Research Article

Periodic Solutions for Impulsive Stochastic BAM Neural Networks with Time-Varying Delays in Leakage Terms

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By using an integral inequality, we establish some sufficient conditions for the existence and p-exponential stability of periodic solutions for a class of impulsive stochastic BAM neural networks with time-varying delays in leakage terms. Moreover, we present an example to illustrate the feasibility of our results.

1. Introduction

Since it was proposed by Kosko (see [1]), the bidirectional associative memory (BAM) neural networks have attracted considerable attentions due to their extensive applications in classification of patterns, associative memories, image processing, and other areas. In the past few years, many scholars have obtained lots of good results on the dynamical behaviors analysis of BAM neural networks. The reader may see [2–8] and the references therein.

But in a real nervous system, it is usually unavoidably affected by external perturbations which are in many cases of great uncertainty and hence may be treated as random. As pointed out by Haykin [9], in real nervous systems, synaptic transmission is a noisy process brought on by random fluctuations from the release of neurotransmitters and other probabilistic causes. And the stability of neural networks could be stabilized or destabilized by some stochastic inputs [10]. Therefore, it is significant and of prime importance to consider the dynamics of stochastic neural networks. With respect to stochastic neural networks, there are many works on the stability. For example, in [11–17], the scholars studied the stability of different classes of stochastic neural networks. For other results on stochastic neural networks, the reader may see [18–23] and the references therein.

However, the above results are mainly on the stability of considered stochastic neural networks. And it is well known that studies on neural dynamical systems not only involve a discussion of stability properties, but also involve many dynamic behaviors such as periodic oscillatory behavior. On the other hand, the neural networks are often subject to impulsive effects that in turn affect dynamical behaviors of the systems. Moreover, a leakage delay, which is the time delay in the leakage term of the systems and a considerable factor affecting dynamics for the worse in the systems, is being put to use in the problem of stability for neural networks. However, so far, very little attention has been paid to neural networks with time delay in the leakage (or "forgetting") term. Such time delays in the leakage term are difficult to handle but have great impact on the dynamical behavior of neural networks. Therefore, it is meaningful to consider neural networks with time delays in the leakage term [24–32].

But to the best of our knowledge, there are few papers published on studying the existence of periodic solutions of impulsive stochastic neural networks with time delay in the leakage term. Motivated by the previous discussions, in this paper, we consider the following impulsive stochastic BAM neural networks:

$$dx_i(t) = \left[-a_i(t)\, x_i(t - \tau_i(t)) \right.$$

$$\left. + \sum_{j=1}^{m} c_{ij}(t)\, f_j\big(y_j(t)\big) + I_i(t) \right] dt$$

$$+ \sum_{j=1}^{m} \sigma_{1ij} \left(y_j \left(t \right) \right) dw_{1j} \left(t \right),$$

$$t \geq t_0, \quad t \neq t_k,$$

$$x_i \left(t_k^+ \right) = \alpha_{ik} x_i \left(t_k \right), \quad t > 0, \quad t = t_k,$$

$$dy_j \left(t \right) = \left[-b_j \left(t \right) y_j \left(t - \sigma_j \left(t \right) \right) \right.$$

$$+ \sum_{i=1}^{n} \overline{d}_{ji} \left(t \right) g_i \left(x_i \left(t \right) \right) + J_j \left(t \right) \left] dt \right.$$

$$+ \sum_{i=1}^{n} \sigma_{2ji} \left(x_i \left(t \right) \right) dw_{2i} \left(t \right),$$

$$t \geq t_0, \quad t \neq t_k,$$

$$y_j \left(t_k^+ \right) = \gamma_{jk} y_j \left(t_k \right), \quad t > 0, \quad t = t_k,$$

$$(1)$$

where $i = 1, 2, \ldots, n$, $j = 1, 2, \ldots, m$ (n and m are the number of neurons in layers) and $x_i(t)$ and $y_j(t)$ denote the activations of the ith neuron and the jth neuron at time t; a_i and b_j represent the rate with which the ith neuron and jth neuron will reset their potential to the resting state in isolation when they are disconnected from the network and the external inputs at time t; $0 < \tau_i < \overline{\tau}_i$ and $0 < \sigma_j < \overline{\sigma}_j$ denote the leakage delays; f_j and g_i are the input-output functions (the activation functions); c_{ij} and d_{ji} are elements of feedback templates at time t; I_i and J_j denote biases of the ith neuron and the jth neuron at time t, $i = 1, 2, \ldots, n$, $j = 1, 2, \ldots, m$; $w_1(t) = (w_{11}(t), \ldots, w_{1m}(t))^T$ and $w_2(t) = (w_{21}(t), \ldots, w_{2n}(t))^T$ are n-dimensional Brownian motions defined on complete probability space $(\Omega, F, \{F_t\}_{t>0}, P)$; here, we denote by F the associated σ-algebra generated by $\{w_k(t)\}$ with the probability measure P, $k = 1, 2$; σ_{1ij} and σ_{2ji} are Borel measurable functions; $\sigma_1 = (\sigma_{1ij})_{n \times m}$ and $\sigma_2 = (\sigma_{2ji})_{m \times m}$ are diffusion coefficient matrices; $x_i(t_k^+) = \alpha_{ik} x_i(t_k)$ and $y_j(t_k^+) = \gamma_{jk} y_j(t_k)$ are impulses at moment t, which describe that the evolution processes experience abrupt change of state at the moments of time t_k, where $t_0 < t_1 < t_2 < \cdots$ and $\lim_{k \to \infty} t_k = \infty$.

Our main purpose in this paper is using an integral inequality, which is from a lemma in [33], to establish some sufficient conditions on the existence and p-exponential stability of the periodic solutions of (1).

Let $(\Omega, F, \{F_t\}_{t>0}, P)$ be a complete probability space with a filtration $\{F_t\}_{t>0}$ satisfying the usual conditions; that is, $\{F_t\}_{t>0}$ is right continuous and F_0 contains all P-null sets. Denote by $PC_{F_0}^b(R, R^{n+m})$ the family of F_0-measurable, R^n valued random variables $z(t)$, where $z(t)$ is an piecewise-continuous stochastic process; that is, $z(s)$ is continuous for all but at most countable points $t \in R$ and at these points $s \in R$, $z(s^+)$ and $z(s^-)$ exist, $z(s^-) = z(s)$. For $z \in PC_{F_0}^b(R, R^{n+m})$, define the norm $\|z\| = \max_{0 \leq t \leq \omega} (E|z(t)|_1^p)^{1/p}$, where $p > 1$

is an integer, $|z(t)|_1 = \left(\sum_{i=1}^{n+m} |z_i(t)|^2 \right)^{1/2}$; $E(\cdot)$ stands for the correspondent expectation operator with respect to the given probability measure P.

For convenience, for an ω-periodic function $f : R \to R$, denote $\overline{f} = \max_{0 \leq t \leq \omega} |f(t)|$, $\underline{f} = \min_{0 \leq t \leq \omega} |f(t)|$. The initial condition of (1) is

$$x_i(s) = \varphi_i(s), \quad y_j(s) = \psi_j(s), \quad s \leq t_0, \qquad (2)$$

where $\varphi_i, \psi_j \in PC_{F_0}^b((-\delta, t_0], R)$, $\delta = \max\{\max_{1 \leq i \leq n} \overline{\tau}_i, \max_{1 \leq j \leq m} \overline{\sigma}_j\}$, $i = 1, 2, \ldots, n$, $j = 1, 2, \ldots, m$.

Throughout this paper, we assume that the following conditions hold:

(H_1) $a_i(t) > 0$, $b_j(t) > 0$, $\tau_i(t)$, $\sigma_j(t)$, $c_{ij}(t)$, $d_{ji}(t)$, $I_i(t)$, and $J_j(t)$ are all periodic continuous functions with period ω for $t \in R$, $i = 1, 2, \ldots, n$, $j = 1, 2, \ldots, m$;

(H_2) f_j, g_i, σ_{1ij}, and σ_{2ji} are Lipschitz-continuous with Lipschitz constants $L_j^f > 0$, $L_i^g > 0$, $l_{1ij} > 0$, $l_{2ji} > 0$ and $f_j(0) = g_i(0) = \sigma_{1ij}(0) = \sigma_{2ji}(0) = 0$, $i = 1, 2, \ldots, n$, $j = 1, 2, \ldots, m$;

(H_3) $\{\alpha_{ik}\}$ and $\{\gamma_{jk}\}$ are real sequences and $\prod_{t_0 < t_k < t} \alpha_{ik}$ and $\prod_{t_0 < t_k < t} \gamma_{jk}$ are ω-periodic, $i = 1, 2, \ldots, n$, $j = 1, 2, \ldots, m$, $k = 1, 2, \ldots$.

Remark 1. From (H_3), there exist constants $\overline{\alpha}, \underline{\alpha}, \overline{\gamma}$, and $\underline{\gamma}$ such that

$$0 \leq \underline{\alpha} \leq \left| \prod_{t_0 < t_k < t} \alpha_{ik} \right| \leq \overline{\alpha},$$

$$0 \leq \underline{\gamma} \leq \left| \prod_{t_0 < t_k < t} \gamma_{jk} \right| \leq \overline{\gamma},$$

$$(3)$$

where $i = 1, 2, \ldots, n$, $j = 1, 2, \ldots, m$, and $k = 1, 2, \ldots$.

This paper is organized as follows. In Section 2, we introduce some definitions and state some preliminary results which are needed in later sections. In Section 3, we state and prove our results. In Section 4, we give an example to illustrate the feasibility of our results obtained in the previous section.

2. Preliminaries

In this section, we introduce some definitions and state some preliminary results.

Definition 2. A stochastic process $x_t(s)$ is said to be periodic with period ω if its finite dimensional distributions are periodic with period ω; that is, for any positive integer m and any moments of time $t_1, t_2, \ldots, t_m$, the joint distribution of the random variables $x_{t_1 + k\omega}(s), x_{t_2 + k\omega}(s), \ldots, x_{t_m + k\omega}(s)$ is independent of k, $k = \pm 1, \pm 2, \ldots$.

Lemma 3 (see [34]). *If $x(t)$ is an ω-periodic stochastic process, then its mathematical expectation and variance are ω-periodic.*

Definition 4. The solution $x_t(t_0, \varphi)$ of (1) is said to be

(i) *p-uniformly bounded* if for each $\alpha > 0$, $t_0 \in R$, there exists a positive constant $\theta = \theta(\alpha)$ which is independent of t_0 such that $\|\varphi\|^p \leq \alpha$ implies $E(\|x_t(t_0, \varphi)\|^p) \leq \theta$, $t \geq t_0$,

(ii) *p-point dissipative* if there exists a constant $N > 0$ such that for any point $\varphi \in PC_{F_0}^b([-\tau, 0], R^n)$, there exists $T(t_0, \varphi)$ such that for each $\alpha > 0$, $t_0 \in R$, there exists a positive constant $\theta = \theta(\alpha)$ which is independent of t_0 such that $E(\|x_t(t_0, \varphi)\|^p) \leq N$, $t \geq t_0 + T(t_0, \varphi)$.

Lemma 5 (see [35]). *Under conditions (H_1)–(H_3), assume that the solution of (1) is p-uniformly bounded and p-point dissipative for $p > 2$; then (1) has an ω-periodic solution.*

Lemma 6 (see [36]). *For any $x \in R_+^n$ and $p > 0$,*

$$|x|^p \leq n^{((p/2)-1)\vee 0} \sum_{i=1}^{n} x_i^p,$$

$$\left(\sum_{i=1}^{n} x_i\right)^p \leq n^{(p-1)\vee 0} \sum_{i=1}^{n} x_i^p. \tag{4}$$

Definition 7. The periodic solution $x(t, t_0, \varphi)$ with initial value $\varphi \in PC_{F_0}^b([-\tau, 0], R^n)$ of (1) is said to be *p-exponential stable* if there are constants $\lambda > 0$ and $M > 1$ such that any solution $y(t, t_0, \varphi_1)$ with initial value $\varphi_1 \in PC_{F_0}^b([-\tau, 0], R^n)$ of (1) satisfies

$$E\left(|x - y|_1^p\right) \leq M\|\varphi - \varphi_1\|^p e^{-\lambda(t-t_0)}, \quad t \geq t_0. \tag{5}$$

Lemma 8 (see [33]). *Let $u(t) \in C(R, R_+^n)$ be a solution of the delay integral inequality*

$$u(t) \leq M_1 e^{-\delta(t-t_0)} [\varphi]_\tau^+$$

$$+ \int_{t_0}^{t} e^{-C_1(t-s)} A_1 u(s)\, ds$$

$$+ \int_{t_0}^{t} e^{-C_1(t-s)} B_1 [u(s)]_\tau^+ ds + J_1, \tag{6}$$

$$t \geq t_0,$$

$$u(t) \leq \varphi(t), \quad \forall t \in [t_0 - \tau, t_0],$$

where A_1, B_1, C_1, and $M_1 \in R_+^{n \times n}$, $J_1 \geq 0$ is a constant vector, and $\varphi(t) \in C([t_0 - \tau, t_0], R_+^n)$. If $\rho(\Pi = C_1^{-1}(A_1 + B_1)) < 1$, then there are constants $0 < \lambda \leq \delta$ and $N \geq 1$ such that

$$u(t) \leq Nze^{-\lambda(t-t_0)} + (I - \Pi)^{-1} J_1, \quad t \geq t_0, \tag{7}$$

where z satisfies $[\varphi]_\tau^+ \leq z$.

Lemma 9 (see [33]). *Assume that all conditions of Lemma 9 hold. If $J_1 = 0$, then all solutions of inequality of (6) exponentially converge to zero.*

By Lemmas 8 and 9, we have the following corollary.

Corollary 10. *Let $u(t) \in C(R, R_+)$ be a solution of the delay integral inequality*

$$u(t) \leq M_1 e^{-\delta(t-t_0)} [\varphi]_\tau^+$$

$$+ \int_{t_0}^{t} e^{-C_1(t-s)} A_1 u(s)\, ds$$

$$+ \int_{t_0}^{t} e^{-C_1(t-s)} B_1 [u(s)]_\tau^+ ds + J_1, \tag{8}$$

$$t \geq t_0,$$

$$u(t) \leq \varphi(t), \quad \forall t \in [t_0 - \tau, t_0],$$

where A_1, B_1, C_1, and $M_1 \in R_+$, $J_1 \geq 0$ is a constant, and $\varphi(t) \in C([t_0 - \tau, t_0], R_+)$. If $(A_1 + B_1)/C_1 < 1$, then there are constants $0 < \lambda \leq \delta$ and $N \geq 1$ such that

$$u(t) \leq Nze^{-\lambda(t-t_0)} + \left(1 - \frac{A_1 + B_1}{C_1}\right)^{-1} J_1, \tag{9}$$

$$t \geq t_0,$$

where z satisfies $[\varphi]_\tau^+ \leq z$. Moreover, if $J_1 = 0$, then all solutions of inequality of (8) exponentially converge to zero.

Under our assumptions, we consider the following system:

$$du_i(t) = \left[-a_i(t) u_i(t - \tau_i(t)) \right.$$

$$+ \prod_{t_0 < t_k < t} \alpha_{ik}^{-1} \sum_{j=1}^{m} c_{ij}(t) f_j \left(\prod_{t_0 < t_k < t} \gamma_{jk} v_j(t) \right)$$

$$\left. + \prod_{t_0 < t_k < t} \alpha_{ik}^{-1} I_i(t) \right] dt$$

$$+ \prod_{t_0 < t_k < t} \alpha_{ik}^{-1} \sum_{j=1}^{m} \sigma_{1ij} \left(\prod_{t_0 < t_k < t} \gamma_{jk} v_j(t) \right) dw_{1j}(t),$$

$$t \geq t_0,$$

$$dv_j(t) = \Bigg[-b_j(t)\,v_j\big(t - \sigma_j(t)\big)$$

$$+ \prod_{t_0 < t_k < t} \gamma_{jk}^{-1} \sum_{i=1}^{n} d_{ji}(t)\, g_i \Bigg(\prod_{t_0 < t_k < t} \alpha_{ik} u_i(t) \Bigg)$$

$$+ \prod_{t_0 < t_k < t} \gamma_{jk}^{-1} J_j(t) \Bigg] dt$$

$$+ \prod_{t_0 < t_k < t} \gamma_{jk}^{-1} \sum_{i=1}^{n} \sigma_{2ji} \Bigg(\prod_{t_0 < t_k < t} \alpha_{ik} u_i(t) \Bigg) dw_{2i}(t),$$

$$t \geq t_0,$$

$$(10)$$

where $i = 1, 2, \ldots, n$, $j = 1, 2, \ldots, m$.

Similar to Lemma 2.1 in paper [33], we have the following lemma.

Lemma 11. *Let* (H_1)–(H_3) *hold. Then,*

(i) *if* $(u_1(t), \ldots, u_n(t), v_1(t), \ldots, v_m(t))^T$ *is a solution of* (10), *then* $(\prod_{t_0 < t_k < t} \alpha_{1k} u_1(t), \ldots, \prod_{t_0 < t_k < t} \alpha_{nk} u_n(t),$ $\prod_{t_0 < t_k < t} \gamma_{1k} v_1(t), \ldots, \prod_{t_0 < t_k < t} \gamma_{mk} v_m(t))^T$ *is a solution of* (1);

(ii) *if* $(x_1(t), \ldots, x_n(t), y_1(t), \ldots, y_m(t))^T$ *is a solution of* (1), *then* $(\prod_{t_0 < t_k < t} \alpha_{ik}^{-1} x_1(t), \ldots, \prod_{t_0 < t_k < t} \alpha_{ik}^{-1} x_n(t),$ $\prod_{t_0 < t_k < t} \gamma_{jk}^{-1} y_1(t), \ldots, \prod_{t_0 < t_k < t} \gamma_{mk}^{-1} y_m(t))^T$ *is a solution of* (10).

3. Main Results

In this section, we will state and prove the sufficient conditions for the existence and p-exponential stability of periodic solution of (1).

Theorem 12. *Let* (H_1)–(H_3) *hold. Suppose further that*

(H_4) *there exists an integer* $p > 2$ *such that* $(\epsilon + c)/\theta < 1$, *where*

$$\theta = \min \left\{ \min_{1 \leq i \leq n} \underline{a}_i, \min_{1 \leq j \leq m} \underline{b}_j \right\},$$

$$c = 6^{p-1} \max \left\{ \max_{1 \leq i \leq n} \overline{a}_i^p, \max_{1 \leq j \leq m} \overline{b}_j^p \right\},$$

$$\epsilon = \max \{ c, \epsilon_1, \epsilon_2 \},$$

$$\epsilon_1 = 6^{p-1} \sum_{i=1}^{n} \Bigg[\underline{a}_i^{1-p} \Bigg(\frac{\overline{\gamma}}{\underline{\alpha}} \sum_{j=1}^{m} \big(\overline{c}_{ij} L_j^f \big)^{p/(p-1)} \Bigg)^{p-1}$$

$$+ l_p n^{(p/2)-1} \left(\frac{2\underline{a}_i(p-1)}{p-2} \right)^{1-(p/2)}$$

$$\times \Bigg(\frac{\overline{\gamma}}{\underline{\alpha}} \sum_{j=1}^{m} \big(l_{1ij} \big)^{p/(p-1)} \Bigg)^{p-1} \Bigg],$$

$$\epsilon_2 = 6^{p-1} \sum_{j=1}^{m} \Bigg[\underline{b}_j^{1-p} \Bigg(\frac{\overline{\alpha}}{\underline{\gamma}} \sum_{i=1}^{n} \big(\overline{d}_{ji} L_i^g \big)^{p/(p-1)} \Bigg)^{p-1}$$

$$+ l_p m^{(p/2)-1} \left(\frac{2\underline{b}_j(p-1)}{p-2} \right)^{1-(p/2)}$$

$$\times \Bigg(\frac{\overline{\alpha}}{\underline{\gamma}} \sum_{i=1}^{n} \big(l_{2ji} \big)^{p/(p-1)} \Bigg)^{p-1} \Bigg].$$

$$(11)$$

Then (1) *has an* ω-*periodic solution, which is* p-*exponentially stable.*

Proof. We can rewrite (10) as follows:

$$du_i(t) = \Bigg[-a_i(t)\,u_i(t) + a_i(t)\,u_i(t)$$

$$- a_i(t)\,u_i\big(t - \tau_i(t)\big) + \prod_{t_0 < t_k < t} \alpha_{ik}^{-1} \sum_{j=1}^{m} c_{ij}(t)\, f_j$$

$$\times \Bigg(\prod_{t_0 < t_k < t} \gamma_{jk} v_j(t) \Bigg) + \prod_{t_0 < t_k < t} \alpha_{ik}^{-1} I_i(t) \Bigg] dt$$

$$+ \prod_{t_0 < t_k < t} \alpha_{ik}^{-1} \sum_{j=1}^{m} \sigma_{1ij} \Bigg(\prod_{t_0 < t_k < t} \gamma_{jk} v_j(t) \Bigg) dw_{1j}(t),$$

$$t \geq t_0, \quad i = 1, 2, \ldots, n,$$

$$dv_j(t) = \Bigg[-b_j(t)\,v_j(t) + b_j(t)\,v_j(t)$$

$$- b_j(t)\,v_j\big(t - \sigma_j(t)\big) + \prod_{t_0 < t_k < t} \gamma_{jk}^{-1} \sum_{i=1}^{n} d_{ji}(t)\, g_i$$

$$\times \Bigg(\prod_{t_0 < t_k < t} \alpha_{ik} u_i(t) \Bigg) + \prod_{t_0 < t_k < t} \gamma_{jk}^{-1} J_j(t) \Bigg] dt$$

$$+ \prod_{t_0 < t_k < t} \gamma_{jk}^{-1} \sum_{i=1}^{n} \sigma_{2ji} \left(\prod_{t_0 < t_k < t} \alpha_{ik} u_i(t) \right) dw_{2i}(t),$$

$$t \geq t_0, \quad j = 1, 2, \ldots, m. \tag{12}$$

By the method of variation parameter, for $t \geq t_0$, $i = 1, 2, \ldots, n$, from the first equation of (12), we have the following:

$$u_i(t) = u_i(t_0) e^{-\int_{t_0}^{t} a_i(\vartheta) d\vartheta}$$

$$+ \int_{t_0}^{t} e^{-\int_{s}^{t} a_i(\vartheta) d\vartheta}$$

$$\times \left[a_i(s) u_i(s) - a_i(s) u_i(s - \tau_i(s)) \right.$$

$$+ \prod_{t_0 < t_k < t} \alpha_{ik}^{-1} \sum_{j=1}^{m} c_{ij}(s) f_j$$

$$\times \left(\prod_{t_0 < t_k < t} \gamma_{jk} v_j(s) \right) + \prod_{t_0 < t_k < t} \alpha_{ik}^{-1} I_i(s) \right] ds$$

$$+ \int_{t_0}^{t} e^{-\int_{s}^{t} a_i(\vartheta) d\vartheta} \prod_{t_0 < t_k < t} \alpha_{ik}^{-1} \sum_{j=1}^{m} \sigma_{1ij}$$

$$\times \left(\prod_{t_0 < t_k < t} \gamma_{jk} v_j(s) \right) dw_{1j}(s). \tag{13}$$

For $i = 1, 2, \ldots, n$, denote

$$F_{1i} = u_i(t_0) e^{-\int_{t_0}^{t} a_i(\vartheta) d\vartheta},$$

$$F_{2i} = \int_{t_0}^{t} e^{-\int_{s}^{t} a_i(\vartheta) d\vartheta} a_i(s) u_i(s) ds,$$

$$F_{3i} = \int_{t_0}^{t} e^{-\int_{s}^{t} a_i(\vartheta) d\vartheta} a_i(s) u_i(s - \tau_i(s)) ds,$$

$$F_{4i} = \int_{t_0}^{t} e^{-\int_{s}^{t} a_i(\vartheta) d\vartheta} \prod_{t_0 < t_k < t} \alpha_{ik}^{-1} I_i(s) ds,$$

$$F_{5i} = \int_{t_0}^{t} e^{-\int_{s}^{t} a_i(\vartheta) d\vartheta} \prod_{t_0 < t_k < t} \alpha_{ik}^{-1}$$

$$\times \sum_{j=1}^{m} c_{ij}(s) f_j \left(\prod_{t_0 < t_k < t} \gamma_{jk} v_j(s) \right) ds,$$

$$F_{6i} = \int_{t_0}^{t} e^{-\int_{s}^{t} a_i(\vartheta) d\vartheta} \prod_{t_0 < t_k < t} \alpha_{ik}^{-1}$$

$$\times \sum_{j=1}^{m} \sigma_{1ij} \left(\prod_{t_0 < t_k < t} \gamma_{jk} v_j(s) \right) dw_{1j}(s). \tag{14}$$

Considering expectations, using Lemma 6, for $i = 1, 2, \ldots, n$, we have

$$E|u_i(t)|^p \leq 6^{p-1} E \left(|F_{1i}|^p + |F_{2i}|^p + |F_{3i}|^p \right.$$

$$\left. + |F_{4i}|^p + |F_{5i}|^p + |F_{6i}|^p \right). \tag{15}$$

For $i = 1, 2, \ldots, n$, we evaluate the first term of (15) as follows:

$$E|F_{1i}|^p = E \left| u_i(t_0) e^{-\int_{t_0}^{t} a_i(\vartheta) d\vartheta} \right|^p$$

$$\leq E \left| u_i(t_0) e^{-\underline{a_i}(t - t_0)} \right|^p \tag{16}$$

$$\leq e^{-p\underline{a_i}(t - t_0)} E|u_i(t_0)|^p.$$

For the second term of (15), we have

$$E|F_{2i}|^p = E \left| \int_{t_0}^{t} e^{-\int_{s}^{t} a_i(\vartheta) d\vartheta} a_i(s) u_i(s) ds \right|^p$$

$$\leq E \left(\int_{t_0}^{t} e^{-\int_{s}^{t} a_i(\vartheta) d\vartheta} |a_i(s)| |u_i(s)| ds \right)^p$$

$$\leq E \left(\int_{t_0}^{t} \overline{a}_i e^{-\underline{a_i}(t-s)} |u_i(s)| ds \right)^p \tag{17}$$

$$= \overline{a}_i^p \int_{t_0}^{t} e^{-p\underline{a_i}(t-s)} E|u_i(s)|^p ds,$$

$$i = 1, 2, \ldots, n.$$

For the third term of (15), we have

$$E|F_{3i}|^p = E \left| \int_{t_0}^{t} e^{-\int_{s}^{t} a_i(\vartheta) d\vartheta} a_i(s) u_i(s - \tau_i(s)) ds \right|^p$$

$$\leq E \left(\int_{t_0}^{t} e^{-\int_{s}^{t} a_i(\vartheta) d\vartheta} |a_i(s)| |u_i(s - \tau_i(s))| ds \right)^p$$

$$\leq E \left(\int_{t_0}^{t} \overline{a}_i e^{-\underline{a_i}(t-s)} |u_i(s - \tau_i(s))| ds \right)^p$$

$$= \overline{a}_i^p \int_{t_0}^t e^{-p\underline{a}_i(t-s)} E |u_i(s - \tau_i(s))|^p ds,$$

$$i = 1, 2, \ldots, n. \tag{18}$$

As for the fourth term of (15), for $i = 1, 2, \ldots, n$, we have

$$E|F_{4i}|^p = E \left| \int_{t_0}^t e^{-\int_s^t a_i(\vartheta)d\vartheta} \prod_{t_0 < t_k < t} \alpha_{ik}^{-1} I_i(s) ds \right|^p$$

$$\leq E \left| \int_{t_0}^t e^{-\underline{a}_i(t-s)} \prod_{t_0 < t_k < t} \alpha_{ik}^{-1} I_i(s) ds \right|^p \tag{19}$$

$$\leq \left(\frac{\overline{I}_i}{\underline{a}_i \underline{\alpha}} \right)^p.$$

For the fifth term of (15), by Hölder inequality, we have

$$E|F_{5i}|^p = E \left| \int_{t_0}^t e^{-\int_s^t a_i(\vartheta)d\vartheta} \prod_{t_0 < t_k < t} \alpha_{ik}^{-1} \right.$$

$$\left. \times \sum_{j=1}^m c_{ij}(s) f_j \left(\prod_{t_0 < t_k < t} \gamma_{jk} v_j(s) \right) ds \right|^p$$

$$\leq E \left(\int_{t_0}^t e^{-\underline{a}_i(t-s)} \left| \prod_{t_0 < t_k < t} \alpha_{ik}^{-1} \right| \sum_{j=1}^m |c_{ij}(s)| \right.$$

$$\left. \times \left| f_j \left(\prod_{t_0 < t_k < t} \gamma_{jk} v_j(s) \right) \right| ds \right)^p$$

$$\leq E \left(\int_{t_0}^t e^{-\underline{a}_i(t-s)} \right.$$

$$\times \left| \prod_{t_0 < t_k < t} \alpha_{ik}^{-1} \right| \sum_{j=1}^m |c_{ij}(s)| L_j^f$$

$$\left. \times \left| \prod_{t_0 < t_k < t} \gamma_{jk} v_j(s) \right| ds \right)^p$$

$$\leq E \left(\int_{t_0}^t e^{-\underline{a}_i(t-s)} \frac{\overline{\gamma}}{\underline{\alpha}} \sum_{j=1}^m |c_{ij}(s)| \right.$$

$$\left. \times L_j^f |v_j(s)| ds \right)^p$$

$$= E \left(\int_{t_0}^t \left(e^{-\underline{a}_i(t-s)} \right)^{(p-1)/p} \left(e^{-\underline{a}_i(t-s)} \right)^{1/p} \right.$$

$$\left. \times \frac{\overline{\gamma}}{\underline{\alpha}} \sum_{j=1}^m |c_{ij}(s)| L_j^f |v_j(s)| ds \right)^p$$

$$\leq E \left(\left(\int_{t_0}^t e^{-\underline{a}_i(t-s)} ds \right)^{p-1} \right.$$

$$\left. \times \int_{t_0}^t e^{-\underline{a}_i(t-s)} \left(\frac{\overline{\gamma}}{\underline{\alpha}} \sum_{j=1}^m \overline{c}_{ij} L_j^f |v_j(s)| \right)^p ds \right)$$

$$\leq \underline{a}_i^{1-p} \int_{t_0}^t e^{-\underline{a}_i(t-s)} \left(\frac{\overline{\gamma}}{\underline{\alpha}} \sum_{j=1}^m (\overline{c}_{ij} L_j^f)^{p/(p-1)} \right)^{p-1}$$

$$\times \sum_{j=1}^m E |v_j(s)|^p ds, \quad i = 1, 2, \ldots, n. \tag{20}$$

As for the last term of (15), using Proposition 1.9 in [37] and Hölder inequality, for $i = 1, 2, \ldots, n$, we have

$$E|F_{6i}|^p = E \left| \int_{t_0}^t e^{-\int_s^t a_i(\vartheta)d\vartheta} \prod_{t_0 < t_k < t} \alpha_{ik}^{-1} \right.$$

$$\left. \times \sum_{j=1}^m \sigma_{1ij} \left(\prod_{t_0 < t_k < t} \gamma_{jk} v_j(s) \right) dw_{1j}(s) \right|^p$$

$$\leq l_p \left[\int_{t_0}^t \left(e^{-p\underline{a}_i(t-s)} E \left| \prod_{t_0 < t_k < t} \alpha_{ik}^{-2} \sum_{j=1}^n \sigma_{1ij}^2 \right. \right. \right.$$

$$\left. \left. \left. \times \left(\prod_{t_0 < t_k < t} \gamma_{jk} v_j(s) \right) \right|^{p/2} \right)^{2/p} ds \right]^{p/2}$$

$$\leq l_p n^{(p/2)-1} \left[\int_{t_0}^t \left(e^{-p\underline{a}_i(t-s)} \right. \right.$$

$$\left. \left. \times E \left(\frac{\overline{\gamma}}{\underline{\alpha}} \sum_{j=1}^n l_{1ij} |v_j(s)| \right)^p \right)^{2/p} ds \right]^{p/2}$$

$$= l_p n^{(p/2)-1} \left[\int_{t_0}^t \left(e^{-(p-1)\underline{a}_i(t-s)} e^{-\underline{a}_i(t-s)} \right. \right.$$

$$\left. \left. \times E \left(\frac{\overline{\gamma}}{\underline{\alpha}} \sum_{j=1}^n l_{1ij} |v_j(s)| \right)^p \right)^{2/p} ds \right]^{p/2}$$

$$\leq l_p n^{(p/2)-1} \left(\int_{t_0}^t e^{(-2\underline{a}_i(p-1)/(p-2))(t-s)} ds \right)^{(p/2)-1}$$

$$\times \left(\int_{t_0}^t e^{-\underline{a}_i(t-s)} E\left(\frac{\overline{\gamma}}{\underline{\alpha}} \sum_{j=1}^n l_{1ij} \left| v_j(s) \right| \right)^p ds \right)$$

$$\leq l_p n^{(p/2)-1} \left(\frac{2\underline{a}_i(p-1)}{p-2} \right)^{1-(p/2)}$$

$$\times \left(\int_{t_0}^t e^{-\underline{a}_i(t-s)} \right.$$

$$\left. \times E\left(\frac{\overline{\gamma}}{\underline{\alpha}} \sum_{j=1}^m l_{1ij} \left| v_j(s) \right| \right)^p ds \right)$$

$$\leq l_p n^{(p/2)-1} \left(\frac{2\underline{a}_i(p-1)}{p-2} \right)^{1-(p/2)}$$

$$\times \left(\int_{t_0}^t e^{-\underline{a}_i(t-s)} \left(\frac{\overline{\gamma}}{\underline{\alpha}} \sum_{j=1}^m \left(l_{1ij} \right)^{p/(p-1)} \right)^{p-1} \right.$$

$$\left. \times \sum_{j=1}^m E\left| v_j(s) \right|^p ds \right),$$

(21)

where $l_p = (p(p-1)/2)^{p/2}$. Therefore, for $i = 1, 2, \ldots, n$, we have

$$E\left| u_i(t) \right|^p$$

$$\leq 6^{p-1} \left\{ e^{-p\underline{a}_i(t-t_0)} E\left| u_i(t_0) \right|^p \right.$$

$$+ \overline{a}_i^p \int_{t_0}^t e^{-p\underline{a}_i(t-s)} E\left| u_i(s) \right|^p ds$$

$$+ \overline{a}_i^p \int_{t_0}^t e^{-p\underline{a}_i(t-s)} E\left| u_i(s - \tau_i(s)) \right|^p ds$$

$$+ \underline{a}_i^{1-p} \int_{t_0}^t e^{-\underline{a}_i(t-s)}$$

$$\times \left(\frac{\overline{\gamma}}{\underline{\alpha}} \sum_{j=1}^m \left(\overline{c}_{ij} L_j^f \right)^{p/(p-1)} \right)^{p-1}$$

$$\times \sum_{j=1}^n E\left| v_j(s) \right|^p ds$$

$$+ l_p n^{(p/2)-1} \left(\frac{2\underline{a}_i(p-1)}{p-2} \right)^{1-(p/2)}$$

$$\times \left(\int_{t_0}^t e^{-\underline{a}_i(t-s)} \left(\frac{\overline{\gamma}}{\underline{\alpha}} \sum_{j=1}^m \left(l_{1ij} \right)^{p/(p-1)} \right)^{p-1} \right.$$

$$\left. \times \sum_{j=1}^m E\left| v_j(s) \right|^p ds \right) + \left(\frac{\overline{I}_i}{\underline{a}_i \underline{\alpha}} \right)^p \right\}$$

$$= 6^{p-1} \left\{ e^{-p\underline{a}_i(t-t_0)} E\left| u_i(t_0) \right|^p \right.$$

$$+ \overline{a}_i^p \int_{t_0}^t e^{-p\underline{a}_i(t-s)} E\left| u_i(s) \right|^p ds$$

$$+ \overline{a}_i^p \int_{t_0}^t e^{-p\underline{a}_i(t-s)} E\left| u_i(s - \tau_i(s)) \right|^p ds$$

$$+ \int_{t_0}^t e^{-\underline{a}_i(t-s)}$$

$$\times \left[\underline{a}_i^{1-p} \left(\frac{\overline{\gamma}}{\underline{\alpha}} \sum_{j=1}^m \left(\overline{c}_{ij} L_j^f \right)^{p/(p-1)} \right)^{p-1} \right.$$

$$+ l_p n^{(p/2)-1} \left(\frac{2\underline{a}_i(p-1)}{p-2} \right)^{1-(p/2)}$$

$$\left. \times \left(\frac{\overline{\gamma}}{\underline{\alpha}} \sum_{j=1}^m \left(l_{1ij} \right)^{p/(p-1)} \right)^{p-1} \right]$$

$$\times \sum_{j=1}^m E\left| v_j(s) \right|^p ds + \left(\frac{\overline{I}_i}{\underline{a}_i \underline{\alpha}} \right)^p \right\}$$

$$\leq 6^{p-1} \left\{ e^{-\underline{a}_i(t-t_0)} E\left| u_i(t_0) \right|^p \right.$$

$$+ \overline{a}_i^p \int_{t_0}^t e^{-\underline{a}_i(t-s)} E\left| u_i(s) \right|^p ds$$

$$+ \overline{a}_i^p \int_{t_0}^t e^{-\underline{a}_i(t-s)} E\left| u_i(s - \tau_i(s)) \right|^p ds$$

$$+ \int_{t_0}^t e^{-\underline{a}_i(t-s)}$$

$$\times \left[\underline{a}_i^{1-p} \left(\frac{\overline{\gamma}}{\underline{\alpha}} \sum_{j=1}^m \left(\overline{c}_{ij} L_j^f \right)^{p/(p-1)} \right)^{p-1} \right.$$

$$+ l_p n^{(p/2)-1} \left(\frac{2\underline{a}_i(p-1)}{p-2} \right)^{1-(p/2)}$$

$$\times \left(\frac{\overline{\gamma}}{\underline{\alpha}} \sum_{j=1}^{m} (l_{1ij})^{p/(p-1)} \right)^{p-1} \right]$$

$$\times \sum_{j=1}^{m} E|v_j(s)|^p ds + \left(\frac{\overline{I}_i}{\underline{a}_i \underline{\alpha}} \right)^p \right\}. \tag{22}$$

Similarly, for $t \geq t_0$, $j = 1, 2, \ldots, m$, from the second equation of (12), we can obtain the following:

$$E|v_j(t)|^p \leq 6^{p-1} \left\{ e^{-\underline{b}_j(t-t_0)} E|v_j(t_0)|^p \right.$$

$$+ \overline{b}_j^p \int_{t_0}^{t} e^{-\underline{b}_j(t-s)} E|v_j(s)|^p ds$$

$$+ \overline{b}_j^p \int_{t_0}^{t} e^{-\underline{b}_j(t-s)} E|v_j(s-\sigma_j(s))|^p ds$$

$$+ \int_{t_0}^{t} e^{-\underline{b}_j(t-s)}$$

$$\times \left[\underline{b}_j^{1-p} \left(\frac{\overline{\alpha}}{\underline{\gamma}} \sum_{i=1}^{n} (\overline{d}_{ji} L_i^g)^{p/(p-1)} \right)^{p-1} \right.$$

$$+ l_p m^{(p/2)-1} \left(\frac{2\underline{b}_j(p-1)}{p-2} \right)^{1-(p/2)}$$

$$\times \left(\frac{\overline{\alpha}}{\underline{\gamma}} \sum_{i=1}^{n} (l_{2ji})^{p/(p-1)} \right)^{p-1} \right]$$

$$\times \sum_{i=1}^{n} E|u_i(s)|^p ds + \left(\frac{\overline{J}_j}{\underline{b}_j \underline{\gamma}} \right)^p \right\}. \tag{23}$$

Hence, by (22) and (23), we have that

$$\sum_{i=1}^{n} E|u_i(t)|^p + \sum_{j=1}^{m} E|v_j(t)|^p$$

$$\leq 6^{p-1} \sum_{i=1}^{n} \left\{ e^{-\underline{a}_i(t-t_0)} E|u_i(t_0)|^p \right.$$

$$+ \overline{a}_i^p \int_{t_0}^{t} e^{-\underline{a}_i(t-s)} E|u_i(s)|^p ds$$

$$+ \overline{a}_i^p \int_{t_0}^{t} e^{-\underline{a}_i(t-s)} E|u_i(s-\tau_i(s))|^p ds$$

$$+ \int_{t_0}^{t} e^{-\underline{a}_i(t-s)}$$

$$\times \left[\underline{a}_i^{1-p} \left(\frac{\overline{\gamma}}{\underline{\alpha}} \sum_{j=1}^{m} (\overline{c}_{ij} L_j^f)^{p/(p-1)} \right)^{p-1} \right.$$

$$+ l_p n^{(p/2)-1} \left(\frac{2\underline{a}_i(p-1)}{p-2} \right)^{1-(p/2)}$$

$$\times \left(\frac{\overline{\gamma}}{\underline{\alpha}} \sum_{j=1}^{m} (l_{1ij})^{p/(p-1)} \right)^{p-1} \right]$$

$$\times \sum_{j=1}^{m} E|v_j(s)|^p ds + \left(\frac{\overline{I}_i}{\underline{a}_i \underline{\alpha}} \right)^p \right\}$$

$$+ 6^{p-1} \sum_{j=1}^{m} \left\{ e^{-\underline{b}_j(t-t_0)} E|v_j(t_0)|^p \right.$$

$$+ \overline{b}_j^p \int_{t_0}^{t} e^{-\underline{b}_j(t-s)} E|v_j(s)|^p ds$$

$$+ \overline{b}_j^p \int_{t_0}^{t} e^{-\underline{b}_j(t-s)} E|v_j(s-\sigma_j(s))|^p ds$$

$$+ \int_{t_0}^{t} e^{-\underline{b}_j(t-s)}$$

$$\times \left[\underline{b}_j^{1-p} \left(\frac{\overline{\alpha}}{\underline{\gamma}} \sum_{i=1}^{n} (\overline{d}_{ji} L_i^g)^{p/(p-1)} \right)^{p-1} \right.$$

$$+ l_p m^{(p/2)-1} \left(\frac{2\underline{b}_j(p-1)}{p-2} \right)^{1-(p/2)}$$

$$\times \left(\frac{\overline{\alpha}}{\underline{\gamma}} \sum_{i=1}^{n} (l_{2ji})^{p/(p-1)} \right)^{p-1} \right]$$

$$\times \sum_{i=1}^{n} E|u_i(s)|^p ds + \left(\frac{\overline{J}_j}{\underline{b}_j \underline{\gamma}} \right)^p \right\}. \tag{24}$$

Set $V(t) = \sum_{i=1}^{n} E|u_i(t)|^p + \sum_{j=1}^{m} E|v_j(t)|^p$. By (24), we have that

$$V(t) \leq 6^{p-1} e^{-\theta(t-t_0)} V(t_0) + \int_{t_0}^{t} \epsilon e^{-\theta(t-s)} V(s) ds$$

$$+ \int_{t_0}^{t} c e^{-\theta(t-s)} V_\delta^+(s) ds + H, \tag{25}$$

where $V_\delta^+(s) = \max_{-\delta \le s \le 0}|V(t + s)|$, $H = 6^{p-1}$ $\max\{\{\max_{1 \le i \le n}(\overline{I}_i/\underline{a}_i\underline{\alpha})^p\}, \{\max_{1 \le j \le m}(\overline{J}_j/\underline{b}_j\underline{\gamma})^p\}\}$. By (H_4) and Corollary 10, the solutions of (10) are p-uniformly bounded and it also shows that the family of all solutions of (10) is p-point dissipative. Then, it follows from Lemma 5 that (10) has an ω-periodic solution $(u_1^*(t), \ldots, u_n^*(t), v_1^*(t), \ldots, v_m^*(t))^T$.

Suppose that $(u_1(t), \ldots, u_n(t), v_1(t), \ldots, v_m(t))^T$ is an arbitrary solution of (10). Then it follows from (10) that

$$
\begin{aligned}
& d\left(u_i(t) - u_i^*(t)\right) \\
& \quad = -a_i(t)\left(u_i\left(t - \tau_i(t)\right) - u_i^*\left(t - \tau_i(t)\right)\right) \\
& \qquad + \prod_{t_0 < t_k < t} \alpha_{ik}^{-1} \sum_{j=1}^m c_{ij}(t) \\
& \qquad \times \left[f_j\left(\prod_{t_0 < t_k < t} \gamma_{jk} v_j(t)\right)\right. \\
& \qquad \qquad \left. - f_j\left(\prod_{t_0 < t_k < t} \gamma_{jk} v_j^*(t)\right)\right] dt \\
& \qquad + \prod_{t_0 < t_k < t} \alpha_{ik}^{-1} \\
& \qquad \times \left[\sum_{j=1}^m \sigma_{1ij}\left(\prod_{t_0 < t_k < t} \gamma_{jk} v_j(t)\right)\right. \\
& \qquad \qquad \left. - \sum_{j=1}^m \sigma_{1ij}\left(\prod_{t_0 < t_k < t} \gamma_{jk} v_j^*(t)\right)\right] dw_{1j}(t), \\
& \qquad \qquad t \ge t_0, \quad i = 1, 2, \ldots, n, \\
& d\left(v_j(t) - v_j^*(t)\right) \\
& \quad = -b_j(t)\left(v_j\left(t - \sigma_j(t)\right) - v_j^*\left(t - \sigma_j(t)\right)\right) \\
& \qquad + \prod_{t_0 < t_k < t} \gamma_{jk}^{-1} \sum_{i=1}^n d_{ji}(t) \\
& \qquad \times \left[g_i\left(\prod_{t_0 < t_k < t} \alpha_{ik} u_i(t)\right)\right. \\
& \qquad \qquad \left. - g_i\left(\prod_{t_0 < t_k < t} \alpha_{ik} u_i^*(t)\right)\right] dt \\
& \qquad + \prod_{t_0 < t_k < t} \gamma_{jk}^{-1} \sum_{i=1}^n \left[\sigma_{2ji}\left(\prod_{t_0 < t_k < t} \alpha_{ik} u_i(t)\right)\right.
\end{aligned}
$$

$$
\left. - \sigma_{2ji}\left(\prod_{t_0 < t_k < t} \alpha_{ik} u_i^*(t)\right)\right] dw_{2i}(t),
$$

$$
t \ge t_0, \quad j = 1, 2, \ldots, m. \tag{26}
$$

Let $U(t) = \sum_{i=1}^n |u_i(t) - u_i^*(t)|^p + \sum_{j=1}^m |v_j(t) - v_j^*(t)|^p$. Proceeding as the proof of the existence of periodic solution of (10), from (26), we obtain that

$$
\begin{aligned}
U(t) \le\ & 6^{p-1} e^{-\theta(t-t_0)} U(t_0) + \int_{t_0}^t \epsilon e^{-\theta(t-s)} U(s)\, ds \\
& + \int_{t_0}^t c e^{-\theta(t-s)} U_\delta^+(s)\, ds.
\end{aligned} \tag{27}
$$

By (H_4) and Corollary 10, the periodic solution of (10) is p-exponentially stable; that is, the periodic solution of (1) is p-exponentially stable. Therefore, (1) has an ω-periodic solution, which is p-exponentially stable. This completes the proof of Theorem 12. □

4. An Example

In this section, we present an example to illustrate the feasibility of our results obtained in the previous section.

Let $n = m = 2$. Consider the following impulsive stochastic BAM neural network:

$$
\begin{aligned}
dx_i(t) = & \left[-a_i(t) x_i\left(t - \tau_i(t)\right)\right. \\
& \left. + \sum_{j=1}^2 c_{ij}(t) f_j\left(y_j(t)\right) + I_i(t)\right] dt \\
& + \sum_{j=1}^2 \sigma_{1ij}\left(y_j(t)\right) dw_{1j}(t), \\
& t \ge t_0, \quad t \ne t_k, \\
x_i(t_k^+) = & \alpha_{ik} x_i(t_k), \quad t > 0, \ t = t_k, \\
dy_j(t) = & \left[-b_j(t) y_j\left(t - \sigma_j(t)\right)\right. \\
& \left. + \sum_{i=1}^2 d_{ji}(t) g_i\left(x_i(t)\right) + J_j(t)\right] dt \\
& + \sum_{i=1}^2 \sigma_{2ji}\left(x_i(t)\right) dw_{2i}(t), \\
& t \ge t_0, \quad t \ne t_k, \\
y_j(t_k^+) = & \gamma_{jk} y_j(t_k), \quad t > 0, \ t = t_k,
\end{aligned} \tag{28}
$$

where $i, j = 1, 2$ and the coefficients are as follows:

$$a_1(t) = 1.15 + 0.05 \sin \frac{\pi}{2} t,$$

$$a_2(t) = 0.85 + 0.01 \cos \frac{\pi}{2} t,$$

$$b_1(t) = 0.86 + 0.01 \sin \frac{\pi}{2} t,$$

$$b_2(t) = 0.74 + 0.01 \cos \frac{\pi}{2} t,$$

$$c_{11}(t) = 0.04 \sin \frac{\pi}{2} t,$$

$$c_{12}(t) = 0.02 \cos \frac{\pi}{2} t,$$

$$c_{21}(t) = 0.03 \sin \frac{\pi}{2} t,$$

$$c_{22}(t) = 0.01 \sin \frac{\pi}{2} t,$$

$$d_{11}(t) = 0.04 + 0.03 \cos \frac{\pi}{2} t,$$

$$d_{12}(t) = 0.01 \sin \frac{\pi}{2} t,$$

$$d_{21}(t) = 0.05 + 0.01 \sin \frac{\pi}{2} t,$$

$$d_{22}(t) = 0.02 \cos \frac{\pi}{2} t,$$

$$f_1(u) = 0.03 \sin u,$$

$$f_2(u) = 0.04 \cos u,$$

$$g_1(u) = 0.05 \cos u,$$

$$g_2(u) = 0.07 \sin u,$$

$$\sigma_{111}(u) = 0.02 \sin u,$$

$$\sigma_{112}(u) = 0.03 \cos u,$$

$$\sigma_{121}(u) = 0.01 \sin u,$$

$$\sigma_{122}(u) = 0.04 \cos u,$$

$$\sigma_{211}(u) = 0.01 \cos u,$$

$$\sigma_{212}(u) = 0.04 \sin u,$$

$$\sigma_{221}(u) = 0.03 \sin u,$$

$$\sigma_{222}(u) = 0.04 \cos u,$$

$$\tau_1(t) = \tau_2(t) = \sigma_1(t) = \sigma_2(t) = 0.2 + 0.1 \, \sin \frac{\pi}{2} t,$$

$$I_1(t) = I_2(t) = 0.6 \, \sin \sqrt{2} t,$$

$$J_1(t) = J_2(t) = 0.5 \cos t, \quad i, j = 1, 2,$$

$$\alpha_{1k} = \gamma_{1k} = 3^{\sin(k\pi/2)},$$

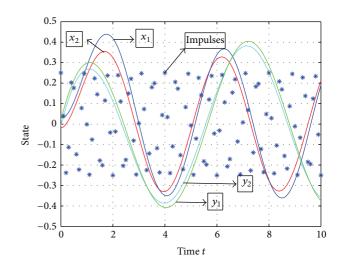$$\alpha_{2k} = \gamma_{2k} = 3^{\cos(k\pi/2)}, \quad k = 1, 2, \ldots .$$

$$\tag{29}$$

FIGURE 1: Transient states of stochastic BAM neural networks.

By calculating, we have

$$\bar{a}_1 = 1.2, \qquad \underline{a}_1 = 1.1,$$

$$\bar{a}_2 = 0.86, \qquad \underline{a}_2 = 0.84,$$

$$\bar{b}_1 = 0.87, \qquad \underline{b}_1 = 0.85,$$

$$\bar{b}_2 = 0.75, \qquad \underline{b}_2 = 0.73,$$

$$\bar{c}_{11} = 0.04, \qquad \bar{c}_{12} = 0.02,$$

$$\bar{c}_{21} = 0.03, \qquad \bar{c}_{22} = 0.01,$$

$$\bar{d}_{11} = 0.07, \qquad \bar{d}_{12} = 0.01,$$

$$\bar{d}_{21} = 0.06, \qquad \bar{d}_{22} = 0.02,$$

$$L_1^f = 0.03, \qquad L_2^f = 0.04,$$

$$L_1^g = 0.05, \qquad L_2^g = 0.07,$$

$$l_{111} = 0.02, \qquad l_{112} = 0.03,$$

$$l_{121} = 0.01, \qquad l_{122} = 0.04,$$

$$l_{211} = 0.01, \qquad l_{212} = 0.04,$$

$$l_{221} = 0.03, \qquad l_{222} = 0.04,$$

$$\bar{\alpha} = \bar{\gamma} = 3, \qquad \underline{\alpha} = \underline{\gamma} = 1.$$

$$\tag{30}$$

Taking $p = 3$, we can verify that all conditions of Theorem 12 are satisfied. Hence, (28) has a 4-periodic solution, which is 3-exponentially stable.

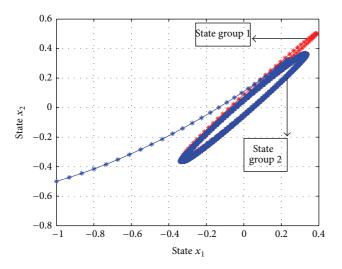By the numerical simulation in Figures 1, 2, and 3 we can show that our results are plausible and effective.
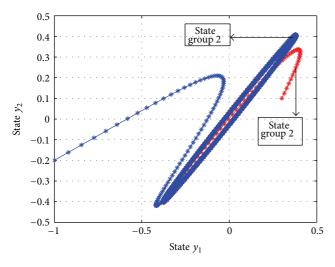
Figure 2: State response of x_1 and x_2.

Figure 3: State response of y_1 and y_2.

Acknowledgment

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