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Research Article

Observer-Based H_∞ Control Design for Nonlinear Networked Control Systems with Limited Information

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This paper is concerned with the problem of designing a robust observer-based H_∞ controller for discrete-time networked systems with limited information. An improved networked control system model is proposed and the effects of random packet dropout, time-varying delay, and quantization are considered simultaneously. Based on the obtained model, a stability criterion is developed by constructing an appropriate Lyapunov-Krasovskii functional and sufficient conditions for the existence of a dynamic quantized output feedback controller which are given in terms of linear matrix inequalities (LMIs) such that the augmented error system is stochastically stable with an H_∞ performance level. An example is presented to illustrate the effectiveness of the proposed method.

1. Introduction

Networked control systems (NCSs) are distributed systems in which communication between sensors, actuators and controllers is supported by a shared real-time network. Compared with conventional point-to-point system connection, this new network-based control scheme reduces system wiring and has low cost, high reliability, information sharing, and remote control [1, 2]. However, NCSs also introduce many new challenges in control system design such as packet dropout, networked-induced delay and signal quantization variable transmission intervals, network security and other communication constraints [3–9]. The authors in [10] proposed different levels of network-induced imperfections and presented the main survey of the methodologies in the analysis and design of NCSs.

The time-delay and packet dropouts occur in various physical, industrial, and engineering systems, which is a source of poor performance and instability of systems. Therefore, the issues of network-induced time-delays and packet dropouts have been considered by many researchers. Markov chain has been used to model the randomness of the network-induced delays (packet dropouts) [11–15]. The Markov chain takes values, which correspond to network-induced delays (packet dropouts), in a finite set based on known probabilities. In addition, the transition probabilities of Markovian jump systems (MJSs) are partly unknown due to the complexity of network [16, 17]. The authors in [16] introduced a nonlinear delayed MJSs model with partially unknown transition probabilities established by multiple channels data transmission framework. An iterative method was proposed in [17] to model NCSs with bounded packet dropout as Markovian jump linear systems with partly unknown transition probabilities. On the other hand, much attention has been devoted to the time-delayed jump linear systems with Markovian jumping parameters [18–20], which applied a less-conservative delay-range-dependent method. The stability analysis and stabilization problems for a class of discrete-time Markov jump linear systems with partially known transition probabilities, and time-varying delays are investigated in [18].

Quantization process cannot be neglected in NCSs. Real communication networks only send finite precision data due to limited bandwidth. Quantizers with coarser quantization densities help in reducing the network congestion. Consequently, network-induced delays can be reduced because less information is transmitted. In [21, 22], the quantizing effects were transformed into sector-bounded uncertainties and the quantization density was given in the control synthesis. For a logarithmic quantizer, the quantization error was modeled as a norm-bounded uncertainty [23]. However, the quantization density in most studies is just neglected or given as sector bounded uncertainties, and the relationship between the quantization density and the network load has not been fully investigated in the aforementioned papers.

Motivated by the above discussion, in this paper, we are aiming at investigating the study of observer-based controller design for nonlinear discrete-time networked systems subject to sensor-controller packet losses, time-varying delays, and output quantization. In this work, the quantization density is designed to be a function of the network load condition which is modeled by a Markov chain [24]. Furthermore, by applying the Lyapunov-Krasovskii functional method, a mode-dependent observer is constructed and the bounded real lemma (BRL) for the resulting error system is derived in terms of LMIs. Also, an improved version of the BRL is given by introducing additional slack matrix variables to eliminate the cross-coupling between system matrices and Lyapunov matrices among different operation modes, which can reduce the computational complexity as far as possible.

The paper is organized as follows. In Section 2, the NCS description, quantization error modeling, packet dropout and time-varying delay formulation are presented. Main results for stability criterion and observer-based controller synthesis are given in Section 3. A numerical example is provided in Section 4. Finally, Section 5 concludes this paper.

Notation. Throughout the paper, the superscripts "−1" and "T" stand for the inverse and transpose of a matrix, respectively; R^n denotes an n-dimensional Euclidean space and the notation $P > 0$ means that P is a real symmetric positive definite matrix. $E\{x\}$ is the expectation of a stochastic variable x. I and 0 represent an identity matrix and a zero matrix with appropriate dimensions in different place, respectively. In symmetric block matrices or complex matrix expressions, we use an asterisk $*$ to represent a term that is induced by symmetry and diag$\{\cdots\}$ stands for a block diagonal matrix. $\|\cdot\|$ refers to the Euclidean norm for vectors and induced 2-norm for matrices. $l_2[k_0,\infty)$ stands for the space of square summable infinite sequence on $[k_0,\infty)$.

2. Problem Formulation

A simple-networked control system is shown in Figure 1. A class of discrete-time nonlinear systems under consideration is described by the following equations:

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) + f(k, x(k)) + Hw(k), \\ y(k) &= Cx(k), \\ z(k) &= Gx(k), \end{aligned} \tag{1}$$

where $x(k) \in R^n$, $u(k) \in R^m$, $z(k) \in R^p$, and $y(k) \in R^s$ are the state vector, control input vector, controlled output vector, and measured output, respectively. $w(k) \in R^d$ is the exogenous disturbance signal belonging to $l_2[0,\infty)$. A, B, H, C, and G are known real matrices with appropriate

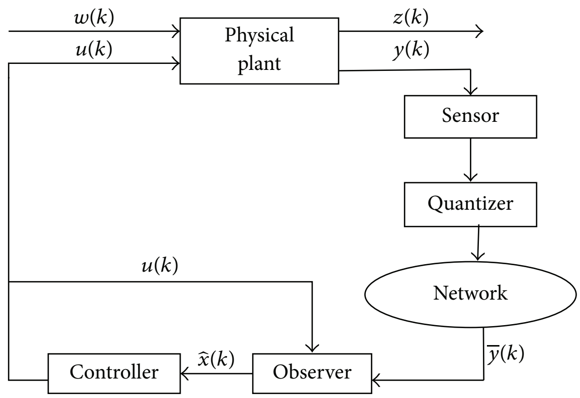

Figure 1: Configuration of the networked control systems with quantizer.

dimensions. The nonlinear function $f(\cdot)$ satisfies $f(\cdot,0) = 0$ and the following sector-bounded condition [25]:

$$[f(k,x(k)) - S_1x(k)]^T [f(k,x(k)) - S_2x(k)] \leq 0, \tag{2}$$

where $S_1, S_2 \in R^{n\times n}$ are known real constant matrices, and $S = S_1 - S_2$ is a symmetric positive definite matrix.

The network load condition is related to the network-induced delay. In [26], a Markov chain has been used to model the network load condition. Varying network load can be modeled by making the Markov chain do a transition and transfer every time over the communication network. Modes 1, 2, and 3 are, respectively, corresponded to light load, medium load, and heavy load.

Then, a discrete homogeneous Markov chain $\{\sigma(k), k\}$ with a finite number of states $\mathfrak{R} = \{1,2,3\}$ is used to model a network load. The transition probability from mode i at k to mode j at time $k+1$ is governed by

$$\pi_{ij} = \text{prob}\{\sigma(k+1) = j \mid \sigma(k) = i\}, \quad i, j \in \mathfrak{R}, \tag{3}$$

where $\pi_{ij} \geq 0$ $(i, j \in \mathfrak{R})$ is the transition probability element and satisfies $\sum_{j=1}^{3} \pi_{ij} = 1$ (for all $i \in \mathfrak{R}$).

In addition, the transition probability considered in this paper is partly accessed, and some elements are unknown. For notation clarity, we denote $\mathfrak{R} = \mathfrak{R}_K^i + \mathfrak{R}_{UK}^i$ (for all $i \in \mathfrak{R}$) with $\mathfrak{R}_K^i \triangleq \{j : \pi_{ij}$ is known$\}$ and $\mathfrak{R}_{UK}^i \triangleq \{j : \pi_{ij}$ is unknown$\}$.

Also, we can obtain

$$\sum_{j\in\mathfrak{R}} \pi_{ij} P_j = P_K^i + \sum_{j\in\mathfrak{R}_{UK}^i} \pi_{ij} P_j, \tag{4}$$

where $P_K^i = \sum_{j\in\mathfrak{R}_K^i} \pi_{ij} P_j$.

In this paper, the quantization density is designed to be a function of the network load condition. Hence, the quantization density function $\delta = \delta(\sigma(k))$ is described as a finite state Markov chain. On the basis of this technique, a networked load dependent on the logarithmic quantizer [23, 24] is proposed as follows:

$$q(v,i) = \begin{cases} \rho^h(i) & \text{if } \frac{1}{1+\delta(i)}\rho^h(i) < v \le \frac{1}{1-\delta(i)}\rho^h(i), \\ & \quad v > 0, \quad h = 0, \pm 1, \pm 2, \ldots \\ 0 & \text{if } v = 0 \\ -q(-v,i) & \text{if } v < 0, \end{cases} \tag{5}$$

where $0 < \rho(i) < 1$ is the quantization density of $q(\cdot,\cdot)$, and $\delta(i)$ is related to $\rho(i)$ by

$$\delta(i) = \frac{1-\rho(i)}{1+\rho(i)}. \tag{6}$$

This means that the smaller $\rho(i)$ results in the smaller $\delta(i)$. For this reason, we will call $\delta(i)$ the quantization density in the rest of this paper.

The associated quantized set U is given by

$$U = \left\{\pm\rho^h(i), h = 0, \pm 1, \pm 2, \ldots\right\} \bigcup \{0\}. \tag{7}$$

Now define the quantization error as

$$e_y(k,i) = q(y(k), i) - y(k) = \Delta_q(k,i)\, y(k), \tag{8}$$

where $y(k)$ is the signal to be quantized and $q(y(k), i)$ is the quantized signal. The uncertainty matrix $\Delta_q(k,i) \triangleq \operatorname{diag}(\Delta_{q1}(k,i), \ldots, \Delta_{qs}(k,i))$ satisfies $\Delta_{q\iota}(k,i) \in [-\delta(i), \delta(i)]$, $\iota = 1, 2, \ldots, s$.

The measured output $y(k)$ which is used as feedback information for controller may not be available all the time due to packet dropouts. To describe these packet dropouts, a stochastic variable following Bernoulli sequence is used. In this paper, the measured output received in the controller side is involved with the effects of quantization, packet losses, and time-varying delay, and it is described by

$$\overline{y}(k) = \theta_k q(y(k-\tau_k)), \tag{9}$$

where $\underline{\tau} = \min \tau_k$ and $\overline{\tau} = \max \tau_k$ are the lower and upper bounds of the delay, respectively. Stochastic variable $\theta_k \in R$ is a Bernoulli distributed white sequence with a probability distribution as follows:

$$\begin{aligned} &\operatorname{Prob}\{\theta_k = 1\} = E\{\theta_k\} = \theta, \\ &\operatorname{Prob}\{\theta_k = 0\} = 1 - E\{\theta_k\} = 1 - \theta, \end{aligned} \tag{10}$$

where $0 \le \theta \le 1$ is a known positive constant which is used to denote the probability that the packet will be transmitted successfully from sensor to controller, and we have

$$E\{\theta_k - \theta\} = 0, \qquad E\left\{(\theta_k - \theta)^2\right\} = \theta(1-\theta) = \overline{\theta}^2. \tag{11}$$

Remark 1. For NCSs, network-induced delay, packet dropout, and quantization error often occur simultaneously due to limited channel capacity and noise. Actually, pattern (9) describes a class of NCSs with data loss, time-varying delay and the quantization error simultaneously, so it is more comprehensive than separate consideration of delay pattern [27] or data loss pattern [28].

In this work, the following dynamic observer-based control scheme is employed for system (1):

$$\begin{aligned} \widehat{x}(k+1) &= A\widehat{x}(k) + Bu(k) + L_{\sigma(k)}(\overline{y}(k) - \widehat{y}(k)) \\ \widehat{y}(k) &= \theta C\widehat{x}(k) \\ u(k) &= K_{\sigma(k)}\widehat{x}(k), \end{aligned} \tag{12}$$

where $\widehat{x}(k) \in R^n$ and $\widehat{y}(k) \in R^s$ represent the state estimation and output estimation vectors of system (1), respectively. $L_{\sigma(k)} \in R^{n\times s}$ and $K_{\sigma(k)} \in R^{m\times n}$ are the observer gain and controller feedback gain to be designed, respectively.

We define the following error variable as

$$e(k) = x(k) - \widehat{x}(k). \tag{13}$$

Combining (12) with (1) and considering (9) and (11), the error system is derived as follows:

$$\begin{aligned} e(k+1) = {}& \theta L_{\sigma(k)}Cx(k) + (A - \theta L_{\sigma(k)}C)\, e(k) \\ & - \theta L_{\sigma(k)}\left(I + \Delta_q(k,\sigma(k))\right)Cx(k-\tau_k) \\ & - (\theta_k - \theta)\, L_{\sigma(k)}\left(I + \Delta_q(k,\sigma(k))\right)Cx(k-\tau_k) \\ & + f(k, x(k)) + Hw(k). \end{aligned} \tag{14}$$

On the other hand, system (1) can be rewritten as

$$\begin{aligned} x(k+1) = {}& \left(A + BK_{\sigma(k)}\right)x(k) - BK_{\sigma(k)}e(k) \\ & + f(k, x(k)) + Hw(k). \end{aligned} \tag{15}$$

By setting $\eta(k) = \left[x^T(k) \;\; e^T(k)\right]^T$, the augmented system can be obtained as follows:

$$\begin{aligned} \eta(k+1) = {}& \Phi_{1\sigma(k)}\eta(k) + \theta\Phi_{2\sigma(k)}\eta(k-\tau_k) \\ & + (\theta_k - \theta)\,\Phi_{2\sigma(k)}\eta(k-\tau_k) \\ & + DF(k, \eta(k)) + \overline{H}w(k), \\ z(k) = {}& \overline{G}\eta(k) \\ \eta(k) = {}& \phi(k); \quad k = -\overline{\tau}, -\overline{\tau}+1, \ldots, 0, \end{aligned} \tag{16}$$

where

$$\begin{aligned} \Phi_{1\sigma(k)} &= \overline{A} + \theta I_1 L_{\sigma(k)}\overline{C}_1 + \overline{B}K_{\sigma(k)}I_2, \\ \Phi_{2\sigma(k)} &= -I_1 L_{\sigma(k)}\left(I + \Delta_q(k,\sigma(k))\right)\overline{C}_2, \\ \overline{G} &= [G \;\; 0], \qquad \overline{H} = \begin{bmatrix} H \\ H \end{bmatrix}, \qquad D = \begin{bmatrix} I \\ I \end{bmatrix}, \\ F(k, \eta(k)) &= f(k, E\eta(k)), \end{aligned} \tag{17}$$

with

$$\overline{A}=\begin{bmatrix}A & 0\\ 0 & A\end{bmatrix},\qquad I_1=\begin{bmatrix}0\\ I\end{bmatrix},\qquad \overline{C}_1=\begin{bmatrix}C & -C\end{bmatrix},$$
$$\overline{B}=\begin{bmatrix}B\\ 0\end{bmatrix},\qquad I_2=\begin{bmatrix}I & -I\end{bmatrix},\tag{18}$$
$$\overline{C}_2=\begin{bmatrix}C & 0\end{bmatrix},\qquad E=\begin{bmatrix}I & 0\end{bmatrix}.$$

Lemma 2 (see [29]). *Let D, E, and F be matrices with appropriate dimensions. Suppose $F^TF\le I$, and then for any scalar $\varepsilon>0$, we have*

$$DFE+E^TF^TD^T\le \varepsilon DD^T+\varepsilon^{-1}E^TE.\tag{19}$$

Definition 3 (see [30]). System (16) with $w(k)\equiv 0$ is said to be stochastically stable if, for every finite $\eta_0=\eta(0)$, the following inequality holds:

$$E\left\{\sum_{k=0}^{\infty}\|\eta(k)\|^2 \mid \eta_0\right\}<\infty.\tag{20}$$

Definition 4 ([31]). For a given scalar $\gamma>0$, system (16) is stochastically stable and has an H_∞ noise attenuation performance index γ if the following conditions are satisfied.

(a) System (16) with $\omega(k)\equiv 0$ is stochastically stable.

(b) Under the zero-initial condition, it holds that

$$\sum_{k=0}^{\infty}E\left\{z^T(k)z(k)\right\}<\gamma^2\sum_{k=0}^{\infty}E\left\{w^T(k)w(k)\right\},\qquad \forall w(k)\in l_2[0,\infty).\tag{21}$$

In this paper, the main objective is to design controller (12) for system (1) in the presence of output quantization, sensor-controller packet losses, and time-varying delays such that system (16) is stochastically stable with a prescribed H_∞ performance index γ.

3. Main Results

Theorem 5. *For a given scalar $\gamma>0$, quantization densities $\rho(i)$ and a packet loss rate $0<\theta<1$, system (16) with partly unknown transition probabilities is stochastically stable with a prescribed H_∞ performance index γ, if there exist matrices $P_i>0$ ($i\in\mathfrak{R}$) and $Q>0$ such that*

$$\begin{bmatrix}\pi_K^i\overline{\Theta}_1^i & \overline{\Gamma}_1^iP_K^i & \overline{\Gamma}_2^iP_K^i\\ * & -P_K^i & 0\\ * & * & -P_K^i\end{bmatrix}<0,$$
$$\begin{bmatrix}\overline{\Theta}_1^i & \overline{\Gamma}_1^iP_j & \overline{\Gamma}_2^iP_j\\ * & -P_j & 0\\ * & * & -P_j\end{bmatrix}<0,\quad \forall j\in\mathfrak{R}_{UK}^i,\tag{22}$$

where

$$\overline{\Theta}_1^i=\begin{bmatrix}(\overline{\tau}-\underline{\tau}+1)Q-P_i-\overline{S}_1+\overline{G}^T\overline{G} & 0 & -\overline{S}_2 & 0\\ * & -Q & 0 & 0\\ * & * & -I & 0\\ * & * & * & -\gamma^2I\end{bmatrix},$$
$$\overline{\Gamma}_1^i=\begin{bmatrix}\Phi_{1i} & \theta\Phi_{2i} & D & \overline{H}\end{bmatrix}^T,\qquad \overline{\Gamma}_2^i=\begin{bmatrix}0 & \overline{\theta}\Phi_{2i} & 0 & 0\end{bmatrix}^T,$$
$$\pi_K^i=\sum_{j\in\mathfrak{R}_K^i}\pi_{ij},\qquad P_K^i=\sum_{j\in\mathfrak{R}_K^i}\pi_{ij}P_j.\tag{23}$$

Proof. Choose a Lyapunov-Krasovskii functional of the form:

$$V(\eta(k),k)=V_1(\eta(k),k)+V_2(\eta(k),k)+V_3(\eta(k),k),\tag{24}$$

with

$$V_1(\eta(k),k)=\eta^T(k)P_{\sigma(k)}\eta(k)$$
$$V_2(\eta(k),k)=\sum_{l=k-\tau_k}^{k-1}\eta^T(l)Q\eta(l)\tag{25}$$
$$V_3(\eta(k),k)=\sum_{h=-\overline{\tau}+1}^{-\underline{\tau}}\sum_{l=k+h}^{k-1}\eta^T(l)Q\eta(l),$$

where $P_{\sigma(k)}>0$ ($\sigma(k)\in\mathfrak{R}$) and $Q>0$ are symmetric positive definite matrices.

Let the mode at times k and $k+1$ be i and j, respectively. That is, $\sigma(k)=i$ and $\sigma(k+1)=j$ for any $i,j\in\mathfrak{R}$. Taking the difference of above functional along the solution of system (16), we have

$$\begin{aligned}E[\Delta V(\eta(k),k)]&=E[V(\eta(k+1),k+1)\mid\eta(k)]-V(\eta(k),k)\\&=\Delta V_1(\eta(k),k)+\Delta V_2(\eta(k),k)+\Delta V_3(\eta(k),k),\end{aligned}\tag{26}$$

with

$$\Delta V_1(\eta(k),k)=\eta^T(k+1)\sum_{j\in\mathfrak{R}}\pi_{ij}P_j\eta(k+1)-\eta^T(k)P_i\eta(k)$$

$$\Delta V_2(\eta(k),k)=\sum_{l=k+1-\tau_{k+1}}^{k}\eta^T(l)Q\eta(l)-\sum_{l=k-\tau_k}^{k-1}\eta^T(l)Q\eta(l)$$

$$
\begin{aligned}
&= \eta^T(k) Q\eta(k) - \eta^T(k-\tau_k) Q\eta(k-\tau_k) \\
&\quad + \sum_{l=k+1-\tau_{k+1}}^{k-1} x^T(l) Qx(l) - \sum_{l=k+1-\tau_k}^{k-1} x^T(l) Qx(l) \\
&\le \eta^T(k) Q\eta(k) - \eta^T(k-\tau_k) Q\eta(k-\tau_k) \\
&\quad + \sum_{l=k+1-\overline{\tau}}^{k-\underline{\tau}} \eta^T(l) Q\eta(l), \\
\Delta V_3(\eta(k), k) &= \sum_{h=-\overline{\tau}+1}^{-\underline{\tau}} \sum_{l=k+1+h}^{k} \eta^T(l) Q\eta(l) \\
&\quad - \sum_{h=-\overline{\tau}+1}^{-\underline{\tau}} \sum_{l=k+h}^{k-1} \eta^T(l) Q\eta(l) \\
&= (\overline{\tau} - \underline{\tau}) \eta^T(k) Q\eta(k) - \sum_{l=k+1-\overline{\tau}}^{k-\underline{\tau}} \eta^T(l) Q\eta(l).
\end{aligned} \tag{27}
$$

Therefore,

$$
\begin{aligned}
&E[\Delta V(\eta(k), k)] \\
&\quad \le \eta^T(k+1) \sum_{j\in\Re} \pi_{ij} P_j \eta(k+1) - \eta^T(k) P_i \eta(k) \\
&\qquad + (\overline{\tau} - \underline{\tau} + 1) \eta^T(k) Q\eta(k) - \eta^T(k-\tau_k) Q\eta(k-\tau_k).
\end{aligned} \tag{28}
$$

From inequality (2), we know

$$
\begin{bmatrix} \eta(k) \\ F(k, \eta(k)) \end{bmatrix}^T \begin{bmatrix} \overline{S}_1 & \overline{S}_2 \\ * & I \end{bmatrix} \begin{bmatrix} \eta(k) \\ F(k, \eta(k)) \end{bmatrix} \le 0, \tag{29}
$$

where

$$
\overline{S}_1 = E^T \frac{S_1^T S_2 + S_2^T S_1}{2} E, \qquad \overline{S}_2 = -E^T \frac{S_1^T + S_2^T}{2}. \tag{30}
$$

Denoting $\zeta(k) = [\eta^T(k), \eta^T(k-\tau_k), F^T(k, \eta(k))]^T$ and combining (28) and (29) lead to

$$
\begin{aligned}
&E[\Delta V(\eta(k), k)] \\
&\quad \le \eta^T(k+1) \sum_{j\in\Re} \pi_{ij} P_j \eta(k+1) \\
&\qquad + \zeta^T(k) \left[\sum_{j\in\Re_K^i} \pi_{ij} + \sum_{j\in\Re_{UK}^i} \pi_{ij} \right] \Theta_1^i \zeta(k) \\
&\quad = \eta^T(k+1) \left[P_K^i + \sum_{j\in\Re_{UK}^i} \pi_{ij} P_j \right] \eta(k+1) \\
&\qquad + \zeta^T(k) \left[\pi_K^i + \sum_{j\in\Re_{UK}^i} \pi_{ij} \right] \Theta_1^i \zeta(k) \\
&\quad = \zeta^T(k) \left\{ \left[\pi_K^i \Theta_1^i + \Theta_2^i \right] + \sum_{j\in\Re_{UK}^i} \pi_{ij} \left(\Theta_1^i + \Theta_3^i \right) \right\} \zeta(k),
\end{aligned} \tag{31}
$$

where

$$
\Theta_1^i = \begin{bmatrix} (\overline{\tau} - \underline{\tau} + 1) Q - P_i - \overline{S}_1 & 0 & -\overline{S}_2 \\ * & -Q & 0 \\ * & * & -I \end{bmatrix},
$$

$$
\Theta_2^i = \begin{bmatrix} \Phi_{1i}^T P_K^i \Phi_{1i} & \theta \Phi_{1i}^T P_K^i \Phi_{2i} & \Phi_{1i}^T P_K^i D \\ * & \left(\theta^2 + \overline{\theta}^2\right) \Phi_{2i}^T P_K^i \Phi_{2i} & \theta \Phi_{2i}^T P_K^i D \\ * & * & D^T P_K^i D \end{bmatrix}, \tag{32}
$$

$$
\Theta_3^i = \begin{bmatrix} \Phi_{1i}^T P_j \Phi_{1i} & \theta \Phi_{1i}^T P_j \Phi_{2i} & \Phi_{1i}^T P_j D \\ * & \left(\theta^2 + \overline{\theta}^2\right) \Phi_{2i}^T P_j \Phi_{2i} & \theta \Phi_{2i}^T P_j D \\ * & * & D^T P_j D \end{bmatrix}.
$$

By Schur complement, (22) implies $\pi_K^i \Theta_1^i + \Theta_2^i < 0$ and $\Theta_1^i + \Theta_3^i < 0$, and then we obtain from (31) that

$$
\begin{aligned}
&E[\Delta V(\zeta(k), k)] \\
&\quad \le -\lambda_{\min} \left[-\pi_K^i \Theta_1^i - \Theta_2^i \right] \zeta^T(k) \zeta(k) \\
&\qquad - \sum_{j\in\Re_{UK}^i} \pi_{ij} \min_j \left\{ \lambda_{\min} \left[-\left(\Theta_1^i + \Theta_3^i \right) \right] \right\} \zeta^T(k) \zeta(k) \\
&\quad \le -(\beta_1 + \beta_2) \zeta^T(k) \zeta(k),
\end{aligned} \tag{33}
$$

where $\beta_1 = \inf_{i\in\Re}\{-\lambda_{\min}[-\pi_K^i \Theta_1^i - \Theta_2^i]\}$ and $\beta_2 = \inf_{i\in\Re}\{-(1-\pi_K^i)\min_j\{\lambda_{\min}[-(\Theta_1^i + \Theta_3^i)]\}\}$. From (33), if we do mathematical expectations in both sides, we obtain that for any $N \ge 0$

$$
\begin{aligned}
&E[V(\zeta(N+1), N+1) - V(\zeta(0), 0)] \\
&\qquad \le -(\beta_1 + \beta_2) \sum_{k=0}^{N} E\left[\zeta^T(k) \zeta(k) \right],
\end{aligned} \tag{34}
$$

which yields

$$
\sum_{k=0}^{\infty} E\left[\zeta^T(k) \zeta(k) \right] \le \frac{1}{\beta_1 + \beta_2} E[V(\zeta(0), 0)] < \infty. \tag{35}
$$

Thus, from Definition 3, the closed-loop system (16) with $w(k) = 0$ is stochastically stable.

Now, to establish an H_∞ performance, consider the following performance index:

$$
J_N \triangleq E\left\{ \sum_{k=0}^{N} \left[z^T(k) z(k) - \gamma^2 w^T(k) w(k) \right] \right\}. \tag{36}
$$

Under zero-initial condition, we have

$$\begin{aligned} J_N &= E\sum_{k=0}^{N}\left[z^T(k)z(k)-\gamma^2 w^T(k)w(k)+\Delta V(\eta(k),k)\right] \\ &\quad - E[V(N+1)] \\ &\le E\sum_{k=0}^{\infty}\left[z^T(k)z(k)-\gamma^2 w^T(k)w(k)+\Delta V(\eta(k),k)\right] \\ &= \sum_{k=0}^{\infty}\xi^T(k)\left[\left[\pi_K^i\overline{\Theta}_1^i+\overline{\Theta}_2^i\right]+\sum_{j\in\mathfrak{R}_{UK}^i}\pi_{ij}\left(\overline{\Theta}_1^i+\overline{\Theta}_3^i\right)\right]\xi(k), \end{aligned} \tag{37}$$

where

$$\begin{aligned} \xi(k) &= \left[\zeta^T(k), w^T(k)\right]^T, \\ \overline{\Theta}_2^i &= \begin{bmatrix} \Phi_{1i}^T P_K^i \Phi_{1i} & \theta\Phi_{1i}^T P_K^i \Phi_{2i} & \Phi_{1i}^T P_K^i D & \Phi_{1i}^T P_K^i \overline{H} \\ * & \left(\theta^2+\overline{\theta}^2\right)\Phi_{2i}^T P_K^i \Phi_{2i} & \theta\Phi_{2i}^T P_K^i D & \theta\Phi_{2i}^T P_K^i \overline{H} \\ * & * & D^T P_K^i D & D^T P_K^i \overline{H} \\ * & * & * & \overline{H}^T P_K^i \overline{H} \end{bmatrix}, \\ \overline{\Theta}_3^i &= \begin{bmatrix} \Phi_{1i}^T P_j \Phi_{1i} & \theta\Phi_{1i}^T P_j \Phi_{2i} & \Phi_{1i}^T P_j D & \Phi_{1i}^T P_j \overline{H} \\ * & \left(\theta^2+\overline{\theta}^2\right)\Phi_{2i}^T P_j \Phi_{2i} & \theta\Phi_{2i}^T P_j D & \theta\Phi_{2i}^T P_j \overline{H} \\ * & * & D^T P_j D & D^T P_j \overline{H} \\ * & * & * & \overline{H}^T P_j \overline{H} \end{bmatrix}. \end{aligned} \tag{38}$$

Similarly, by Schur complement, it can be obtained from (22) that $[\pi_K^i\overline{\Theta}_1^i+\overline{\Theta}_2^i]+\sum_{j\in\mathfrak{R}_{UK}^i}\pi_{ij}(\overline{\Theta}_1^i+\overline{\Theta}_3^i) < 0$, which means that $J_N < 0$.

Then,

$$\sum_{k=0}^{\infty}E\left\{\|z(k)\|^2\right\} \le \gamma^2\sum_{k=0}^{\infty}\|w(k)\|^2. \tag{39}$$

This completes the proof. □

Remark 6. Note that $\beta_1+\beta_2$ will be reduced to only β_1 (resp., β_2) if all the transition probabilities are known (resp., unknown).

Remark 7. The considered systems are more general than these systems with completely known or completely unknown transition probabilities, which can be viewed as two special cases of the ones tackled here; that is, when all the transition probabilities are known or unknown, the underlying systems are the traditional MJSs ($\mathfrak{R}_K^i = \mathfrak{R}$, $\mathfrak{R}_{UK}^i = \varnothing$) or the switched systems under arbitrary switching ($\mathfrak{R}_K^i = \varnothing$, $\mathfrak{R}_{UK}^i = \mathfrak{R}$), respectively. Therefore, observer-based controller with partly unknown transition probabilities of this paper is a more natural assumption to the Markovian jump systems and hence covers the existing ones.

Corollary 8. *For a given scalar $\gamma > 0$, quantization densities $\rho(i)$ and a packet losses rate $0 < \theta < 1$, the closed-loop system* (16) *with completely known transition probabilities is stochastically stable with an H_∞ attenuation level if there exist matrices $P_i > 0$ ($i \in \mathfrak{R}$) and $Q > 0$ such that*

$$\begin{bmatrix} \pi_K^i\overline{\Theta}_1^i & \overline{\Gamma}_1^i P_K^i & \overline{\Gamma}_2^i P_K^i \\ * & -P_K^i & 0 \\ * & * & -P_K^i \end{bmatrix} < 0, \tag{40}$$

where $P_K^i = \sum_{j\in\mathfrak{R}_K^i}\pi_{ij}P_j$ with $\mathfrak{R}_K^i = \mathfrak{R}$ and $\mathfrak{R}_{UK}^i = \varnothing$.

Proof. Similar to the proof line of Theorem 5, the result can be obtained. The detailed proof is omitted here. □

Theorem 5 provides Markov-chain-element-dependent sufficient conditions which guarantee the H_∞ performance as well as the stochastic stability of the augmented error system (16). The following theorem gives sufficient conditions on the existence of a mode-dependent observer-based controller.

Theorem 9. *For a given scalar $\gamma > 0$, quantization densities $\rho(i)$ and a packet loss rate $0 < \theta < 1$, if there exist matrices $\widetilde{B}_i$, $\widetilde{L}_i$, R_i, $P_i > 0$ ($i \in \mathfrak{R}$), $Q > 0$ and scalar $\varepsilon_i > 0$ such that*

$$\begin{bmatrix} \overline{\overline{\Theta}}_1^i & \Sigma_1^i & \Sigma_2^i & 0 \\ * & \Upsilon_j - R_i - R_i^T & 0 & -\theta\widetilde{L}_i \\ * & * & \Upsilon_j - R_i - R_i^T & \overline{\theta}\widetilde{L}_i \\ * & * & * & -\varepsilon_i I \end{bmatrix} < 0, \tag{41}$$

where

$$\begin{aligned} \overline{\overline{\Theta}}_1^i &= \begin{bmatrix} (\overline{\tau}-\underline{\tau}+1)Q - P_i - \overline{S}_1 + \overline{G}^T\overline{G} & 0 & -\overline{S}_2 & 0 \\ * & -Q+\varepsilon_i\delta^2(i)\overline{C}_2^T\overline{C}_2 & 0 & 0 \\ * & * & -I & 0 \\ * & * & * & -\gamma^2 I \end{bmatrix}, \\ \Sigma_1^i &= \left[R_i\overline{A}+\widetilde{B}_iI_2+\theta\widetilde{L}_i\overline{C}_1 \;\; -\theta\widetilde{L}_i\overline{C}_2 \;\; R_iD \;\; R_i\overline{H}\right]^T, \\ \Sigma_2^i &= \left[0 \;\; \overline{\theta}\widetilde{L}_i\overline{C}_2 \;\; 0 \;\; 0\right]^T, \end{aligned} \tag{42}$$

then, there exists a mode-dependent observer-based controller as (12) *such that the augmented system* (16) *is stochastically stable with a prescribed H_∞ performance index γ. Moreover, if*

LMIs (41) *have a feasible solution, the desired controller gains and observer gains are given by*

$$\widetilde{B}_i = R_i\overline{B}K_i, \quad \widetilde{L}_i = R_i I_1 L_i, \quad i \in \mathfrak{R}. \tag{43}$$

Proof. From Theorem 9, inequalities (22) can be rewritten as

$$\begin{bmatrix} \overline{\Theta}_1^i & \overline{\Gamma}_1^i \Upsilon_j & \overline{\Gamma}_2^i \Upsilon_j \\ * & -\Upsilon_j & 0 \\ * & * & -\Upsilon_j \end{bmatrix} < 0, \tag{44}$$

where

$$\Upsilon_j \triangleq \begin{cases} \frac{1}{\pi_K^i} P_K^i, & \forall j \in \mathfrak{R}_K^i \\ P_j, & \forall j \in \mathfrak{R}_{UK}^i. \end{cases} \tag{45}$$

Performing a congruence transformation to (44) by $\text{diag}\{\underbrace{I,\ldots,I}_{4}, \Upsilon_j^{-1}R_i^T, \Upsilon_j^{-1}R_i^T\}$, one has

$$\begin{bmatrix} \overline{\Theta}_1^i & \overline{\Gamma}_1^i R_i^T & \overline{\Gamma}_2^i R_i^T \\ * & -R_i\Upsilon_j^{-1}R_i^T & 0 \\ * & * & -R_i\Upsilon_j^{-1}R_i^T \end{bmatrix} < 0. \tag{46}$$

For an arbitrary matrix R_i, for all $i \in \mathfrak{R}$, we have the following fact:

$$\begin{gathered} \left(\frac{1}{\pi_K^i}P_K^i - R_i\right)\left(\frac{1}{\pi_K^i}P_K^i\right)^{-1}\left(\frac{1}{\pi_K^i}P_K^i - R_i\right)^{-1} \geq 0, \\ \left(P_j - R_i\right)P_j^{-1}\left(P_j - R_i\right)^{-1} \geq 0. \end{gathered} \tag{47}$$

Then, we obtain $\Upsilon_j - R_i^T - R_i \geq -R_i\Upsilon_j^{-1}R_i^T$, so (46) holds if the following condition is satisfied:

$$\begin{bmatrix} \overline{\Theta}_1^i & \overline{\Gamma}_1^i R_i^T & \overline{\Gamma}_2^i R_i^T \\ * & \Upsilon_j - R_i - R_i^T & 0 \\ * & * & \Upsilon_j - R_i - R_i^T \end{bmatrix} < 0. \tag{48}$$

Then, using the condition (8), we have

$$\Delta_q(k,i) = \delta(i)\Delta_i, \tag{49}$$

where $\Delta_i = \text{diag}\{\Delta_{q1}(k,i)/\delta(i),\ldots,\Delta_{qs}(k,i)/\delta(i)\}$ and satisfies $\Delta_i^T\Delta_i \leq I$.

Substituting (49) into (48) and using Lemma 2, we get that (48) is equivalent to

$$\begin{aligned} &\Lambda + \Pi_{1i}\Delta_i^T\Pi_{2i} + \Pi_{2i}^T\Delta_i\Pi_{1i}^T \\ &\quad \leq \Lambda + \varepsilon_i\Pi_{1i}\Pi_{1i}^T + \varepsilon_i^{-1}\Pi_{2i}^T\Pi_{2i} < 0, \end{aligned} \tag{50}$$

where

$$\Lambda = \begin{bmatrix} (\overline{\tau} - \underline{\tau} + 1)Q - P_i - \overline{S}_1 & 0 & -\overline{S}_2 & 0 & (R_i\Phi_{1i})^T & 0 \\ * & -Q & 0 & 0 & -(\theta R_i I_1 L_i \overline{C}_2)^T & (\overline{\theta} R_i I_1 L_i \overline{C}_2)^T \\ * & * & -I & 0 & (R_i D)^T & 0 \\ * & * & * & -\gamma^2 I & (R_i \overline{H})^T & 0 \\ * & * & * & * & \Upsilon_j - R_i - R_i^T & 0 \\ * & * & * & * & * & \Upsilon_j - R_i - R_i^T \end{bmatrix},$$

$$\begin{gathered} \Pi_{1i} = \begin{bmatrix} 0 & \delta(i)\overline{C}_2 & 0 & 0 & 0 & 0 \end{bmatrix}^T, \\ \Pi_{2i} = \begin{bmatrix} 0 & 0 & 0 & 0 & -(\theta R_i I_1 L_i)^T & (\overline{\theta} R_i I_1 L_i)^T \end{bmatrix}. \end{gathered} \tag{51}$$

Applying Schur complement and substituting $\widetilde{B}_i = R_i\overline{B}K_i$ and $\widetilde{L}_i = R_i I_1 L_i$ into (50), we have that (50) is equivalent to (41). Therefore, we obtain that system (16) is stochastically stable with a prescribed H_∞ performance index γ. The proof is completed. □

4. Numerical Example

Consider the following state-space representation of a nonlinear networked system in (1) with the following parameters:

$$\begin{gathered} A = \begin{bmatrix} -0.35 & -0.1 \\ 0.1 & 0.2 \end{bmatrix}, \qquad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \qquad H = \begin{bmatrix} -0.4 \\ 0.2 \end{bmatrix}, \\ C = \begin{bmatrix} 0.1 & 0 \end{bmatrix}, \qquad G = \begin{bmatrix} 0.2 & -0.3 \end{bmatrix}, \\ f(k, x(k)) = \left[f_1^T(k, x(k)) \quad f_2^T(k, x(k))\right]^T, \text{with} \\ f_1(k, x(k)) = -\tanh(x_1) + 0.5x_1 + 0.1x_2, \\ f_2(k, x(k)) = 0.1x_1 - \tanh(x_2) + 0.5x_2. \end{gathered} \tag{52}$$

It is easy to verify that

$$S_1 = \begin{bmatrix} -0.5 & 0.1 \\ 0.1 & -0.1 \end{bmatrix}, \qquad S_2 = \begin{bmatrix} 0.5 & 0.1 \\ 0.1 & 0.5 \end{bmatrix}. \tag{53}$$

For the given system with data loss and time-varying delay, delay bounds are given as $\overline{\tau} = 3$ and $\underline{\tau} = 1$, and the packet losses parameters are selected as $\theta = 0.8$. The network load is modeled by a Markov chain which takes values from a finite set $\{1, 2, 3\}$ and the transition probability matrix is

$$\Omega = \begin{bmatrix} 0.1 & 0.6 & 0.3 \\ ? & 0.4 & ? \\ 0.5 & ? & ? \end{bmatrix}, \tag{54}$$

where ? stands for the unknown elements.

The quantization density for each mode is assumed as $\delta(1) = 0.2$, $\delta(2) = 0.3$, and $\delta(3) = 0.4$. Then the corresponding augmented system (16) can be obtained.

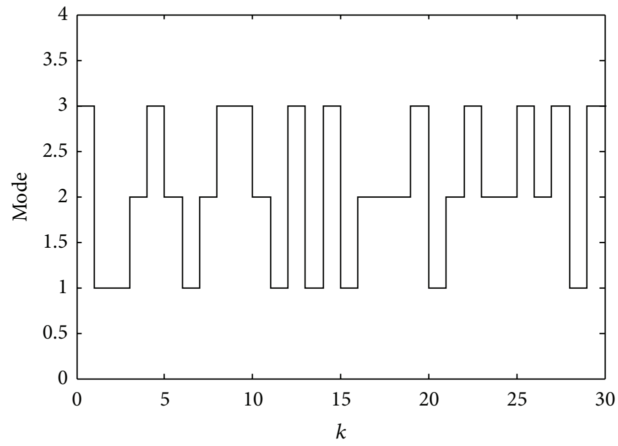

Figure 2: Markovian switching signal.

Taking $\gamma = 1.2$, then solving (41) in Theorem 9, we obtain the observer gain as

$$L_1 = \begin{bmatrix} 0.4278 \\ 0.1599 \end{bmatrix}, \qquad L_2 = \begin{bmatrix} 0.5882 \\ 0.8301 \end{bmatrix}, \qquad L_3 = \begin{bmatrix} -0.7726 \\ -0.5458 \end{bmatrix}, \tag{55}$$

and controller gains can be computed as

$$K_1 = \begin{bmatrix} 0.7088 & -0.2844 \end{bmatrix}, \qquad K_2 = \begin{bmatrix} 0.4107 & -0.2327 \end{bmatrix}, \qquad K_3 = \begin{bmatrix} 0.7023 & -0.2856 \end{bmatrix}. \tag{56}$$

Thus according to Theorem 9, we can conclude that, under the designed controller, the corresponding augmented system (16) is stochastically stable with a given H_∞ performance level $\gamma = 1.2$.

The initial condition is selected as $x(0) = \begin{bmatrix} 1 & -1 \end{bmatrix}$, $e(0) = \begin{bmatrix} 0 & 0 \end{bmatrix}$, and the noise signal $w(k) = 0.01 \exp(-0.01k) \sin(0.02\pi k)$. The switching sequence and the trajectories of plant state $x(k)$ and error $e(k)$ are shown in Figures 2, 3, 4, and 5.

It is clearly observed from the simulation results that the corresponding augmented system (16) is stochastically stable. This demonstrates the effectiveness of the proposed results.

5. Conclusions

In this work, the problem of robust observer-based H_∞ control for discrete-time nonlinear networked systems with limited information is proposed. The quantization density is designed to be a function of the network load condition which is modeled by a Markov chain with partly unknown transition probability. Bernoulli random sequences are used to model packet dropouts in the network. An estimation technique is then developed to obtain the estimation of system state and the observer-based controller is proposed to stabilize the networked controlled systems by using the Lyapunov-Krasovskii functional approach. An example is presented to illustrate the effectiveness of the proposed method.

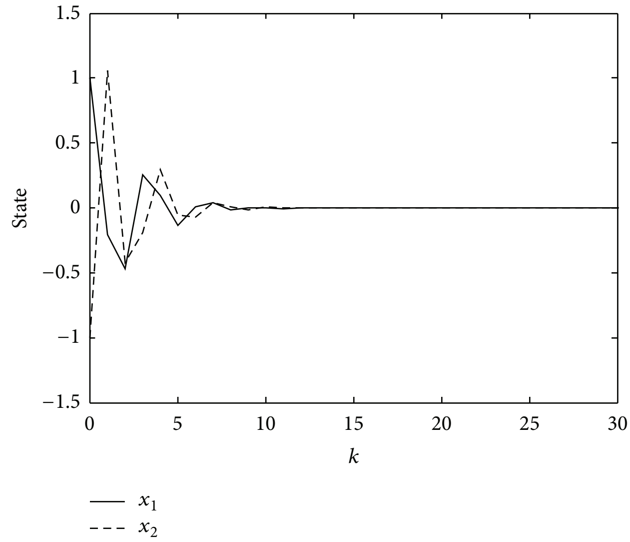

Figure 3: Trajectory of system state $x(k)$.

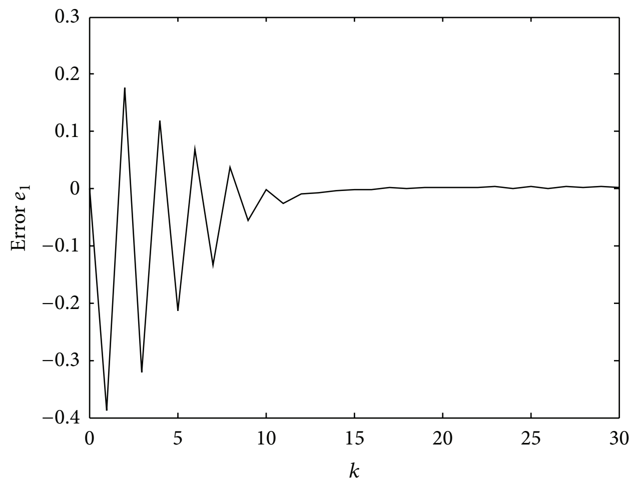

Figure 4: Trajectory of error state $e_1(k)$.

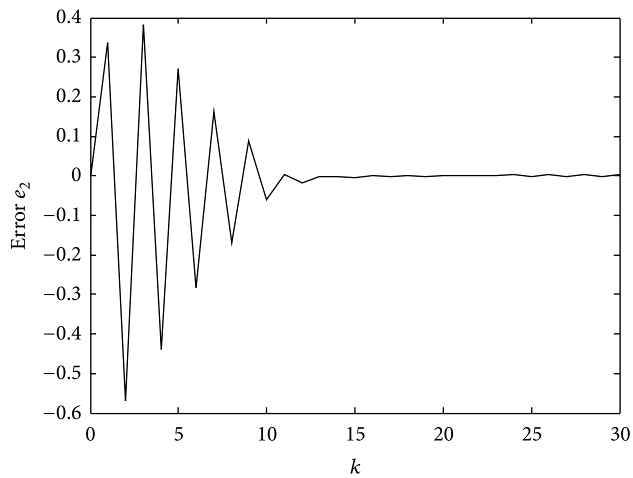

Figure 5: Trajectory of error state $e_2(k)$.

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