

## Research Article

# Stability Analysis for Uncertain Neural Networks of Neutral Type with Time-Varying Delay in the Leakage Term and Distributed Delay

Qi Zhou,<sup>1</sup> Xueying Shao,<sup>2</sup> Jin Zhu,<sup>2</sup> and Hamid Reza Karimi<sup>3</sup>

<sup>1</sup> College of Information Science and Technology, Bohai University, Jinzhou, Liaoning 121013, China

<sup>2</sup> School of Mathematics and Physics, Bohai University, Jinzhou, Liaoning 121013, China

<sup>3</sup> Department of Engineering, Faculty of Engineering and Science, University of Agder, N-4898 Grimstad, Norway

Correspondence should be addressed to Qi Zhou; [zhouqi2009@gmail.com](mailto:zhouqi2009@gmail.com)

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The stability problem is investigated for a class of uncertain networks of neutral type with leakage, time-varying discrete, and distributed delays. Both the parameter uncertainty and the generalized activation functions are considered in this paper. New stability results are achieved by constructing an appropriate Lyapunov-Krasovskii functional and employing the free weighting matrices and the linear matrix inequality (LMI) method. Some numerical examples are given to show the effectiveness and less conservatism of the proposed results.

## 1. Introduction

Neural networks are a complex large-scale power system and have very rich dynamic property. In the past years, neural networks have some applications in areas of associative memory [1], pattern recognition [2], and optimization problems [3–5]. Recently, the stability problem for neural networks has been widely investigated and some results have been reported in [6–14].

Time delays are often encountered in various engineering, biological, and economical systems [15–23]. Due to the finite speed of information processing, the existence of time delays frequently causes oscillation, divergence, or instability in neural networks. Recently, the stability of neural networks with time delays has drawn considerable attention, and many results on stability of neural networks with time delays have been reported in the literature [24–30]. In practice, the time-varying delay often belongs to a given interval, and the lower bound of the delays may not be zero. Recently, some papers investigated the stability conditions for neural networks with interval time-varying delay in the literature, for example [31]. Furthermore, due to the presence of parallel pathways of different axonal sizes and lengths, there may exist a spatial

extent in neural networks, which may cause distributed time delays [32–35]. Recently, the stability problem of neural networks with neutral-type was studied in [36, 37].

More recently, more and more attention has been paid to time delay in the leakage (or “forgetting”) term [38, 39], since there exist some theoretical and technical difficulties when handling the leakage delay [40]. It has been pointed out that the leakage delay has a great impact on the dynamics of neural networks [41, 42]. The authors in [43] showed that time delay in the stabilizing negative feedback term has a tendency to destabilize a system. More recently, the authors in [44] focused on recurrent neural networks with time delay in the leakage term and showed that all the results mentioned about the existence and uniqueness of the equilibrium point are independent of time delays and initial conditions. Then, it can be seen that time delays in leakage terms do not affect the existence and uniqueness of the equilibrium point. Recently, some results about stability analysis for neural networks with leakage delay have been reported in [38, 39, 41]. To mention a few, the work in [14] investigated the problems of delay-dependent stability analysis and strict  $(Q, S, R)$ - $\alpha$ -dissipativity analysis for cellular neural networks with distributed delay. However, it should be mentioned that there

are few results about stability analysis for uncertain neural networks of neutral type with time-varying delay in the leakage term and distributed delay, which motivates this study.

In this paper, the stability problem is investigated for uncertain neural networks of neutral type with time-varying delay in the leakage term and time-varying distributed delay. Firstly, by constructing a new type of Lyapunov functional and developing some novel techniques to handle the delays considered in this paper, some novel robust stability criteria are proposed. Secondly, the proposed conditions can be expressed in terms of linear matrix inequalities (LMIs), which can be easily solved via standard software. Finally, some numerical examples are given to demonstrate the effectiveness and less conservatism of the proposed results.

*Notation.* Throughout this paper, the notations are standard.  $\mathbb{R}^n$  and  $\mathbb{R}^{n \times m}$  denote the  $n$ -dimensional Euclidean space and the set of all  $n \times m$  real matrices, respectively. In this paper, the superscript  $T$  denotes matrix transposition. For real symmetric matrices  $X$  and  $Y$ , the notation  $X \geq Y$  ( $X > Y$  resp.) means that the  $X - Y$  is positive-semidefinite (positive-definite resp.). The notation  $\text{diag}\{\}$  stands for a block-diagonal matrix.  $I$  is the identity matrix with appropriate dimensions. Matrices if not explicitly stated, where the symbol “\*” stands for the symmetric term in a matrix, are assumed to have compatible dimensions.

### 2. Problem Formulation

Consider the following uncertain neural network of neutral type with time delay in the leakage term and distributed delay:

$$\begin{aligned} \dot{x}(t) = & -(W_1 + \Delta W_1(t))x(t - \delta) + (W_2 + \Delta W_2(t))f(x(t)) \\ & + (W_3 + \Delta W_3(t))f(t - \tau(t)) \\ & + (W_4 + \Delta W_4(t)) \int_{t-r(t)}^t f(x(s)) ds \\ & + (W_5 + \Delta W_5(t))\dot{x}(t - h(t)), \end{aligned} \tag{1}$$

where  $x(t) = [x_1(t) \ x_2(t) \ \dots \ x_n(t)]^T \in \mathbb{R}^n$  is the network state vector at time  $t$ ;  $f(t) := f(x(t)) = [f_1(x_1(t)) \ f_2(x_2(t)) \ \dots \ f_n(x_n(t))]^T \in \mathbb{R}^n$  denotes the activation function at time  $t$ ;  $W_1$  is a positive diagonal matrix;  $W_1 = (W_{1ij})_{n \times n}$ ,  $W_2 = (W_{2ij})_{n \times n}$ ,  $W_3 = (W_{3ij})_{n \times n}$ ,  $W_4 = (W_{4ij})_{n \times n}$ , and  $W_5 = (W_{5ij})_{n \times n}$  are known constant matrices;  $\Delta W_1(t)$ ,  $\Delta W_2(t)$ ,  $\Delta W_3(t)$ ,  $\Delta W_4(t)$ , and  $\Delta W_5(t)$ , are unknown matrices;  $\delta$  is leakage delay,  $\tau(t)$  is time-varying discrete delay;  $h(t)$  is neutral delay; and  $r(t)$  is time-varying distributed delay. They satisfy the following conditions:

$$\begin{aligned} 0 \leq \delta, \quad 0 < \tau_1 \leq \tau(t) \leq \tau_2, \quad \dot{\tau}(t) \leq \tau_d, \\ 0 < h(t) \leq h, \quad \dot{h}(t) \leq h_d < 1, \quad 0 < r(t) \leq \bar{r}, \end{aligned} \tag{2}$$

where  $\delta$ ,  $\tau_1$ ,  $\tau_2$ ,  $\tau_d$ ,  $h$ , and  $h_d$  are constants. The time-varying parameter uncertainties  $\Delta W_1(t)$ ,  $\Delta W_2(t)$ ,  $\Delta W_3(t)$ ,  $\Delta W_4(t)$ , and  $\Delta W_5(t)$  are assumed to be of the form

$$\begin{aligned} [\Delta W_1(t) \ \Delta W_2(t) \ \Delta W_3(t) \ \Delta W_4(t) \ \Delta W_5(t)] \\ = HG(t) [E_1 \ E_2 \ E_3 \ E_4 \ E_5], \end{aligned} \tag{3}$$

where  $H, E_1, E_2, E_3, E_4, E_5$  are known constant matrices and  $G(t)$  is an unknown time-varying matrix satisfying

$$G(t)^T G(t) \leq I. \tag{4}$$

Throughout this paper, we make the following assumption:  
(H1) for any  $i = 1, 2, \dots, n$ ,  $f_i(0) = 0$ , there exist constants  $F_i^-, F_i^+$ , for all  $\alpha_1, \alpha_2 \in \mathbb{R}^n$  with  $\alpha_1 \neq \alpha_2$  such that

$$F_i^- \leq \frac{f_i(\alpha_1) - f_i(\alpha_2)}{\alpha_1 - \alpha_2} \leq F_i^+. \tag{5}$$

**Lemma 1** (see [45]). *For any positive symmetric constant matrices  $M \in \mathbb{R}^{n \times n}$ , scalar  $\tau > 0$ , vector function  $\omega(s) \in \mathbb{R}^n$  such that the integrations concerned are well defined; then*

$$\begin{aligned} - \int_{t-\tau}^t \omega^T(s) M \omega(s) ds \\ \leq - \frac{1}{\tau} \left( \int_{t-\tau}^t \omega(s) ds \right)^T M \left( \int_{t-\tau}^t \omega(s) ds \right). \end{aligned} \tag{6}$$

**Lemma 2** ([46] Schur complement). *Given constant matrices  $\Omega_1, \Omega_2$ , and  $\Omega_3$  with appropriate dimensions, where  $\Omega_1^T = \Omega_1$  and  $\Omega_2^T = \Omega_2 > 0$ , then  $\Omega_1 + \Omega_3^T \Omega_2^{-1} \Omega_3 < 0$  if and only if*

$$\begin{bmatrix} \Omega_1 & \Omega_3^T \\ * & -\Omega_2 \end{bmatrix} < 0 \quad \text{or} \quad \begin{bmatrix} -\Omega_2 & \Omega_3 \\ * & \Omega_1 \end{bmatrix} < 0. \tag{7}$$

*In order to present novel stability criteria for neural networks (1), the following notations are defined:*

$$\begin{aligned} F_1 = \text{diag}(F_1^-, F_2^-, \dots, F_n^-), \quad F_2 = \text{diag}(F_1^+, F_2^+, \dots, F_n^+), \\ F_3 = \text{diag}(F_1^- F_1^+, F_2^- F_2^+, \dots, F_n^- F_n^+), \\ F_4 = \text{diag}\left(\frac{F_1^- + F_1^+}{2}, \frac{F_2^- + F_2^+}{2}, \dots, \frac{F_n^- + F_n^+}{2}\right). \end{aligned} \tag{8}$$

### 3. Main Results

In this section, by considering the delay in the leakage term and distributed delay and using some new techniques, novel stability criteria will be proposed for uncertain neural network of neutral-type with time-varying delay in (1). Firstly, considering neutral-type delay, the delay in the leakage term, and distributed delay, we have the following theorem.

**Theorem 3.** *For given scalars  $0 \leq \delta$ ,  $0 < \tau_1 < \tau_2$ ,  $0 < h$ ,  $0 < \bar{r}$ ,  $h_d < 1$ , and  $\tau_d$ , neural network (1) under Assumption (H1) is robustly asymptotically stable, if there exist matrices*

$P_i > 0$  ( $i = 1, 2, \dots, 5$ ),  $Q = [Q_{ij}]_{2 \times 2} > 0$ ,  $K = [K_{ij}]_{2 \times 2} > 0$ ,  $Z_i > 0$  ( $i = 1, 2, \dots, 7$ ),  $D = \text{diag} \{d_1, d_2, \dots, d_n\} > 0$ ,  $L = \text{diag} \{l_1, l_2, \dots, l_n\} > 0$ , positive diagonal matrices  $R$  and  $S$ , and appropriately dimensioned matrices  $N_k, M_k$  ( $k = 1, 2, \dots, 14$ ), such that the following LMIs hold:

$$\begin{bmatrix} \Xi + \Pi + \Pi^T & \sqrt{h}N & MH & \varepsilon E^T \\ * & -Z_7 & 0 & 0 \\ * & * & -\varepsilon I & 0 \\ * & * & * & -\varepsilon I \end{bmatrix} < 0, \quad (10)$$

$$\begin{bmatrix} Z_3 & T \\ * & Z_3 \end{bmatrix} > 0, \quad (9) \quad \text{where}$$

$$\Theta_{12} = \begin{bmatrix} 0 & P_1 - F_1^T D^T + F_2^T L^T & 0 & F_4 R & 0 & W_1 P_1 W_1 & 0 \\ 0 & 0 & 0 & 0 & F_4 S & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -W_1 P_1 W_1 & 0 \end{bmatrix}, \quad \Xi = \begin{bmatrix} \Theta_{11} & \Theta_{12} \\ * & \Theta_{22} \end{bmatrix},$$

$$\Theta_{22} = \begin{bmatrix} -Z_5 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & \Psi_{99} & 0 & D - L & 0 & -P_1 W_1 & 0 \\ * & * & -(1 - h_d) Z_6 & 0 & 0 & 0 & 0 \\ * & * & * & P_5 + \bar{r}^2 Z_4 - R & 0 & 0 & 0 \\ * & * & * & * & -(1 - \tau_d) P_5 - S & 0 & 0 \\ * & * & * & * & * & -P_3 & 0 \\ * & * & * & * & * & * & -Z_4 \end{bmatrix},$$

$$\Theta_{11} = \begin{bmatrix} \Psi_{11} & 0 & Z_2 & 0 & Z_1 + Q_{12} & K_{12} & P_1 W_1 \\ * & \Psi_{22} & Z_3 - T^T & Z_3 - T & 0 & 0 & 0 \\ * & * & P_4 - Z_2 - Z_3 - Q_{22} & T & -Q_{12}^T & 0 & 0 \\ * & * & * & -K_{22} - Z_3 & 0 & -K_{12}^T & 0 \\ * & * & * & * & -Z_1 + Q_{22} - Q_{11} & 0 & 0 \\ * & * & * & * & * & K_{22} - K_{11} & 0 \\ * & * & * & * & * & * & -P_2 \end{bmatrix},$$

$$\Psi_{11} = -P_1 W_1 - W_1^T P_1^T + P_2 + \delta^2 P_3 + Q_{11} + K_{11} - Z_1 - Z_2 - F_3 R + Z_5,$$

$$\Psi_{22} = -(1 - \tau_d) P_4 - Z_3 - Z_3^T - F_3 S + T + T^T, \quad \Psi_{99} = \frac{\tau_1^2}{4} Z_1 + \tau_1^2 Z_2 + \tau_{12}^2 Z_3 + Z_6 + h Z_7,$$

$$\Pi = [N, 0, 0, 0, 0, 0, 0, -N, 0, 0, 0, 0, 0, 0] + M [0, 0, 0, 0, 0, 0, -W_1, 0, -I, W_5, W_2, W_3, 0, W_4],$$

$$N = [N_1^T, N_2^T, N_3^T, N_4^T, N_5^T, N_6^T, N_7^T, N_8^T, N_9^T, N_{10}^T, N_{11}^T, N_{12}^T, N_{13}^T, N_{14}^T]^T,$$

$$M = [M_1^T, M_2^T, M_3^T, M_4^T, M_5^T, M_6^T, M_7^T, M_8^T, M_9^T, M_{10}^T, M_{11}^T, M_{12}^T, M_{13}^T, M_{14}^T]^T,$$

$$E = [0, 0, 0, 0, 0, 0, -E_1, 0, 0, E_5, E_2, E_3, 0, E_4].$$

(11)

*Proof.* By using Newton-Leibniz formulation and considering neural network (1), the following equalities hold for appropriately dimensioned matrices  $N$  and  $M$ :

$$2\eta^T(t) N \left[ x(t) - x(t - h(t)) - \int_{t-h(t)}^t \dot{x}(s) ds \right] = 0,$$

$$2\eta^T(t) M \left[ -(W_1 + \Delta W_1(t)) x(t - \delta) + (W_2 + \Delta W_2(t)) \times f(x(t)) + (W_3 + \Delta W_3(t)) f(t - \tau(t)) \right.$$

$$\left. + (W_4 + \Delta W_4(t)) \int_{t-r(t)}^t f(x(s)) ds + (W_5 + \Delta W_5(t)) \dot{x}(t - h(t)) - \dot{x}(t) \right] = 0, \quad (12)$$

with

$$\eta^T(t) = \left[ x^T(t), x^T(t - \tau(t)), x^T(t - \tau_1), x^T(t - \tau_2), \right.$$

$$\begin{aligned}
& x^T\left(t - \frac{\tau_1}{2}\right) x^T\left(t - \frac{\tau_2}{2}\right), x^T(t - \delta), x^T(t - h(t)), \\
& \dot{x}^T(t), \dot{x}^T(t - h(t)), f^T(x(t)), f^T(x(t - \tau(t))), \\
& \left(\int_{t-\delta}^t x(s) ds\right)^T, \left(\int_{t-r(t)}^t f(x(s)) ds\right)^T \Big]^T.
\end{aligned} \tag{13}$$

For positive diagonal matrices  $R$  and  $S$ , based on Assumption (H1), it can be seen that the following inequalities hold:

$$\begin{aligned}
& \begin{bmatrix} x(t) \\ f(x(t)) \end{bmatrix}^T \begin{bmatrix} F_3 R & -F_4 R \\ -F_4 R & R \end{bmatrix} \begin{bmatrix} x(t) \\ f(x(t)) \end{bmatrix} \leq 0, \\
& \begin{bmatrix} x(t - \tau(t)) \\ f(x(t - \tau(t))) \end{bmatrix}^T \begin{bmatrix} F_3 S & -F_4 S \\ -F_4 S & S \end{bmatrix} \begin{bmatrix} x(t - \tau(t)) \\ f(x(t - \tau(t))) \end{bmatrix} \leq 0.
\end{aligned} \tag{14}$$

Now, choosing the following Lyapunov-Krasovskii functional:

$$V(t) = \sum_{i=1}^7 V_i(t), \tag{15}$$

where

$$\begin{aligned}
V_1(t) &= \left(x(t) - W_1 \int_{t-\delta}^t x(s) ds\right)^T P_1 \left(x(t) - W_1 \int_{t-\delta}^t x(s) ds\right), \\
V_2(t) &= 2 \sum_{i=1}^n d_i \int_0^{x_i(t)} (f_i(s) - F_i^- s) ds \\
&\quad + 2 \sum_{i=1}^n l_i \int_0^{x_i(t)} (F_i^+ s - f_i(s)) ds, \\
V_3(t) &= \int_{t-\delta}^t x^T(s) P_2 x(s) ds + \delta \int_{-\delta}^0 \int_{t+\theta}^t x^T(s) P_3 x(s) ds d\theta, \\
V_4(t) &= \int_{t-(\tau_1/2)}^t \omega_1^T(s) Q \omega_1(s) ds + \int_{t-(\tau_2/2)}^t \omega_2^T(s) K \omega_2(s) ds \\
&\quad + \int_{t-\tau(t)}^{t-\tau_1} x^T(s) P_4 x(s) ds \\
&\quad + \int_{t-\tau(t)}^t f^T(x(s)) P_5 f(x(s)) ds, \\
V_5(t) &= \int_{-\tau_1/2}^0 \int_{t+\theta}^t \frac{\tau_1}{2} \dot{x}^T(s) Z_1 \dot{x}(s) ds d\theta \\
&\quad + \int_{-\tau_1}^0 \int_{t+\theta}^t \tau_1 \dot{x}^T(s) Z_2 \dot{x}(s) ds d\theta \\
&\quad + \int_{-\tau_2}^{-\tau_1} \int_{t+\theta}^t \tau_{12} \dot{x}^T(s) Z_3 \dot{x}(s) ds d\theta, \\
V_6(t) &= \bar{r} \int_{-\bar{r}}^0 \int_{t+\theta}^t f^T(x(s)) Z_4 f(x(s)) ds d\theta,
\end{aligned}$$

$$\begin{aligned}
V_7(t) &= \int_{t-h(t)}^t x^T(s) Z_5 x(s) ds + \int_{t-h(t)}^t \dot{x}^T(s) Z_6 \dot{x}(s) ds \\
&\quad + \int_{-h}^0 \int_{t+\theta}^t \dot{x}^T(s) Z_7 \dot{x}(s) ds d\theta,
\end{aligned} \tag{16}$$

where  $\tau_{12} = \tau_2 - \tau_1$ , and

$$\omega_i(t) = \left[ x^T(t) \quad x^T\left(t - \frac{\tau_i}{2}\right) \right]^T, \quad i = 1, 2. \tag{17}$$

Then, the derivatives of  $V_i(t)$ , ( $i = 1, \dots, 7$ ) with time  $t$  can be obtained as

$$\begin{aligned}
\dot{V}_1(t) &= 2 \left( x(t) - W_1 \int_{t-\delta}^t x(s) ds \right)^T \\
&\quad \times P_1 \left( \dot{x}(t) - W_1 x(t) + W_1 x(t - \delta) \right), \\
\dot{V}_2(t) &= 2 \dot{x}^T(t) D (f(x(t)) - F_1 x(t)) \\
&\quad + 2 \dot{x}^T(t) L (F_2 x(t) - f(x(t))), \\
\dot{V}_3(t) &= x^T(t) P_2 x(t) + x^T(t - \delta) P_2 x(t - \delta) \\
&\quad + \delta^2 x^T(t) P_3 x(t) - \delta \int_{t-\delta}^t x^T(s) P_3 x(s) ds, \\
\dot{V}_4(t) &= \omega_1^T(t) Q \omega_1(t) - \omega_1^T\left(t - \frac{\tau_1}{2}\right) Q \omega_1\left(t - \frac{\tau_1}{2}\right) \\
&\quad + \omega_2^T(t) K \omega_2(t) - \omega_2^T\left(t - \frac{\tau_2}{2}\right) K \omega_2\left(t - \frac{\tau_2}{2}\right) \\
&\quad + x^T(t - \tau_1) P_4 x(t - \tau_1) \\
&\quad - (1 - \dot{\tau}(t)) x^T(t - \tau(t)) P_4 x(t - \tau(t)) \\
&\quad + f^T(x(t)) P_5 f(x(t)) \\
&\quad - (1 - \dot{\tau}(t)) f^T(x(t - \tau(t))) P_5 f(x(t - \tau(t))), \\
\dot{V}_5(t) &= \frac{\tau_1^2}{4} \dot{x}^T(t) Z_1 \dot{x}(t) - \int_{t-(\tau_1/2)}^t \frac{\tau_1}{2} \dot{x}^T(s) Z_1 \dot{x}(s) ds \\
&\quad + \tau_1^2 \dot{x}^T(t) Z_2 \dot{x}(t) - \int_{t-\tau_1}^t \tau_1 \dot{x}^T(s) Z_2 \dot{x}(s) ds \\
&\quad + \tau_{12}^2 \dot{x}^T(t) Z_3 \dot{x}(t) - \int_{t-\tau_2}^{t-\tau_1} \tau_{12} \dot{x}^T(s) Z_3 \dot{x}(s) ds, \\
\dot{V}_6(t) &= \bar{r}^2 f^T(x(t)) Z_4 f(x(t)) \\
&\quad - \int_{t-\bar{r}}^t \bar{r} f^T(x(s)) Z_4 f(x(s)) ds, \\
\dot{V}_7(t) &= x^T(t) Z_5 x(t) - x^T(t - h(t)) Z_5 x(t - h(t)) \\
&\quad + \dot{x}^T(t) Z_6 \dot{x}(t) - (1 - \dot{h}(t)) \dot{x}^T(t - h(t))
\end{aligned}$$

$$\begin{aligned} & \times Z_6 \dot{x}(t-h(t)) \\ & + h \dot{x}^T(t) Z_7 \dot{x}(t) - \int_{t-h}^t \dot{x}^T(s) Z_7 \dot{x}(s) ds. \end{aligned} \tag{18}$$

By Lemma 1, one can have

$$\begin{aligned} & - \int_{t-(\tau_1/2)}^t \frac{\tau_1}{2} \dot{x}^T(s) Z_1 \dot{x}(s) ds \\ & \leq - \left[ x(t) - x\left(t - \frac{\tau_1}{2}\right) \right]^T Z_1 \left[ x(t) - x\left(t - \frac{\tau_1}{2}\right) \right], \end{aligned} \tag{19}$$

$$\begin{aligned} & - \int_{t-\tau_1}^t \tau_1 \dot{x}^T(s) Z_2 \dot{x}(s) ds \\ & \leq - [x(t) - x(t-\tau_1)]^T Z_2 [x(t) - x(t-\tau_1)], \end{aligned} \tag{20}$$

$$\begin{aligned} & - \int_{t-\tau_1}^{t-\tau_2} \tau_{12} \dot{x}^T(s) Z_3 \dot{x}(s) ds \\ & = - \int_{t-\tau(t)}^{t-\tau_1} \tau_{12} \dot{x}^T(s) Z_3 \dot{x}(s) ds \\ & \quad - \int_{t-\tau_2}^{t-\tau(t)} \tau_{12} \dot{x}^T(s) Z_3 \dot{x}(s) ds \\ & \leq - \frac{\tau_{12}}{\tau(t) - \tau_1} \eta^T(t) (e_3 - e_2) Z_3 (e_3 - e_2)^T \eta(t) \\ & \quad - \frac{\tau_{12}}{\tau_2 - \tau(t)} \eta^T(t) (e_2 - e_4) Z_3 (e_2 - e_4)^T \eta(t) \\ & = - \frac{1}{\alpha_1} \eta^T(t) (e_3 - e_2) Z_3 (e_3 - e_2)^T \eta(t) \\ & \quad - \frac{1}{\alpha_2} \eta^T(t) (e_2 - e_4) Z_3 (e_2 - e_4)^T \eta(t) \\ & = - \eta^T(t) (e_3 - e_2) Z_3 (e_3 - e_2)^T \eta(t) \\ & \quad - \eta^T(t) (e_2 - e_4) Z_3 (e_2 - e_4)^T \eta(t) \\ & \quad - \frac{\alpha_2}{\alpha_1} \eta^T(t) (e_3 - e_2) Z_3 (e_3 - e_2)^T \eta(t) \\ & \quad - \frac{\alpha_1}{\alpha_2} \eta^T(t) (e_2 - e_4) Z_3 (e_2 - e_4)^T \eta(t), \end{aligned} \tag{21}$$

where  $\alpha_1 = (\tau(t) - \tau_1)/\tau_{12}$  and  $\alpha_2 = (\tau_2 - \tau(t))/\tau_{12}$ . It can be seen from the condition (9) that

$$\begin{aligned} & - \frac{\alpha_2}{\alpha_1} \eta^T(t) (e_3 - e_2) Z_3 (e_3 - e_2)^T \eta(t) \\ & \quad - \frac{\alpha_1}{\alpha_2} \eta^T(t) (e_2 - e_4) Z_3 (e_2 - e_4)^T \eta(t) \\ & \quad + \eta^T(t) (e_3 - e_2) T (e_2 - e_4)^T \eta(t) \end{aligned}$$

$$\begin{aligned} & + \eta^T(t) (e_2 - e_4) T^T (e_3 - e_2)^T \eta(t) \\ & = - \eta^T(t) \begin{bmatrix} \sqrt{\frac{\alpha_2}{\alpha_1}} (e_3^T - e_2^T) \\ -\sqrt{\frac{\alpha_1}{\alpha_2}} (e_2^T - e_4^T) \end{bmatrix}^T \begin{bmatrix} Z_3 & T \\ T^T & Z_3 \end{bmatrix} \\ & \quad \times \begin{bmatrix} \sqrt{\frac{\alpha_2}{\alpha_1}} (e_3^T - e_2^T) \\ -\sqrt{\frac{\alpha_1}{\alpha_2}} (e_2^T - e_4^T) \end{bmatrix} \eta(t) \leq 0, \end{aligned} \tag{22}$$

which means

$$\begin{aligned} & - \frac{\alpha_2}{\alpha_1} \eta^T(t) (e_3 - e_2) Z_3 (e_3 - e_2)^T \eta(t) \\ & \quad - \frac{\alpha_1}{\alpha_2} \eta^T(t) (e_2 - e_4) Z_3 (e_2 - e_4)^T \eta(t) \\ & \leq - \eta^T(t) (e_3 - e_2) T (e_2 - e_4)^T \eta(t) - \eta^T(t) \\ & \quad \times (e_2 - e_4) T^T (e_3 - e_2)^T \eta(t). \end{aligned} \tag{23}$$

It follows from (21) and (23) that

$$\begin{aligned} & - \int_{t-\tau_1}^{t-\tau_2} \tau_{12} \dot{x}^T(s) Z_3 \dot{x}(s) ds \\ & \leq - \eta^T(t) (e_2 - e_4) T^T (e_3^T - e_2^T) \eta(t) \\ & \quad - \eta^T(t) (e_3 - e_2) T (e_2^T - e_4^T) \eta(t) \\ & \quad - \eta^T(t) (e_3 - e_2) Z_3 (e_3 - e_2)^T \eta(t) \\ & \quad - \eta^T(t) (e_2 - e_4) Z_3 (e_2 - e_4)^T \eta(t) \\ & = - \eta^T(t) \begin{bmatrix} e_3^T - e_2^T \\ e_2^T - e_4^T \end{bmatrix}^T \begin{bmatrix} Z_3 & T \\ T^T & Z_3 \end{bmatrix} \\ & \quad \times \begin{bmatrix} e_3^T - e_2^T \\ e_2^T - e_4^T \end{bmatrix} \eta(t). \end{aligned} \tag{24}$$

Therefore, it is straight forward to obtain that

$$\begin{aligned} \dot{V}(t) & \leq 2 \left( x(t) - W_1 \int_{t-\delta}^t x(s) ds \right)^T \\ & \quad \times P_1 (\dot{x}(t) - W_1 x(t) + W_1 x(t-\delta)) \\ & \quad + 2 \dot{x}^T(t) D (f(x(t)) - F_1 x(t)) \\ & \quad + 2 \dot{x}^T(t) L (F_2 x(t) - f(x(t))) \\ & \quad + x^T(t) P_2 x(t) - x^T(t-\delta) P_2 x(t-\delta) \\ & \quad + \delta^2 x^T(t) P_3 x(s) ds - \left( \int_{t-\delta}^t x(s) ds \right)^T P_3 \int_{t-\delta}^t x(s) ds \\ & \quad + \omega_1^T(t) Q \omega_1(t) - \omega_1^T\left(t - \frac{\tau_1}{2}\right) Q \omega_1\left(t - \frac{\tau_1}{2}\right) \end{aligned}$$

$$\begin{aligned}
& + \omega_2^T(t) K \omega_2(t) - \omega_2^T\left(t - \frac{\tau_2}{2}\right) K \omega_2\left(t - \frac{\tau_2}{2}\right) \\
& + x^T(t - \tau_1) P_4 x(t - \tau_1) \\
& - (1 - \tau_d) x^T(t - \tau(t)) P_4 x(t - \tau(t)) \\
& + f^T(x(t)) P_5 f(x(t)) \\
& - (1 - \tau_d) f^T(x(t - \tau(t))) P_5 f(x(t - \tau(t))) \\
& + \frac{\tau_1^2}{4} \dot{x}^T(t) Z_1 \dot{x}(t) \\
& - \left[ x(t) - x\left(t - \frac{\tau_1}{2}\right) \right]^T Z_1 \left[ x(t) - x\left(t - \frac{\tau_1}{2}\right) \right] \\
& + \tau_1^2 \dot{x}^T(t) Z_2 \dot{x}(t) \\
& - [x(t) - x(t - \tau_1)]^T Z_2 [x(t) - x(t - \tau_1)] \\
& + \tau_{12}^2 \dot{x}^T(t) Z_3 \dot{x}(t) \\
& - \eta^T(t) \begin{bmatrix} e_3^T - e_2^T \\ e_2^T - e_4^T \end{bmatrix}^T \begin{bmatrix} Z_3 & T \\ T^T & Z_3 \end{bmatrix} \begin{bmatrix} e_3^T - e_2^T \\ e_2^T - e_4^T \end{bmatrix} \eta(t) \\
& + \bar{r}^2 f^T(x(t)) Z_4 f(x(t)) \\
& - \int_{t-\bar{r}}^t \bar{r} f^T(x(s)) Z_4 f(x(s)) ds + x^T(t) Z_5 x(t) \\
& - x^T(t - h(t)) Z_5 x(t - h(t)) \\
& + \dot{x}^T(t) Z_6 \dot{x}(t) - (1 - h_d) \dot{x}^T(t - h(t)) Z_6 \dot{x}(t - h(t)) \\
& + h \dot{x}^T(t) Z_7 \dot{x}(t) - \int_{t-h}^t \dot{x}^T(s) Z_7 \dot{x}(s) ds \\
& - \begin{bmatrix} x(t) \\ f(x(t)) \end{bmatrix}^T \begin{bmatrix} F_3 R & -F_4 R \\ -F_4 R & R \end{bmatrix} \begin{bmatrix} x(t) \\ f(x(t)) \end{bmatrix} \\
& - \begin{bmatrix} x(t - \tau(t)) \\ f(x(t - \tau(t))) \end{bmatrix}^T \begin{bmatrix} F_3 S & -F_4 S \\ -F_4 S & S \end{bmatrix} \\
& \times \begin{bmatrix} x(t - \tau(t)) \\ f(x(t - \tau(t))) \end{bmatrix} \\
& + 2\eta^T(t) M \left[ \begin{aligned}
& - (W_1 + \Delta W_1(t)) x(t - \delta) \\
& + (W_2 + \Delta W_2(t)) f(x(t)) \\
& + (W_3 + \Delta W_3(t)) f(t - \tau(t)) \\
& + (W_4 + \Delta W_4(t)) \int_{t-r(t)}^t f(x(s)) ds \\
& + (W_5 + \Delta W_5(t)) \dot{x}(t - h(t)) - \dot{x}(t)
\end{aligned} \right]
\end{aligned}$$

$$\begin{aligned}
& \leq \eta^T(t) (\Xi + \Pi + \Pi^T + hNZ_7^{-1}N^T + \Delta\Xi) \eta(t) \\
& \quad - \int_{t-h(t)}^t (N^T \eta(t) + Z_7 \dot{x}(s))^T \\
& \quad \quad \times Z_7^{-1} (N^T \eta(t) + Z_7 \dot{x}(s)) ds \\
& \leq \eta^T(t) (\Xi + \Pi + \Pi^T + hNZ_7^{-1}N^T + \Delta\Xi) \eta(t) \\
& \leq \eta^T(t) (\Xi + \Pi + \Pi^T + hNZ_7^{-1}N^T \\
& \quad + \varepsilon^{-1} M H H^T M^T + \varepsilon E^T E) \eta(t), \tag{25}
\end{aligned}$$

where  $\Delta\Xi = MHF(t)E + (MHF(t)E)^T$ . By Schur complement, it can be seen from the condition (10) that  $\dot{V}(t) < 0$ , which means that neural network (1) under Assumption (H1) is robustly asymptotically stable. This completes this proof.  $\square$

When neural network (1) without uncertainties, we have

$$\begin{aligned}
\dot{x}(t) = & -W_1 x(t - \delta) + W_2 f(x(t)) + W_3 f(t - \tau(t)) \\
& + W_4 \int_{t-r(t)}^t f(x(s)) ds + W_5 \dot{x}(t - h(t)). \tag{26}
\end{aligned}$$

The stability condition can be easily obtained from Theorem 3 in the following corollary.

**Corollary 4.** Given scalars  $0 \leq \delta$ ,  $0 \leq \tau_1 < \tau_2$ ,  $0 < h$ ,  $0 < \bar{r}$ ,  $h_d < 1$ , and  $\tau_d$ , neural network (26) is asymptotically stable, if there exist matrices  $P_i > 0$  ( $i = 1, 2, \dots, 5$ ),  $Q = [Q_{ij}]_{2 \times 2} > 0$ ,  $K = [K_{ij}]_{2 \times 2} > 0$ ,  $Z_i > 0$  ( $i = 1, 2, \dots, 7$ ),  $D = \text{diag}\{d_1, d_2, \dots, d_n\} > 0$ ,  $L = \text{diag}\{l_1, l_2, \dots, l_n\} > 0$ , positive diagonal matrices  $R$  and  $S$ , and appropriately dimensioned matrices  $N_k, M_k$  ( $k = 1, 2, \dots, 14$ ), such that the following LMIs hold:

$$\begin{bmatrix} Z_3 & T \\ T^T & Z_3 \end{bmatrix} > 0, \quad \begin{bmatrix} \Xi + \Pi + \Pi^T & \sqrt{h}N \\ * & -Z_7 \end{bmatrix} < 0, \tag{27}$$

where  $\Xi, \Pi$  and  $N$  have been defined in Theorem 3.

**Remark 5.** For neural network (1), if time delay in the leakage term is not considered in this paper, the following model can be obtained:

$$\begin{aligned}
\dot{x}(t) = & -W_1 x(t) + W_2 f(x(t)) + W_3 f(t - \tau(t)) \\
& + W_4 \int_{t-r(t)}^t f(x(s)) ds + W_5 \dot{x}(t - h(t)). \tag{28}
\end{aligned}$$

In order to present stability criterion for neural network (28), we choose the following Lyapunov-Krasovskii functional:

$$V(t) = x^T(t) P x(t) + \sum_{i=2}^7 V_i(t), \tag{29}$$

where  $V_i$  ( $i = 2, 3, 4, 5, 6, 7$ ) is defined in (15). From the proof of Theorem 3, the following stability condition for neural network (28) can be obtained.

**Corollary 6.** For given scalars  $0 \leq \tau_1 < \tau_2$ ,  $0 < h$ ,  $0 < \bar{r}$ ,  $h_d < 1$ , and  $\tau_d$ , neural network (1) under Assumption (H1) is asymptotically stable, if there exist matrices  $P_i > 0$  ( $i = 1, 4, \dots, 5$ ),  $Q = [Q_{ij}]_{2 \times 2} > 0$ ,  $K = [K_{ij}]_{2 \times 2} > 0$ ,  $Z_i > 0$  ( $i = 1, 2, \dots, 7$ ),  $D = \text{diag} \{d_1, d_2, \dots, d_n\} > 0$ ,  $L = \text{diag} \{l_1, l_2, \dots, l_n\} > 0$ , positive diagonal matrices  $R$  and  $S$ , and

appropriately dimensioned matrices  $\widehat{N}_k, \widehat{M}_k$  ( $k = 1, 2, \dots, 12$ ), such that the following LMIs hold:

$$\begin{bmatrix} Z_3 & T \\ T^T & Z_3 \end{bmatrix} > 0, \quad \begin{bmatrix} \widehat{\Xi} + \widehat{\Pi} + \widehat{\Pi}^T & \sqrt{h}\widehat{N} \\ * & -Z_7 \end{bmatrix} < 0, \quad (30)$$

where

$$\begin{aligned} \widehat{\Xi} &= \begin{bmatrix} \widehat{\Theta}_{11} & \widehat{\Theta}_{12} \\ * & \widehat{\Theta}_{22} \end{bmatrix}, \quad \widehat{\Theta}_{12} = \begin{bmatrix} 0 & P_1 - F_1^T D^T + F_2^T L^T & 0 & F_4 R & 0 & 0 \\ 0 & 0 & 0 & 0 & F_4 S & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \\ \widehat{\Theta}_{11} &= \begin{bmatrix} \widehat{\Psi}_{11} & 0 & Z_2 & 0 & Z_1 + Q_{12} & K_{12} \\ * & \widehat{\Psi}_{22} & Z_3 - T^T & Z_3 - T & 0 & 0 \\ * & * & P_4 - Z_2 - Z_3 - Q_{22} & T & -Q_{12}^T & 0 \\ * & * & * & -K_{22} - Z_3 & 0 & -K_{12}^T \\ * & * & * & * & -Z_1 + Q_{22} - Q_{11} & 0 \\ * & * & * & * & * & K_{22} - K_{11} \end{bmatrix}, \\ \widehat{\Psi}_{11} &= Q_{11} + K_{11} - Z_1 - Z_2 - F_3 R + Z_5, \quad \widehat{\Psi}_{22} = -(1 - \tau_d) P_4 - Z_3 - Z_3^T - F_3 S + T + T^T, \\ \widehat{\Psi}_{88} &= \left( \frac{\tau_1^2}{4} \right) Z_1 + \tau_1^2 Z_2 + \tau_{12}^2 Z_3 + Z_6 + h Z_7, \\ \widehat{\Pi} &= [\widehat{N}, 0, 0, 0, 0, 0, -\widehat{N}, 0, 0, 0, 0, 0] + \widehat{M} [-W_1, 0, 0, 0, 0, 0, 0, -I, W_5, W_2, W_3, W_4], \\ \widehat{\Theta}_{22} &= \begin{bmatrix} -Z_5 & 0 & 0 & 0 & 0 & 0 \\ * & \widehat{\Psi}_{88} & 0 & D - L & 0 & 0 \\ * & * & -(1 - h_d) Z_6 & 0 & 0 & 0 \\ * & * & * & P_5 + \bar{r}^2 Z_4 - R & 0 & 0 \\ * & * & * & * & -(1 - \tau_d) P_5 - S & 0 \\ * & * & * & * & * & -Z_4 \end{bmatrix}, \\ \widehat{N} &= [N_1^T, N_2^T, N_3^T, N_4^T, N_5^T, N_6^T, N_7^T, N_8^T, N_9^T, N_{10}^T, N_{11}^T, N_{12}^T]^T, \\ \widehat{M} &= [M_1^T, M_2^T, M_3^T, M_4^T, M_5^T, M_6^T, M_7^T, M_8^T, M_9^T, M_{10}^T, M_{11}^T, M_{12}^T]^T. \end{aligned} \quad (31)$$

*Remark 7.* When neural network (1) without time-varying neutral delay  $h(t)$ , the following model can be obtained:

$$\begin{aligned} \dot{x}(t) &= -W_1 x(t - \delta) + W_2 f(x(t)) \\ &+ W_3 f(t - \tau(t)) + W_4 \int_{t-r(t)}^t f(x(s)) ds. \end{aligned} \quad (32)$$

In order to present stability criterion for neural network (32), we choose the following Lyapunov-Krasovskii functional:

$$V(t) = \sum_{i=1}^6 V_i(t), \quad (33)$$

where  $V_i$  ( $i = 1, \dots, 6$ ) are defined in (15). Following the same line of Theorem 3, the following corollary can be presented.

**Corollary 8.** Given scalars  $0 \leq \delta$ ,  $0 \leq \tau_1 < \tau_2$ ,  $0 < \bar{r}$ , and  $\tau_d$ , the neural network (32) under Assumptions (H1)-(H2) is asymptotically stable, if there exist matrices  $P_i > 0$  ( $i = 1, 2, \dots, 5$ ),  $Q = [Q_{ij}]_{2 \times 2} > 0$ ,  $K = [K_{ij}]_{2 \times 2} > 0$ ,  $Z_i > 0$  ( $i = 1, 2, \dots, 4$ ),  $D = \text{diag} \{d_1, d_2, \dots, d_n\} > 0$ ,  $L = \text{diag} \{l_1, l_2, \dots, l_n\} > 0$ , positive diagonal matrices  $R$  and  $S$ , and appropriately dimensioned matrices  $\check{M}_k$  ( $k = 1, 2, \dots, 12$ ), such that the following LMIs hold:

$$\begin{bmatrix} Z_3 & T \\ T^T & Z_3 \end{bmatrix} > 0, \quad \check{\Xi} + \check{\Pi} + \check{\Pi}^T < 0, \quad (34)$$

where

$$\begin{aligned} \check{\Psi}_{11} &= -P_1W_1 - W_1^T P_1^T + P_2 + \delta^2 P_3 + Q_{11} + K_{11} - Z_1 - Z_2 - F_3R, \\ \check{\Psi}_{22} &= -(1 - \tau_d)P_4 - Z_3 - Z_3^T - F_3S + T + T^T, \quad \check{\Psi}_{88} = \left(\frac{\tau_1^2}{4}\right)Z_1 + \tau_1^2 Z_2 + \tau_{12}^2 Z_3, \\ \widehat{\Pi} &= \widehat{M} [0, 0, 0, 0, 0, 0, -W_1, -I, W_2, W_3, 0, W_4], \\ \widehat{M} &= [M_1^T, M_2^T, M_3^T, M_4^T, M_5^T, M_6^T, M_7^T, M_8^T, M_9^T, M_{10}^T, M_{11}^T, M_{12}^T]^T, \\ \check{\Theta}_{11} &= \begin{bmatrix} \check{\Psi}_{11} & 0 & Z_2 & 0 & Z_1 + Q_{12} & K_{12} \\ * & \check{\Psi}_{22} & Z_3 - T^T & Z_3 - T & 0 & 0 \\ * & * & P_4 - Z_2 - Z_3 - Q_{22} & T & -Q_{12}^T & 0 \\ * & * & * & -K_{22} - Z_3 & 0 & -K_{12}^T \\ * & * & * & * & -Z_1 + Q_{22} - Q_{11} & 0 \\ * & * & * & * & * & K_{22} - K_{11} \end{bmatrix}, \\ \check{\Theta}_{12} &= \begin{bmatrix} P_1W_1 & P_1 - F_1^T D^T + F_2^T L^T & F_4R & 0 & W_1P_1W_1 & 0 \\ 0 & 0 & 0 & F_4S & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad \check{\Xi} = \begin{bmatrix} \check{\Theta}_{11} & \check{\Theta}_{12} \\ * & \check{\Theta}_{22} \end{bmatrix}, \\ \check{\Theta}_{22} &= \begin{bmatrix} -P_2 & 0 & 0 & 0 & -W_1P_1W_1 & 0 \\ * & \check{\Psi}_{88} & D - L & 0 & -P_1W_1 & 0 \\ * & * & P_5 + \bar{r}^2 Z_4 - R & 0 & 0 & 0 \\ * & * & * & -(1 - \tau_d)P_5 - S & 0 & 0 \\ * & * & * & * & -P_3 & 0 \\ * & * & * & * & * & -Z_4 \end{bmatrix}. \end{aligned} \tag{35}$$

Consider the following neural network with time-varying delay:

$$\dot{x}(t) = -W_1x(t) + W_2f(x(t)) + W_3f(t - \tau(t)). \tag{36}$$

By choosing the Lyapunov-Krasovskii functional

$$V(t) = x^T(t)Px(t) + V_2(t) + V_4(t) + V_6(t), \tag{37}$$

where  $V_i$  ( $i = 2, 4, 6$ ) is defined in (15). The novel stability condition for neural network with time-varying delay (36) can be presented from Theorem 3 in the following corollary.

**Corollary 9.** For given scalars  $0 \leq \tau_1 < \tau_2$ , and  $\tau_d$ , neural network (36) under Assumptions (H1)-(H2) is asymptotically

stable, if there exist matrices  $P_i > 0$  ( $i = 1, 4, 5$ ),  $Q = [Q_{ij}]_{2 \times 2} > 0$ ,  $K = [K_{ij}]_{2 \times 2} > 0$ ,  $Z_i > 0$  ( $i = 1, 2, 3$ ),  $D = \text{diag} \{d_1, d_2, \dots, d_n\} > 0$ ,  $L = \text{diag} \{l_1, l_2, \dots, l_n\} > 0$ , positive diagonal matrices  $R$  and  $S$ , and appropriately dimensioned matrices  $\bar{M}_k$  ( $k = 1, 2, \dots, 9$ ), such that the following LMIs hold:

$$\begin{bmatrix} Z_3 & T \\ T^T & Z_3 \end{bmatrix} > 0, \quad \bar{\Xi} + \bar{\Pi} + \bar{\Pi}^T < 0, \tag{38}$$

where

$$\bar{\Theta}_{11} = \begin{bmatrix} \bar{\Psi}_{11} & 0 & Z_2 & 0 & Z_1 + Q_{12} \\ * & \bar{\Psi}_{22} & Z_3 - T^T & Z_3 - T & 0 \\ * & * & P_4 - Z_2 - Z_3 - Q_{22} & T & -Q_{12}^T \\ * & * & * & -K_{22} - Z_3 & 0 \\ * & * & * & * & -Z_1 + Q_{22} - Q_{11} \end{bmatrix},$$



$$\begin{aligned} \bar{\Theta}_{12} &= \begin{bmatrix} K_{12} & P_1 - F_1^T D^T + F_2^T L^T & F_4 R & 0 \\ 0 & 0 & 0 & F_4 S \\ 0 & 0 & 0 & 0 \\ -K_{12}^T & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, & \bar{\Psi}_{77} &= \frac{\tau_1^2}{4} Z_1 + \tau_1^2 Z_2 + \tau_{12}^2 Z_3, \\ \bar{\Theta}_{22} &= \begin{bmatrix} K_{22} - K_{11} & 0 & 0 & 0 \\ * & \bar{\Psi}_{77} & D - L & 0 \\ * & * & P_5 - R & 0 \\ * & * & * & -(1 - \tau_d) P_5 - S \end{bmatrix}, & \bar{\Xi} &= \begin{bmatrix} \bar{\Theta}_{11} & \bar{\Theta}_{12} \\ * & \bar{\Theta}_{22} \end{bmatrix}, \\ \bar{\Psi}_{11} &= Q_{11} + K_{11} - Z_1 - Z_2 - F_3 R, & \bar{\Psi}_{22} &= -(1 - \tau_d) P_4 - Z_3 - Z_3^T - F_3 S + T + T^T, \\ \bar{\Pi} &= \bar{M} [-W_1, 0, 0, 0, 0, -I, W_2, W_3], & \bar{M} &= [M_1^T, M_2^T, M_3^T, M_4^T, M_5^T, M_6^T, M_7^T, M_8^T, M_9^T]^T. \end{aligned} \tag{39}$$

### 4. Numerical Examples

In this section, two numerical examples are provided to show the effectiveness of the proposed results.

*Example 1.* Consider uncertain neural network of neutral type with leakage delay time-varying delay, and distributed delay as follows:

$$\begin{aligned} \dot{x}(t) &= -(W_1 + \Delta W_1(t)) x(t - \delta) + (W_2 + \Delta W_2(t)) f(x(t)) \\ &+ (W_3 + \Delta W_3(t)) f(t - \tau(t)) \\ &+ (W_4 + \Delta W_4(t)) \int_{t-r(t)}^t f(x(s)) ds \\ &+ (W_5 + \Delta W_5(t)) \dot{x}(t - h(t)), \end{aligned} \tag{40}$$

where

$$\begin{aligned} W_1 &= \text{diag}\{1.6305, 1.9221, 2.5973, 1.3775\}, \\ W_2 &= \begin{bmatrix} -2.5573 & -1.3813 & 1.9574 & -1.1398 \\ -1.0226 & -0.8845 & 0.5045 & -0.2111 \\ 1.0378 & 1.5532 & 0.6645 & 1.1902 \\ -0.3896 & 0.7079 & -0.3398 & -2.2543 \end{bmatrix}, \\ W_3 &= \begin{bmatrix} 0.2853 & -0.0793 & 0.4694 & 0.5354 \\ -0.5955 & 1.3352 & -0.9036 & 0.5529 \\ -0.1497 & -0.6065 & -0.1641 & -0.2037 \\ -0.4348 & -1.3474 & -0.8275 & -2.2543 \end{bmatrix}, \\ W_4 &= \begin{bmatrix} 0.0265 & 0.1157 & 0.0578 & -0.0930 \\ 0.3186 & -0.1363 & -0.0859 & 0.0742 \\ 0.2037 & -0.2049 & -0.0112 & 0.1475 \\ -0.3161 & -0.2469 & -0.0736 & -2.2543 \end{bmatrix}, \\ W_5 &= \begin{bmatrix} -0.3054 & 0.3682 & 0.1761 & -0.0235 \\ -0.0546 & -0.2089 & -0.0754 & 0.2668 \\ 0.4563 & 0.0023 & 0.1440 & 0.6928 \\ -0.0115 & -0.2349 & 0.2004 & 0.1574 \end{bmatrix}, \end{aligned}$$

$$\begin{aligned} F_1 &= \text{diag}\{0, 0, 0, 0\}, \\ F_2 &= \text{diag}\{1.0275, 0.9960, 0.3223, 0.2113\}, \\ H &= 0.1 \times I, \quad E_1 = 0.1 \times I, \quad E_2 = 0.2 \times I, \\ E_3 &= 0.3 \times I, \quad E_4 = 0.4 \times I, \quad E_5 = 0.5 \times I. \end{aligned} \tag{41}$$

By using Matlab LMI Toolbox, from Theorem 3, it can be found that the uncertain neural network (1) under Assumption (H1) is robustly asymptotically stable for  $0 < \delta = \tau_d = h_d < 0.7$ . To calculate the maximum allowable  $\tau_2 = h = \bar{r}$  for different  $\delta = \tau_d = h_d$  in this paper, the numerical results in Table 1 illustrate the effectiveness of the proposed results.

*Example 2.* Consider the neural network with time-varying delay as follows:

$$\dot{x}(t) = -W_1 x(t) + W_2 f(x(t)) + W_3 f(t - \tau(t)). \tag{42}$$

$$W_1 = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}, \quad W_2 = \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix}, \quad W_3 = \begin{bmatrix} 0.88 & 1 \\ 1 & 1 \end{bmatrix}, \tag{43}$$

$$F_2 = \begin{bmatrix} 0.4 & 0 \\ 0 & 0.8 \end{bmatrix}, \quad F_1 = \text{diag}\{0, 0\}.$$

By using Matlab LMI Toolbox, it can be found from Corollary 9 that the maximum allowable  $\tau_2$  can be obtained to guarantee the stability of neural network with time-varying delay in (42) for different  $\tau_d$ . Compared with the previous results proposed in [28], it is clear that the new stability condition in Corollary 9 is less conservative than the one in [28] (see Table 2).

### 5. Conclusion

In this paper, the problem of stability analysis for neural networks of neutral type with time-varying delay in the leakage term and distributed delay has been studied. By constructing appropriate Lyapunov-Krasovskii functional and employing some advanced methods, some novel stability criteria have been proposed in terms of LMIs, which can be easily solved

TABLE 1: Allowable upper bound  $\tau_2 = h = \bar{r}$  for different  $\delta = \tau_d = h_d$ .

Method	$\tau_1$	$\delta = \tau_d = h_d = 0.1$	$\delta = \tau_d = h_d = 0.3$	$\delta = \tau_d = h_d = 0.5$
Theorem 3	$\tau_1 = 0.1$	$\tau_2 = h = \bar{r} = 0.2200$	$\tau_2 = h = \bar{r} = 0.1904$	$\tau_2 = h = \bar{r} = 0.1428$
Theorem 3	$\tau_1 = 0.2$	$\tau_2 = h = \bar{r} = 0.2951$	$\tau_2 = h = \bar{r} = 0.2687$	$\tau_2 = h = \bar{r} = 0.2257$

TABLE 2: Allowable upper bound  $\tau_2$  for different  $\tau_d$ .

Methods	$\tau_1$	$\tau_d = 0.1$	$\tau_d = 0.5$	$\tau_d = 0.9$
[28]	$\tau_1 = 1$	$\tau_2 = 3.3068$	$\tau_2 = 2.5802$	$\tau_2 = 2.2736$
Corollary 9	$\tau_1 = 1$	$\tau_2 = 3.6720$	$\tau_2 = 2.6649$	$\tau_2 = 2.3779$
[28]	$\tau_1 = 2$	$\tau_2 = 3.3125$	$\tau_2 = 2.7500$	$\tau_2 = 2.6468$
Corollary 9	$\tau_1 = 2$	$\tau_2 = 3.7871$	$\tau_2 = 2.9085$	$\tau_2 = 2.8008$

by standard software. Two examples have been given to illustrate the effectiveness and merit of the proposed results. It should be mentioned that the leakage delay handling method proposed in this paper can also be used to investigate the systems with the delay in the leakage term, for example, the fault-tolerant control systems [47, 48].

## Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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