J$ and at these points, $\psi(s^+,x)$ and $\psi(s^-,x)$ exist, $\psi(s^+,x) = \psi(s^-,x)\}$, where $\psi(s^+,x)$ and $\psi(s^-,x)$ denote the right-hand and left-hand limit of the function $\psi(s,x)$, respectively. Especially, let $\mathrm{PC}_\Omega = \mathrm{PC}([-\tau,0]\times\Omega, L^2(\Omega))$. For any $\psi(t,x) = (\psi_1(t,x),\ldots,\psi_n(t,x)) \in \mathrm{PC}_\Omega$, suppose that $|\psi_i(t,x)|_\tau = \sup_{-\tau<s\leq 0}|\psi_i(t+s,x)|$ exists as a finite number

and introduce the norm $\|\psi(t)\|_2 = (\sum_{i=1}^{n} \|\psi_i(t)\|_2^2)^{1/2}$, where $\|\psi_i(t)\|_2 = (\int_\Omega |\psi_i(t,x)|^2 dx)^{1/2}$.

Throughout the paper, we make the following assumptions.

(H1) There exists a positive diagonal matrix $F = \operatorname{diag}(F_1, F_2, \ldots, F_n)$, and $G = \operatorname{diag}(G_1, G_2, \ldots, G_n)$ such that

$$F_j = \sup_{x \neq y} \left| \frac{f_j(x) - f_j(y)}{x - y} \right|, \qquad G_j = \sup_{x \neq y} \left| \frac{g_j(x) - g_j(y)}{x - y} \right| \tag{2}$$

for all $x \neq y$, $j = 1, 2, \ldots, n$.

(H2) $c_i(t) > 0$, $a_{ij}(t), b_{ij}(t), \alpha_{ij}(t), \beta_{ij}(t), T_{ij}(t), H_{ij}(t), v_i(t), I_i(t)$, and $\tau_j(t) \geq 0$ are periodic function with a common positive period ω for all $t \geq t_0$, $i, j = 1, 2, \ldots, n$.

(H3) For $\omega > 0$, $i = 1, 2, \ldots, n$, there exists $q \in Z_+$ such that $t_k + \omega = t_{k+q}$, $I_{ik}(u_i) = I_{i(k+q)}(u_i)$ and $I_{ik}(u_i(t_k, x))$ are Lipschitz continuous in $\mathbb{R}^n$.

Definition 1. The model in (1) is said to be globally exponentially periodic if (i) there exists one ω-periodic solution and (ii) all other solutions of the model converge exponentially to it as $t \to +\infty$.

Definition 2 (see [26]). Let $\mathbf{C} = ([t-\tau, t], \mathbb{R}^n)$, where $\tau \geq 0$ and $F(t, x, y) \in \mathbf{C}(\mathbb{R}^+ \times \mathbb{R}^n \times \mathbf{C}, \mathbb{R}^n)$. Then the function $F(t, x, y) = (f_1(t, x, y), f_2(t, x, y), \ldots, f_n(t, x, y))^{\mathrm{T}}$ is called an M-function, if (i) for every $t \in \mathbb{R}^+$, $x \in \mathbb{R}^n$, $y^{(1)} \in \mathbf{C}$, there holds $F(t, x, y^{(1)}) \leq F(t, x, y^{(2)})$, for $y^{(1)} \leq y^{(2)}$, where $y^{(1)} = (y_1^{(1)}, \ldots, y_n^{(1)})^{\mathrm{T}}$ and $y^{(2)} = (y_1^{(2)}, \ldots, y_n^{(2)})^{\mathrm{T}}$; (ii) every ith element of F satisfies $f_i(t, x^{(1)}, y) \leq f_i(t, x^{(2)}, y)$ for any $y \in \mathbf{C}$, $t \geq t_0$, where arbitrary $x^{(1)}$ and $x^{(2)}$ $(x^{(1)} \leq x^{(2)})$ belong to $\mathbb{R}^n$ and have the same ith component $x_i^{(1)} = x_i^{(2)}$. Here, $x^{(1)} = (x_1^{(1)}, \ldots, x_n^{(1)})^{\mathrm{T}}$, $x^{(2)} = (x_1^{(2)}, \ldots, x_n^{(2)})^{\mathrm{T}}$.

Definition 3 (see [26]). A real matrix $A = (a_{ij})_{n\times n}$ is said to be a nonsingular M-matrix if $a_{ij} \leq 0$ $(i \neq j; i, j = 1, \ldots, n)$ and all successive principal minors of A are positive.

Lemma 4 (see [13]). *Let u and u^* be two states of the model in* (1)*, then we have*

$$\left| \bigwedge_{j=1}^{n} \alpha_{ij}(t) f_j(u_j) - \bigwedge_{j=1}^{n} \alpha_{ij}(t) f_j(u_j^*) \right| \leq \sum_{j=1}^{n} \left|\alpha_{ij}(t)\right| \cdot \left| f_j(u_j) - f_j(u_j^*) \right|,$$

$$\left| \bigvee_{j=1}^{n} \beta_{ij}(t) f_j(u_j) - \bigvee_{j=1}^{n} \beta_{ij}(t) f_j(u_j^*) \right| \leq \sum_{j=1}^{n} \left|\beta_{ij}(t)\right| \cdot \left| f_j(u_j) - f_j(u_j^*) \right|. \tag{3}$$

Lemma 5 (see [26]). *Assume that $F(t, x, y)$ is an M-function, and (i) $x(t) < y(t)$, $t \in [t-\tau, t_0]$, (ii) $D^+ y(t) > F(t, y(t), y^s(t))$, $D^+ x(t) \leq F(t, x(t), x^s(t))$, $t \geq t_0$, where $x^s(t) = \sup_{-\tau \leq s \leq 0} x(t+s)$, $y^s(t) = \sup_{-\tau \leq s \leq 0} y(t+s)$. Then $x(t) < y(t), t \geq t_0$.*

3. Main Results and Proofs

We should first point out that, under assumptions **(H1)**, **(H2)**, and **(H3)**, the FIRDDCNN model (1) has at least one ω-periodic solution of [26]. The proof of the existence of the ω-periodic solution of (1) can be carried out similar to [26, 28] by the nonlinear functional analysis methods such as topological degree and here is omitted. We will mainly discuss the uniqueness of the periodic solution and its exponential stability.

Theorem 6. *Assume that* **(H1)**–**(H3)** *holds. Furthermore, assume that the following conditions hold*

(H4) *$\underline{C} - \overline{A}F - (\overline{\alpha} + \overline{\beta})G$ is a nonsingular M-matrix.*

(H5) *The impulsive operators $h_k(u) = u + I_k(u)$ is Lipschitz continuous in $\mathbb{R}^n$; that is, there exists a nonnegative diagnose matrix $\Gamma_k = \operatorname{diag}(\gamma_{1k}, \ldots, \gamma_{nk})$ such that $|h_k(u) - h_k(u^*)| \leq \Gamma_k |u - u^*|$ for all $u, u^* \in \mathbb{R}^n$, $k \in N^+$, where $|h_k(u)| = (|h_{1k}(u_1)|, \ldots, |h_{nk}(u_n)|)^{\mathrm{T}}$, $I_k(u) = (I_{1k}(u_1), \ldots, I_{nk}(u_n))^{\mathrm{T}}$.*

(H6) *$\eta = \sup_{k\in N^+}\{\ln \eta_k/(t_k - t_{k-1})\} < \lambda$, where $\eta_k = \max_{1\leq i\leq n}\{1, \gamma_{ik}\}, k \in N^+$.*

Then the model (1) is global exponential periodic and the exponential convergence rate index $\lambda - \eta$ and λ can be estimated by

$$\xi_i(\lambda - \underline{c}_i) + \sum_{j=1}^{n} \xi_j \left(\left|\overline{a}_{ij}\right| F_j + e^{\tau\lambda} \left(\left|\overline{\alpha}_{ij}\right| + \left|\overline{\beta}_{ij}\right| \right) G_j \right) < 0 \tag{4}$$

$$i = 1, \ldots, n,$$

where $\underline{C} = \operatorname{diag}(\underline{c}_1, \ldots, \underline{c}_n)$ and $\xi_i > 0$, $\overline{A} = (|\overline{a}_{ij}|)_{n\times n}$, $\overline{\alpha} = (|\overline{\alpha}_{ij}|)_{n\times n}$, $\overline{\beta} = (|\overline{\beta}_{ij}|)_{n\times n}$, satisfies $-\xi_i \underline{c}_i + \sum_{j=1}^{n} \xi_i (|\overline{a}_{ij}| F_i + (|\overline{\alpha}_{ij}| + |\overline{\beta}_{ij}|) G_i) < 0$.

Proof. For any $\phi, \psi \in \mathrm{PC}_\Omega$, let $u(t, x, \phi) = (u_1(t, x, \phi), \ldots, u_n(t, x, \phi))^{\mathrm{T}}$ be a periodic solution of the system (1) starting from ϕ and $u(t,x,\psi) = (u_1(t,x,\psi),\ldots,u_n(t,x,\psi))^{\mathrm{T}}$, a solution of the system (1) starting from ψ. Define

$$u_t(\phi,x) = u(t+s,x,\phi),\qquad u_t(\psi,x) = u(t+s,x,\psi),\quad s\in[-\tau,0],\tag{5}$$

and we can see that $u_t(\phi,x), u_t(\psi,x)\in \mathrm{PC}_\Omega$ for all $t>0$. Let $U_i = u_i(t,x,\phi) - u_i(t,x,\psi)$, then from (1) we get

$$\begin{aligned}\frac{\partial U_i}{\partial t} &= \sum_{l=1}^{m}\frac{\partial}{\partial x_l}\left(D_i\frac{\partial U_i}{\partial x_l}\right) - c_i(t)U_i + \sum_{j=1}^{n}a_{ij}(t)\\ &\quad\times\left[f_j(u_j(t,x,\phi)) - f_j(u_j(t,x,\psi))\right]\\ &\quad+\left[\bigwedge_{j=1}^{n}\alpha_{ij}(t)f_j(u_j(t-\tau_j(t),x,\phi)) - \bigwedge_{j=1}^{n}\alpha_{ij}(t)f_j(u_j(t-\tau_j(t),x,\psi))\right]\\ &\quad+\left[\bigvee_{j=1}^{n}\beta_{ij}(t)f_j(u_j(t-\tau_j(t),x,\phi)) - \bigvee_{j=1}^{n}\beta_{ij}(t)f_j(u_j(t-\tau_j(t),x,\psi))\right]\end{aligned}\tag{6}$$

for all $t\neq t_k$, $x\in\Omega$, $i=1,\ldots,n$.

Multiplying both sides of (6) by U_i and integrating it in Ω, we have

$$\begin{aligned}\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\int_\Omega U_i^2\mathrm{d}x &= \int_\Omega U_i\sum_{l=1}^{m}\frac{\partial}{\partial x_l}\left(D_i\frac{\partial U_i}{\partial x_l}\right)\mathrm{d}x\\ &\quad - c_i(t)\int_\Omega U_i^2dx + \sum_{j=1}^{n}a_{ij}(t)\int_\Omega U_i\\ &\quad\times\left[f_j(u_j(t,x,\phi) - f_j(u_j(t,x,\psi)))\right]\mathrm{d}x\\ &\quad+\int_\Omega U_i\left[\bigwedge_{j=1}^{n}\alpha_{ij}(t)f_j(u_j(t-\tau_j(t),x,\phi)) - \bigwedge_{j=1}^{n}\alpha_{ij}(t)f_j(u_j(t-\tau_j(t),x,\psi))\right]\mathrm{d}x\\ &\quad+\int_\Omega U_i\left[\bigvee_{j=1}^{n}\beta_{ij}(t)f_j(u_j(t-\tau_j(t),x,\phi)) - \bigvee_{j=1}^{n}\beta_{ij}(t)f_j(u_j(t-\tau_j(t),x,\psi))\right]\mathrm{d}x\end{aligned}\tag{7}$$

for $t\neq t_k$, $x\in\Omega$, $i=1,\ldots,n$. By boundary condition and Green Formula, we can get

$$\int_\Omega U_i\sum_{l=1}^{m}\frac{\partial}{\partial x_l}\left(D_i\frac{\partial U_i}{\partial x_l}\right)dx \le -D_i\int_\Omega(\nabla U_i)^2\mathrm{d}x.\tag{8}$$

Then, from (8), (9), **(H1)**-**(H2)**, Lemma 4, and the Holder inequality,

$$\begin{aligned}\frac{d}{dt}\|U_i\|_2^2 &\le -2\underline{c}_i\|U_i\|_2^2 + 2\sum_{j=1}^{n}\left|\overline{a}_{ij}\right|F_j\|U_i\|_2\left\|U_j\right\|_2\\ &\quad+2\sum_{j=1}^{n}\left(\left|\overline{\alpha}_{ij}\right|+\left|\overline{\beta}_{ij}\right|\right)G_j\|U_i\|_2\\ &\quad\times\left\|u_j(t-\tau_j(t),x,\phi) - u_j(t-\tau_j(t),x,\psi)\right\|_2,\quad t\neq t_k.\end{aligned}\tag{9}$$

Thus,

$$\begin{aligned}D^+\|U_i\|_2 &\le -\underline{c}_i\|U_i\|_2 + \sum_{j=1}^{n}\left|\overline{\alpha}_{ij}\right|F_j\left\|U_j\right\|_2\\ &\quad+\sum_{j=1}^{n}\left(\left|\overline{\alpha}_{ij}\right|+\left|\overline{\beta}_{ij}\right|\right)G_j\\ &\quad\times\left\|u_j(t-\tau_j(t),x,\phi) - u_j(t-\tau_j(t),x,\psi)\right\|_2,\quad t\neq t_k\end{aligned}\tag{10}$$

for $i=1,\ldots,n$. Since $\underline{C} - (\overline{A}F + (\overline{\alpha}+\overline{\beta})G)$ is a nonsingular M-matrix, there exists a vector $\xi = (\xi_1,\ldots,\xi_n)^{\mathrm{T}} > 0$ such that

$$-\xi_i\underline{c}_i + \sum_{j=1}^{n}\xi_j\left(\left|\overline{a}_{ij}\right|F_j + \left(\left|\overline{\alpha}_{ij}\right|+\left|\overline{\beta}_{ij}\right|\right)G_j\right) < 0.\tag{11}$$

Considering functions

$$\begin{aligned}\Psi_i(y) &= \xi_i(y-\underline{c}_i)\\ &\quad+\sum_{j=1}^{n}\xi_j\left(\left|\overline{a}_{ij}\right|F_j + e^{\tau y}\left(\left|\overline{\alpha}_{ij}\right|+\left|\overline{\beta}_{ij}\right|\right)G_j\right),\quad i=1,\ldots,n,\end{aligned}\tag{12}$$

we know from (11) that $\Psi_i(0) < 0$ and $\Psi_i(y)$ is continuous. Since $\mathrm{d}\Psi_i(y)/\mathrm{d}y > 0$, $\Psi_i(y)$ is strictly monotonically increasing, there exists a scalar $\lambda_i > 0$ such that

$$\begin{aligned}\Psi_i(\lambda_i) &= \xi_i(\lambda_i-\underline{c}_i)\\ &\quad+\sum_{j=1}^{n}\xi_j\left(\left|\overline{a}_{ij}\right|F_j + e^{\tau\lambda_i}\left(\left|\overline{\alpha}_{ij}\right|+\left|\overline{\beta}_{ij}\right|\right)G_j\right) = 0,\quad i=1,\ldots,n.\end{aligned}\tag{13}$$

Choosing $0 < \lambda < \min\{\lambda_1, \ldots, \lambda_n\}$, we have

$$\xi_i (\lambda_i - \underline{c}_i) + \sum_{j=1}^{n} \xi_j \left(\left|\overline{a}_{ij}\right| F_j + e^{\tau\lambda_i} \left(\left|\overline{\alpha}_{ij}\right| + \left|\overline{\beta}_{ij}\right| \right) G_j \right) < 0, \quad i = 1, \ldots, n. \tag{14}$$

That is,

$$\lambda\xi - \left(\underline{C} - \overline{A}F\right)\xi + \left(\overline{\alpha} + \overline{\beta}\right) G\xi e^{-\lambda t} < 0. \tag{15}$$

Furthermore, choose a positive scalar p large enough such that

$$pe^{-\lambda t}\xi > (1, 1, \ldots, 1)^{\mathrm{T}}, \quad t \in [-\tau, 0]. \tag{16}$$

For any $\varepsilon > 0$, let

$$r(t) = pe^{-\lambda t} \left(\|\phi - \psi\|_2 + \varepsilon \right) \xi, \quad t_0 \le t < t_1. \tag{17}$$

From (15)–(17), we obtain

$$\begin{aligned} D^+ r(t) &> -\left(\underline{C} - \overline{A}F\right) r(t) + \left(\overline{\alpha} + \overline{\beta}\right) G r^s(t) \\ &=: V\left(t, r(t), r^s(t)\right), \quad t_0 \le t < t_1, \end{aligned} \tag{18}$$

where $r^s(t) = (r_1^s(t), \ldots, r_n^s(t))^{\mathrm{T}}$ and $r_i^s(t) = \sup_{-\tau \le s \le 0} pe^{-\lambda(t+s)} (\|\phi - \varphi\|_2 + \varepsilon)\xi_i$. It is easy to verify that $V(t, r(t), r^s(t))$ is an M-function. It follows also from (16) and (17) that

$$\begin{aligned} \|U_i\|_2 \le \|\phi - \varphi\|_2 &< pe^{-\lambda t}\xi_i \|\phi - \varphi\|_2 \\ &< r_i(t), \quad t \in [-\tau, 0], \ i = 1, 2, \ldots, n. \end{aligned} \tag{19}$$

Denote

$$\begin{aligned} U^{\diamond} &:= \left(\|u_1(t, x, \phi) - u_1(t, x, \psi)\|_2, \ldots, \right. \\ &\qquad \left. \|u_n(t, x, \phi) - u_n(t, x, \psi)\|_2 \right)^{\mathrm{T}}, \\ U^{\diamond(s)} &:= \left(\|u_1(t, x, \phi) - u_1(t, x, \psi)\|_2^{(s)}, \ldots, \right. \\ &\qquad \left. \|u_n(t, x, \phi) - u_n(t, x, \psi)\|_2^{(s)} \right)^{\mathrm{T}}, \end{aligned} \tag{20}$$

where $\|U_i\|_2^{(s)} = \sup_{-\tau \le s \le 0} \|u_i(t+s, x, \phi) - u_i(t+s, x, \psi)\|_2$, then

$$U^{\diamond} < r(t), \quad t \in [-\tau, 0]. \tag{21}$$

From (10), we can obtain

$$\begin{aligned} D^+ U^{\diamond} &\le -\left(\underline{C} - \overline{A}F\right) U^{\diamond} + \left(\overline{\alpha} + \overline{\beta}\right) G U^{\diamond(s)} \\ &= V\left(t, U^{\diamond}, U^{\diamond(s)}\right), \quad t \ne t_k. \end{aligned} \tag{22}$$

Now, it follows from (18)–(22) and Lemma 5 that

$$U^{\diamond} < r(t) = pe^{-\lambda t} \left(\|\phi - \psi\|_2 + \varepsilon \right) \xi, \quad t_0 \le t < t_1. \tag{23}$$

Letting $\varepsilon \to 0$, we have

$$U^{\diamond} \le p\xi \|\phi - \psi\|_2 e^{-\lambda t}, \quad t_0 \le t < t_1. \tag{24}$$

And moreover, from (24), we get

$$\left(\sum_{i=1}^{n} \|U_i\|_2^2 \right)^{1/2} \le p \left(\sum_{i=1}^{n} \xi_i^2 \right)^{1/2} \|\phi - \psi\|_2 e^{-\lambda t}, \quad t_0 \le t < t_1. \tag{25}$$

Let $\widetilde{M} = p(\sum_{i=1}^{n} \xi_i^2)^{1/2}$, then $\widetilde{M} \ge 1$. Define $W(t) = \|u_t(x, \phi) - u_t(x, \psi)\|_2$; it follows from (25) and the definitions of $u_t(\phi, x)$ and $u_t(\psi, x)$ that

$$\begin{aligned} W(t) &= \|u_t(x, \phi) - u_t(x, \psi)\|_2 \\ &\le \widetilde{M} \|\phi - \psi\|_2 e^{-\lambda t}, \quad t_0 \le t < t_1. \end{aligned} \tag{26}$$

It is easily observed that

$$W(t) \le \widetilde{M} \|\phi - \psi\|_2 e^{-\lambda t}, \quad -\tau \le t \le t_0 = 0. \tag{27}$$

Because (26) holds, we can suppose that for $l \le k$ inequality

$$W(t) \le \eta_0 \cdots \eta_{l-1} \widetilde{M} \|\phi - \psi\|_2 e^{-\lambda t}, \quad t_{l-1} \le t < t_l \tag{28}$$

holds, where $\eta_0 = 1$. When $l = k + 1$, we note **(H5)** that

$$\begin{aligned} W(t_k) &= \left\| u_{t_k}(x, \phi) - u_{t_k}(x, \psi) \right\|_2 \\ &= \left\| h_k\left(u_{t_k}^-(x, \phi)\right) - h_k\left(u_{t_k}^-(x, \psi)\right) \right\|_2 \\ &\le \rho\left(\Gamma_k^2\right) \left\| u_{t_k}^-(x, \phi) - u_{t_k}^-(x, \psi) \right\|_2 \\ &= \rho\left(\Gamma_k^2\right) W\left(t_k^-\right) \\ &\le \eta_0 \cdots \eta_{l-1} \rho\left(\Gamma_k^2\right) \widetilde{M} \|\phi - \psi\|_2 e^{-\lambda t_k} \\ &\le \eta_0 \cdots \eta_{l-1} \eta_k \rho\left(\Gamma_k^2\right) \widetilde{M} \|\phi - \psi\|_2 e^{-\lambda t_k}, \end{aligned} \tag{29}$$

where $\rho(\Gamma_k^2)$ is the spectral radius of Γ_k^2. Let $M = \max\{\widetilde{M}, \rho(\Gamma_k^2)\widetilde{M}\}$, by (28), (29), and $\eta \ge 1$, we obtain

$$W(t) \le \eta_0 \cdots \eta_{l-1} \eta_k M \|\phi - \psi\|_2 e^{-\lambda t}, \quad t_k - \tau \le t \le t_k. \tag{30}$$

Combining (10), (17), (30), and Lemma 5, we get

$$W(t) \le \eta_0 \cdots \eta_{l-1} \eta_k M \|\phi - \psi\|_2 e^{-\lambda t}, \quad t_k \le t < t_{k+1}, \ k \in N^+. \tag{31}$$

Applying mathematical induction, we conclude that

$$W(t) \le \eta_0 \cdots \eta_{l-1} M \|\phi - \psi\|_2 e^{-\lambda t}, \quad t_{k-1} \le t < t_k, \ k \in N^+. \tag{32}$$

From **(H6)** and (32), we have

$$\begin{aligned} W(t) &\le e^{\eta t_1} e^{\eta(t_2-t_1)} \cdots e^{\eta(t_{k-1}-t_{k-2})} \\ &\quad \times M\|\phi-\psi\|_2 e^{-\lambda t} \le M\|\phi-\psi\|_2 e^{\eta t} e^{-\lambda t} \\ &= M\|\phi-\psi\|_2 e^{-(\lambda-\eta)t}, \\ &\qquad t_{k-1} \le t < t_k,\ k \in N^+. \end{aligned} \tag{33}$$

This means that

$$\begin{aligned} &\|u_t(x,\phi) - u_t(x,\psi)\|_2 \\ &\quad \le M\|\phi-\psi\|_2 e^{-(\lambda-\eta)t} \\ &\quad \le M\|\phi-\psi\|_2 e^{-(\lambda-\eta)(t-\tau)}, \quad t \ge t_0, \end{aligned} \tag{34}$$

choosing a positive integer N such that

$$Me^{-(\lambda-\eta)(N\omega-\tau)} \le \frac{1}{6}. \tag{35}$$

Define a Poincare mapping $\mathfrak{D} : \Gamma \to \Gamma$ by

$$\mathfrak{D}(\phi) = u_\omega(x,\phi), \tag{36}$$

Then

$$\mathfrak{D}^N(\phi) = u_{N\omega}(x,\phi). \tag{37}$$

Setting $t = N\omega$ in (34), from (35) and (37), we have

$$\left\|\mathfrak{D}^N(\phi) - \mathfrak{D}^N(\psi)\right\|_2 \le \frac{1}{6}\|\phi-\psi\|_2, \tag{38}$$

which implies that $\mathfrak{D}^N$ is a contraction mapping. Thus, there exists a unique fixed point $\phi^* \in \Gamma$ such that

$$\mathfrak{D}^N(\mathfrak{D}(\phi^*)) = \mathfrak{D}\left(\mathfrak{D}^N(\phi^*)\right) = \mathfrak{D}(\phi^*). \tag{39}$$

From (37), we know that $\mathfrak{D}(\phi^*)$ is also a fixed point of $\mathfrak{D}^N$, and then it follows from the uniqueness of the fixed point that

$$\mathfrak{D}(\phi^*) = \phi^*, \quad \text{that is, } u_\omega(x,\phi^*) = \phi^*. \tag{40}$$

Let $u(t,x,\phi^*)$ be a solution of the model (1), then $u(t+\omega,x,\phi^*)$ is also a solution of the model (1). Obviously,

$$u_{t+\omega}(x,\phi^*) = u_t(u_\omega(x,\phi^*)) = u_t(x,\phi^*), \tag{41}$$

for all $t \ge t_0$. Hence, $u(t+\omega,x,\phi^*) = u(t,x,\phi^*)$, which shows that $u(t,x,\phi^*)$ is exactly one ω-periodic solution of model (1). It is easy to see that all other solutions of model (1) converge to this periodic solution exponentially as $t \to +\infty$, and the exponential convergence rate index is $\lambda-\eta$. The proof is completed. □

Remark 7. When $c_i(t) = c_i$, $a_{ij}(t) = a_{ij}$, $b_{ij}(t) = b_{ij}$, $\alpha_{ij}(t) = \alpha_{ij}$, $\beta_{ij}(t) = \beta_{ij}$, $T_{ij}(t) = T_{ij}$, $H_{ij}(t) = H_{ij}$, $v_i(t) = v_i$, $I_i(t) = I_i$, and $\tau_t = \tau_i$($c_i, a_{ij}, b_{ij}, \alpha_{ij}, \beta_{ij}, T_{ij}, H_{ij}, v_i, I_i$, and τ_i are constants), then the model (1) is changed into

$$\begin{aligned} \frac{\partial u_i(t,x)}{\partial t} &= \sum_{l=1}^{m} \frac{\partial}{\partial x_l}\left(D_i \frac{\partial u_i(t,x)}{\partial x_l}\right) \\ &\quad - c_i u_i(t,x) + \sum_{j=1}^{n} a_{ij} f_j\left(u_j(t,x)\right) \\ &\quad + \sum_{j=1}^{n} b_{ij} v_j + J_i \\ &\quad + \bigwedge_{j=1}^{n} \alpha_{ij} g_j\left(u_j\left(t-\tau_j(t),x\right)\right) \\ &\quad + \bigvee_{j=1}^{n} \beta_{ij} g_j\left(u_j\left(t-\tau_j(t),x\right)\right) \\ &\quad + \bigwedge_{j=1}^{n} T_{ij} v_j + \bigvee_{j=1}^{n} H_{ij} v_j, \quad t \ne t_k,\ x \in \Omega, \\ u_i(t_k^+,x) - u_i(t_k^-,x) &= I_{ik}(u_i(t_k^-,x)), \\ &\qquad t = t_k,\ k \in \mathbb{Z}_+,\ x \in \Omega, \\ \frac{\partial u_i(t,x)}{\partial n} &= 0, \quad t \ge t_0,\ x \in \partial\Omega, \\ u_i(t_0+s,x) &= \psi_i(s,x), \quad -\tau_j \le s \le 0,\ x \in \Omega. \end{aligned} \tag{42}$$

For any positive constant $\omega \ge 0$, we have $c_i(t+\omega) = c_i(t)$, $a_{ij}(t+\omega) = a_{ij}(t)$, $b_{ij}(t+\omega) = b_{ij}(t)$, $\alpha_{ij}(t+\omega) = \alpha_{ij}(t)$, $\beta_{ij}(t+\omega) = \beta_{ij}(t)$, $T_{ij}(t+\omega) = T_{ij}(t)$, $H_{ij}(t+\omega) = H_{ij}(t)$, $v_i(t+\omega) = v_i(t)$, $I_i(t+\omega) = I_i(t)$, and $\tau_i(t+\omega) = \tau_i(t)$ for $t \ge t_0$. Thus, the sufficient conditions in Theorem 6 are satisfied.

Remark 8. If $I_k(\cdot) = 0$, the model (1) is changed into

$$\begin{aligned} \frac{\partial u_i(t,x)}{\partial t} &= \sum_{l=1}^{m} \frac{\partial}{\partial x_l}\left(D_i \frac{\partial u_i(t,x)}{\partial x_l}\right) \\ &\quad - c_i(t) u_i(t,x) + \sum_{j=1}^{n} a_{ij}(t) f_j\left(u_j(t,x)\right) \\ &\quad + \sum_{j=1}^{n} b_{ij}(t) v_j(t) + J_i(t) \\ &\quad + \bigwedge_{j=1}^{n} \alpha_{ij}(t) g_j\left(u_j\left(t-\tau_j(t),x\right)\right) \\ &\quad + \bigvee_{j=1}^{n} \beta_{ij}(t) g_j\left(u_j\left(t-\tau_j(t),x\right)\right) \end{aligned}$$

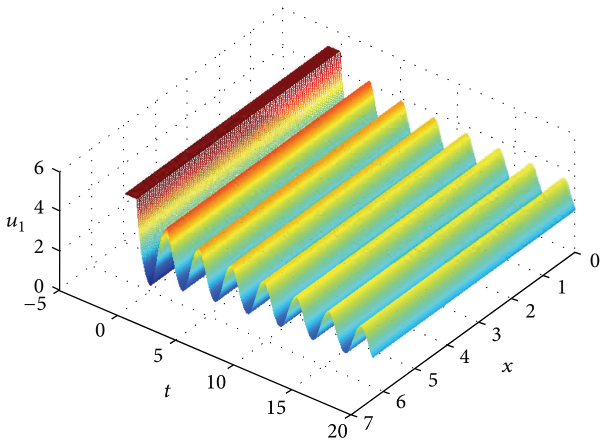

Figure 1: State response $u1(t,x)$ of model (44) without impulsive effects.

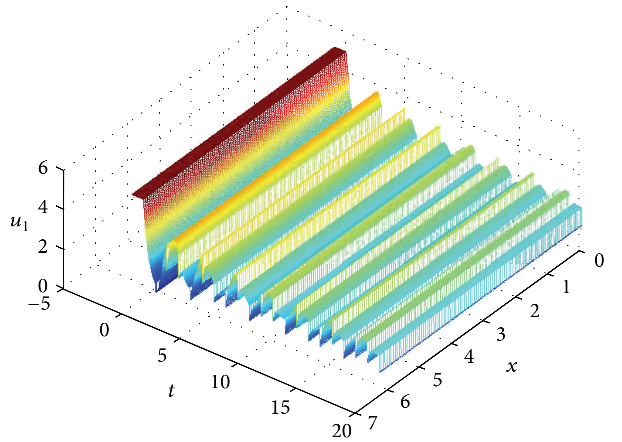

Figure 2: State response $u1(t,x)$ of model (44) with impulsive effects.

$$
\begin{aligned}
&+\bigwedge_{j=1}^{n} T_{ij}(t)\, v_j(t) + \bigvee_{j=1}^{n} H_{ij}(t)\, v_j(t), \\
&\qquad\qquad t \neq t_k,\ x \in \Omega, \\
&\frac{\partial u_i(t,x)}{\partial n} = 0, \quad t \geq t_0,\ x \in \partial\Omega, \\
&u_i(t_0+s,x) = \psi_i(s,x), \quad -\tau_j \leq s \leq 0,\ x \in \Omega,
\end{aligned} \tag{43}
$$

which has been discussed in [22]. As Song and Wang have pointed out, the model (43) is more general than some well-studied fuzzy neural networks. For example, when $c_i(t) > 0, a_{ij}(t), b_{ij}(t), \alpha_{ij}(t), \beta_{ij}(t), T_{ij}(t), H_{ij}(t), v_i(t)$, and $I_i(t)$ are all constants, the model in (43) reduces the model which has been studied by Huang [19]. Moreover, if $D_i = 0$, $\tau_i(t) = 0$, $f_i(\theta) = g_i(\theta) = (1/2)(|\theta+1| - |\theta-1|)$, $(i = 1,\ldots,n)$, then model (42) covers the model studied by Yang et al. [4, 5] as a special case. If $D_i = 0$ and $\tau_j(t)$ is assumed to be differentiable for $i, j = 1,2,\ldots,n$, then model (43) can be specialized to the model investigated in Liu and Tang [12] and Yuan et al. [13]. Obviously, our results are less conservative than that of the above-mentioned literature, because they do not consider impulsive effects.

4. Numerical Examples

Example 9. Consider a two-neuron FIRDDCNN model:

$$
\begin{aligned}
\frac{\partial u_i(t,x)}{\partial t} &= \sum_{l=1}^{m} \frac{\partial}{\partial x_l}\left(D_i \frac{\partial u_i(t,x)}{\partial x_l}\right) \\
&\quad - c_i(t)\, u_i(t,x) \\
&\quad + \sum_{j=1}^{2} a_{ij}(t)\, f_j\left(u_j(t,x)\right) \\
&\quad + \sum_{j=1}^{2} b_{ij}(t)\, v_j(t) + J_i(t) \\
&\quad + \bigwedge_{j=1}^{2} \alpha_{ij}(t)\, g_j\left(u_j\left(t-\tau_j(t),x\right)\right) \\
&\quad + \bigvee_{j=1}^{2} \beta_{ij}(t)\, g_j\left(u_j\left(t-\tau_j(t),x\right)\right) \\
&\quad + \bigwedge_{j=1}^{2} T_{ij}(t)\, v_j(t) + \bigvee_{j=1}^{2} H_{ij}(t)\, v_j(t), \\
&\qquad\qquad t \neq t_k,\ x \in \Omega, \\
u_i(t_k^+,x) &= (1-\gamma_{ik})\, u_i(t_k^-,x), \\
&\qquad t = t_k,\ k \in \mathbb{Z}_+,\ x \in \Omega, \\
\frac{\partial u_i(t,x)}{\partial n} &= 0, \quad t \geq t_0,\ x \in \partial\Omega, \\
u_i(t_0+s,x) &= \psi_i(s,x), \\
&\quad -\tau_j \leq s \leq 0,\ x \in \Omega,
\end{aligned} \tag{44}
$$

where $i = 1,2$. $c_1(t) = 26$, $c_2(t) = 20.8$, $a_{11}(t) = -1-\cos(t)$, $a_{12}(t) = 1+\cos(t)$, $a_{21}(t) = 1+\sin(t)$, $a_{22}(t) = -1-\sin(t)$, $D_1 = 8$, $D_2 = 4$, $\partial u_i(t,x)/\partial n = 0$ $(t \geq t_0, x = 0, 2\pi)$, $\gamma_{1k} = 0.4$, $\gamma_{2k} = 0.2$, $\psi_1(\cdot) = \psi_1(\cdot) = 5$, $b_{11}(t) = b_{21}(t) = \cos(t)$, $b_{12}(t) = b_{22}(t) = -\cos(t)$, $J_1(t) = J_2(t) = 1$, $H_{11}(t) = H_{21}(t) = \sin(t)$, $H_{12}(t) = H_{22}(t) = -1+\sin(t)$, $T_{11}(t) = T_{21}(t) = -\sin(t)$, $T_{12}(t) = T_{22}(t) = 2+\sin(t)$, $\tau_1(t) = \tau_2(t) = 1$, $f_j(u_j) = u_j(t,x)$ $(j = 1,2)$, $g_j(u_j(t-1,x)) = u_j(t-1,x)e^{-u_j(t-1,x)}$ $(j = 1,2)$, $\alpha_{11}(t) = -12.8$, $\alpha_{21}(t) = \alpha_{12}(t) = -1+\cos(t)$, $\alpha_{22}(t) = -10$, $\beta_{11}(t) = 12.8$, $\beta_{12}(t) = -1+\sin(t) = \beta_{21}(t)$, $\beta_{22}(t) =$

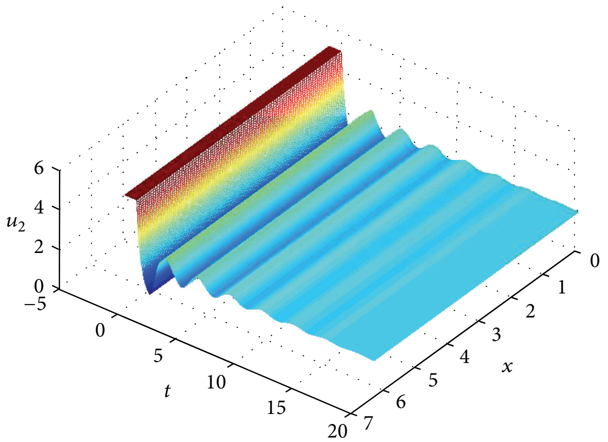

FIGURE 3: State response $u2(t, x)$ of model (44) without impulsive effects.

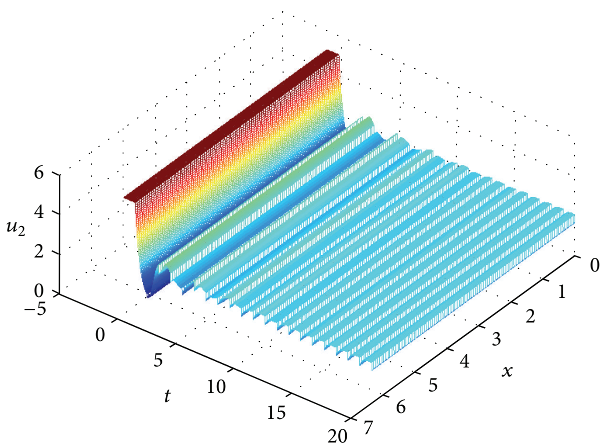

FIGURE 4: State response $u2(t, x)$ of model (44) with impulsive effects.

10, $v_j(t) = \sin(t)$. We assume that there exists $q = 6$ such that $t_k + 2\pi = t_{k+q}$. Obviously, f_1, f_2, g_1, and g_2 satisfy the assumption **(H1)** with $F_1 = F_2 = G_1 = G_2 = 1$ and **(H2)** and **(H3)** are satisfied with a common positive period 2π

$$\underline{C} - \left(\overline{A} + \overline{\alpha} + \overline{\beta}\right) F = \begin{bmatrix} \frac{2}{5} & 0 \\ 0 & \frac{4}{5} \end{bmatrix} \tag{45}$$

is a nonsingular M-matrix. The conditions of Theorem 6 are satisfied, hence there exists exactly one 2ω-periodic solution of the model and all other solutions of the model converge exponentially to it as $t \to +\infty$. Furthermore, the exponential converging index can be calculated as $\lambda = 0.021$, because here $\eta_k = 1$ and $\eta = 0$. The simulation results are shown in Figures 1, 2, 3, and 4, respectively.

5. Conclusions

In this paper, periodicity and global exponential stability of a class of FIRDDCNN model with variable both coefficients and delays have been investigated. By using Halanay's delay differential inequality, M-matrix theory, and analytic methods, some new sufficient conditions have been established to guarantee the existence, uniqueness, and global exponential stability of the periodic solution. Moreover, the exponential convergence rate index can be estimated. An example and its simulation have been given to show the effectiveness of the obtained results. In particular, the differentiability of the time-varying delays has been removed. The dynamic behaviors of fuzzy neural networks with the property of exponential periodicity are of great importance in many areas such as learning systems.

Acknowledgments

The authors would like to thank the editors and the anonymous reviewers for their valuable comments and constructive suggestions. This research is supported by the Natural Science Foundation of Guangxi Autonomous Region (no. 2012GXNSFBA053003), the National Natural Science Foundations of China (60973048, 61272077, 60974025, 60673101, and 60939003), National 863 Plan Project (2008 AA04Z401, 2009 AA043404), the Natural Science Foundation of Shandong Province (no. Y2007G30), the Scientific and Technological Project of Shandong Province (no. 2007GG3WZ04016), the Science Foundation of Harbin Institute of Technology (Weihai) (HIT (WH) 200807), the Natural Scientific Research Innovation Foundation in Harbin Institute of Technology (HIT. NSRIF. 2001120), the China Postdoctoral Science Foundation (2010048 1000), and the Shandong Provincial Key Laboratory of Industrial Control Technique (Qingdao University).

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