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Research Article

Lyapunov-Type Inequality for a Class of Discrete Systems with Antiperiodic Boundary Conditions

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A class of higher-order 3-dimensional discrete systems with antiperiodic boundary conditions is investigated. Based on the existence of the positive solution of linear homogeneous system, several new Lyapunov-type inequalities are established.

1. Introduction

Lyapunov-type inequalities have been proved to be very useful in oscillation theory, disconjugacy, eigenvalue problems, and numerous other applications in the theory of differential and difference equations [1–3]. In recent years, there are many literatures which improved and extended the classical Lyapunov inequality including continuous and discrete cases [4–6]. Guseinov and Kaymakçalan [7] considered the following discrete Hamiltonian system:

$$\begin{aligned} \Delta x(t) &= a(t)\, x(t+1) + b(t)\, u(t), \\ \Delta u(t) &= -c(t)\, x(t+1) - a(t)\, u(t), \end{aligned} \tag{1}$$

where Δ denotes the forward difference operator, with the coefficients $a(t)$ satisfying the condition $1 - a(t) \neq 0$, $t \in Z$. They [7] presented some Lyapunov-type inequalities for discrete linear scalar Hamiltonian systems when the coefficient $c(t)$ is not necessarily nonnegative value. Applying these inequalities, they [7] obtained some stability criteria for discrete Hamiltonian systems.

For simplicity, the following assumptions are introduced:

$$1 - \alpha(n) > 0, \quad \forall n \in Z, \tag{2}$$

$$\begin{aligned} &x(a) = 0, \quad \text{or} \quad x(a)\, x(a+1) < 0, \\ &x(b) = 0, \quad \text{or} \quad x(b)\, x(b+1) < 0, \\ &\max_{a \le n \le b} |x(n)| > 0, \quad a, b \in Z. \end{aligned} \tag{3}$$

Recently, Zhang and Tang [8] also considered the discrete linear Hamiltonian system:

$$\begin{aligned} \Delta x(n) &= \alpha(n)\, x(n+1) + \beta(n)\, y(n), \\ \Delta y(n) &= -\gamma(n)\, x(n+1) - \alpha(n)\, y(n), \end{aligned} \tag{4}$$

where $\alpha(n)$, $\beta(n)$, and $\gamma(n)$ are real-valued functions defined on Z and Δ denotes the forward difference operator defined by $\Delta x(n) = x(n+1) - x(n)$, $\beta(n) \ge 0$. They [8] obtained the following interesting Lyapunov-type inequality.

Theorem A. *Suppose that* (2) *holds, and let* $a, b \in Z$ *with* $a < b - 1$. *Assume* (4) *has a real solution* $(x(n), y(n))$ *such that* (3) *holds. Then one has the following inequality:*

$$\sum_{n=a}^{b-1} |\alpha(n)| + \left[\sum_{n=a}^{b} \beta(n) \sum_{n=a}^{b-1} \gamma^{+}(n)\right]^{1/2} \ge 2. \tag{5}$$

In 2012, the following assumptions are introduced in [9].

(H1) $r_1(n), r_2(n), f_1(n)$, *and* $f_2(n)$ *are real-valued functions, and* $r_1(n) > 0$, *and* $r_2(n) > 0$.

(H2) $1 < p_1, p_2 < \infty, \alpha_1, \alpha_2, \beta_1, \beta_2 > 0$ *satisfy* $\alpha_1/p_1 + \alpha_2/p_2 = 1$ *and* $\beta_1/p_1 + \beta_2/p_2 = 1$.

(H3) $r_i(n)$ *and* $f_i(n)$ *are real-valued functions and* $r_i(n) > 0$ *for* $i = 1, 2, \ldots, m$. *Furthermore,* $1 < p_i < \infty$ *and* $\alpha_i(n) > 0$ *satisfy* $\sum_{i=1}^{m} (\alpha_i/p_i) = 1$.

Under the boundary value conditions, Zhang and Tang [9] considered the following quasilinear difference systems with hypotheses (H1) and (H2):

$$\begin{aligned} &-\Delta\left(r_1(n)|\Delta u(n)|^{p_1-2}\Delta u(n)\right)\\ &\quad= f_1(n)|u(n+1)|^{\alpha_1-2}|v(n+1)|^{\alpha_2}u(n+1),\\ &-\Delta\left(r_2(n)|\Delta v(n)|^{p_1-2}\Delta v(n)\right)\\ &\quad= f_2(n)|u(n+1)|^{\beta_1}|v(n+1)|^{\beta_2-2}v(n+1),\end{aligned}\tag{6}$$

and the quasilinear difference systems involving the $(p_1, p_2, \ldots, p_m)$*-Laplacian:*

$$\begin{aligned} &-\Delta\left(r_1(n)\left|\Delta u_1(n)\right|^{p_1-2}\Delta u_1(n)\right)\\ &= f_1(n)\left|u_1(n+1)\right|^{\alpha_1-2}\\ &\quad\times\left|u_2(n+1)\right|^{\alpha_2}\cdots\left|u_m(n+1)\right|^{\alpha_m}u_1(n+1),\\ &\quad-\Delta\left(r_2(n)\left|\Delta u_2(n)\right|^{p_2-2}\Delta u_2(n)\right)\\ &= f_2(n)\left|u_1(n+1)\right|^{\alpha_1}\\ &\quad\times\left|u_2(n+1)\right|^{\alpha_2-2}\cdots\left|u_m(n+1)\right|^{\alpha_m}u_2(n+1),\\ &\vdots\\ &-\Delta\left(r_m(n)\left|\Delta u_m(n)\right|^{p_m-2}\Delta u_m(n)\right)\\ &\quad= f_m(n)\left|u_1(n+1)\right|^{\alpha_1}\\ &\qquad\times\left|u_2(n+1)\right|^{\alpha_2}\cdots\left|u_m(n+1)\right|^{\alpha_m-2}u_m(n+1).\end{aligned}\tag{7}$$

Some Lyapunov-type inequalities are established in [9].

Recently, antiperiodic problems have received considerable attention as antiperiodic boundary conditions appear in numerous situations [10–12]. For the sake of convenience, in this paper, one will only consider the following higher-order 3-dimensional discrete system:

$$\begin{aligned} &\left|\Delta^m x(n)\right|^{p_1-2}\Delta^m x(n)\\ &\quad+ f_1(n)\psi_{q_{1,1}}(x(n))\psi_{q_{1,2}}(y(n))\psi_{q_{1,3}}(z(n)) = 0,\\ &\left|\Delta^m y(n)\right|^{p_2-2}\Delta^m y(n)\\ &\quad+ f_2(n)\psi_{q_{2,1}}(x(n))\psi_{q_{2,2}}(y(n))\psi_{q_{2,3}}(z(n)) = 0,\\ &\left|\Delta^m z(n)\right|^{p_3-2}\Delta^m z(n)\\ &\quad+ f_3(n)\psi_{q_{3,1}}(x(n))\psi_{q_{3,2}}(y(n))\psi q_{3,3}(z(n)) = 0,\end{aligned}\tag{8}$$

where $1 < p_k < +\infty$ *for* $k = 1,2,3$; $q_{i,j}$ *are nonnegative constants for* $i, j = 1,2,3$; $\psi_q(u) = |u|^{q-1}u$ *for* $q > 0$ *with* $\psi_0(u) = \text{sign}(u) = \pm 1$ *for* $q = 0$.

Obviously, the results obtained in [9] required that $\alpha_1/p_1 + \alpha_2/p_2 = 1$ *and* $\beta_1/p_1 + \beta_2/p_2 = 1$ *or* $\sum_{i=1}^{m}(\alpha_i/p_i) = 1$. *The order of the quasilinear difference systems considered in [9] is less than 3. In this paper, one will remove the unreasonably severe constraints* $\alpha_1/p_1 + \alpha_2/p_2 = 1$ *and* $\beta_1/p_1 + \beta_2/p_2 = 1$ *or* $\sum_{i=1}^{m}(\alpha_i/p_i) = 1$ *in [9]. one will introduce the antiperiodic boundary conditions instead of boundary conditions in [9]. In this paper, one will establish some new Lyapunov-type inequalities for higher-order 3-dimensional discrete system* (8) *by a method different from that in [9] under the following antiperiodic boundary conditions:*

$$\begin{aligned} \Delta^i x(a) + \Delta^i x(b) &= \Delta^i y(a) + \Delta^i y(b)\\ &= \Delta^i z(a) + \Delta^i z(b) = 0,\\ &\qquad i = 0, 1, \ldots, m-1.\end{aligned}\tag{9}$$

The similar results for higher-order m-dimensional discrete system are easy to obtain.

Throughout this paper, $p_i > 1$ *and* p_i' *is a conjugate exponent; that is,* $1/p_i + 1/p_i' = 1, i = 1,2,3$.

2. Main Results

Theorem 1. *Let* $a < b$, *and assume that there exists a positive solution* (e_1, e_2, e_3) *of the following linear homogeneous system:*

$$\begin{aligned} \left(q_{1,1} + 1 - p_1\right)e_1 + q_{2,1}e_2 + q_{3,1}e_3 &= 0,\\ q_{1,2}e_1 + \left(q_{2,2} + 1 - p_2\right)e_2 + q_{3,2}e_3 &= 0,\\ q_{1,3}e_1 + q_{2,3}e_2 + \left(q_{3,3} + 1 - p_3\right)e_3 &= 0.\end{aligned}\tag{10}$$

If $(x(n), y(n), z(n))$ *is a nonzero solution of* (8) *satisfying the antiperiodic boundary conditions* (9), *then*

$$\begin{aligned} &\prod_{k=1}^{3}\left(\sum_{n=a}^{b-1}\left|f_k(n)\right|^{p_k/(p_k-1)}\right)^{(1-1/p_k)e_k}\\ &\quad\geq (b-a)^{\sum_{i=1}^{3}\sum_{j=1}^{3}(q_{i,j}/p_j)e_i}\left(\frac{2}{b-a}\right)^{m\sum_{i=1}^{3}(p_i-1)e_i}.\end{aligned}\tag{11}$$

Proof. Let $(x(n), y(n)$, and $z(n))$ be a nonzero solution of (8). By the antiperiodic boundary conditions (9), $x(a) + x(b) = 0$. For $n \in Z[a,b]$, we have

$$\begin{aligned} x(n) &= \frac{1}{2}\sum_{k=a}^{n-1}\left[x(k+1) - x(k)\right] - \frac{1}{2}\sum_{k=n}^{b-1}\left[x(k+1) - x(k)\right]\\ &= \frac{1}{2}\sum_{k=a}^{n-1}\Delta x(k) - \frac{1}{2}\sum_{k=n}^{b-1}\Delta x(k).\end{aligned}\tag{12}$$

Using discrete Hölder inequality gives

$$\begin{aligned} |x(n)| &\leq \frac{1}{2}\sum_{k=a}^{b-1}\left|\Delta x(k)\right|\\ &\leq \frac{1}{2}(b-a)^{1/p_1'}\left(\sum_{k=a}^{b-1}\left|\Delta x(k)\right|^{p_1}\right)^{1/p_1}.\end{aligned}\tag{13}$$

Similarly,

$$\begin{aligned}\left|\Delta^i x(n)\right| &\le \frac{1}{2}\sum_{k=a}^{b-1}\left|\Delta^{i+1}x(k)\right| \\ &\le \frac{1}{2}(b-a)^{1/p_1'}\left(\sum_{k=a}^{b-1}\left|\Delta^{i+1}x(k)\right|^{p_1}\right)^{1/p_1}.\end{aligned} \tag{14}$$

Then

$$\left|\Delta^i x(n)\right|^{p_1} \le \left(\frac{1}{2}\right)^{p_1}(b-a)^{p_1/p_1'}\left(\sum_{k=a}^{b-1}\left|\Delta^{i+1}x(k)\right|^{p_1}\right). \tag{15}$$

Summing (15) from a to $b-1$, we have

$$\begin{aligned}&\sum_{n=a}^{b-1}\left|\Delta^i x(n)\right|^{p_1} \\ &\quad\le (b-a)\left(\frac{1}{2}\right)^{p_1}(b-a)^{\frac{p_1}{p_1'}}\left(\sum_{k=a}^{b-1}\left|\Delta^{i+1}x(k)\right|^{p_1}\right);\end{aligned} \tag{16}$$

that is,

$$\left(\sum_{k=a}^{b-1}\left|\Delta^i x(k)\right|^{p_1}\right)^{1/p_1} \le \frac{b-a}{2}\left(\sum_{k=a}^{b-1}\left|\Delta^{i+1}x(k)\right|^{p_1}\right)^{1/p_1}. \tag{17}$$

So

$$\begin{aligned}|x(n)| &\le \frac{1}{2}(b-a)^{1/p_1'}\left(\sum_{k=a}^{b-1}|\Delta x(k)|^{p_1}\right)^{1/p_1} \\ &\le \frac{1}{2}(b-a)^{1/p_1'}\left(\frac{b-a}{2}\right)^{m-1}\left(\sum_{k=a}^{b-1}|\Delta^m x(k)|^{p_1}\right)^{1/p_1}.\end{aligned} \tag{18}$$

Similarly,

$$\left|y(n)\right| \le \frac{1}{2}(b-a)^{1/p_2'}\left(\frac{b-a}{2}\right)^{m-1}\left(\sum_{k=a}^{b-1}|\Delta^m y(k)|^{p_2}\right)^{1/p_2}, \tag{19}$$

$$|z(n)| \le \frac{1}{2}(b-a)^{1/p_3'}\left(\frac{b-a}{2}\right)^{m-1}\left(\sum_{k=a}^{b-1}|\Delta^m z(k)|^{p_3}\right)^{1/p_3}. \tag{20}$$

Multiplying the first equation of (8) by $\Delta^m x(n)$ and using inequalities (18)–(20), we have

$$\begin{aligned}&\left|\Delta^m x(n)\right|^{p_1} \\ &= \left|-f_1(n)\psi_{q_{1,1}}(x(n))\psi_{q_{1,2}}(y(n))\psi_{q_{1,3}}(z(n))\Delta^m x(n)\right| \\ &= \left|f_1(n)\right||x(n)|^{q_{1,1}}\left|y(n)\right|^{q_{1,2}}|z(n)|^{q_{1,3}}\left|\Delta^m x(n)\right| \\ &\le \left[\frac{1}{2}(b-a)^{1/p_1'}\left(\frac{b-a}{2}\right)^{m-1}\right. \\ &\qquad\left.\times\left(\sum_{k=a}^{b-1}|\Delta^m x(k)|^{p_1}\right)^{1/p_1}\right]^{q_{1,1}} \\ &\quad\times\left[\frac{1}{2}(b-a)^{1/p_2'}\left(\frac{b-a}{2}\right)^{m-1}\right. \\ &\qquad\left.\times\left(\sum_{k=a}^{b-1}|\Delta^m y(k)|^{p_2}\right)^{1/p_2}\right]^{q_{1,2}} \\ &\quad\times\left[\frac{1}{2}(b-a)^{1/p_3'}\left(\frac{b-a}{2}\right)^{m-1}\right. \\ &\qquad\left.\times\left(\sum_{k=a}^{b-1}|\Delta^m z(k)|^{p_3}\right)^{1/p_3}\right]^{q_{1,3}} \\ &\quad\times\left|f_1(n)\right|\left|\Delta^m x(n)\right|.\end{aligned} \tag{21}$$

Then

$$\begin{aligned}&\sum_{n=a}^{b-1}\left|\Delta^m x(n)\right|^{p_1} \\ &\le (b-a)^{-\sum_{j=1}^{3}(q_{1,j}/p_j)}\left(\frac{b-a}{2}\right)^{m(\sum_{j=1}^{3}q_{1,j})} \\ &\quad\times\left(\sum_{k=a}^{b-1}|\Delta^m x(k)|^{p_1}\right)^{q_{1,1}/p_1}\left(\sum_{k=a}^{b-1}|\Delta^m y(k)|^{p_2}\right)^{q_{1,2}/p_2} \\ &\quad\times\left(\sum_{k=a}^{b-1}|\Delta^m z(k)|^{p_3}\right)^{q_{1,3}/p_3}\sum_{n=a}^{b-1}\left|f_1(n)\right|\left|\Delta^m x(n)\right| \\ &\le (b-a)^{-\sum_{j=1}^{3}(q_{1,j}/p_j)}\left(\frac{b-a}{2}\right)^{m(\sum_{j=1}^{3}q_{1,j})} \\ &\quad\times\left(\sum_{k=a}^{b-1}|\Delta^m x(k)|^{p_1}\right)^{q_{1,1}/p_1}\left(\sum_{k=a}^{b-1}|\Delta^m y(k)|^{p_2}\right)^{q_{1,2}/p_2}\end{aligned}$$

$$\times\left(\sum_{k=a}^{b-1}\left|\Delta^m z(k)\right|^{p_3}\right)^{q_{1,3}/p_3}\left(\sum_{n=a}^{b-1}\left|f_1(n)\right|^{p_1'}\right)^{1/p_1'}\times\left(\sum_{n=a}^{b-1}\left|\Delta^m x(n)\right|^{p_1}\right)^{1/p_1}. \tag{22}$$

So

$$\left(\sum_{k=a}^{b-1}\left|\Delta^m x(k)\right|^{p_1}\right)^{(q_{1,1}+1)/p_1-1}\left(\sum_{k=a}^{b-1}\left|\Delta^m y(k)\right|^{p_2}\right)^{q_{1,2}/p_2}\times\left(\sum_{k=a}^{b-1}\left|\Delta^m z(k)\right|^{p_3}\right)^{q_{1,3}/p_3}\left(\sum_{n=a}^{b-1}\left|f_1(n)\right|^{p_1'}\right)^{1/p_1'}\geq(b-a)^{\sum_{j=1}^{3}(q_{1,j}/p_j)}\left(\frac{2}{b-a}\right)^{m(\sum_{j=1}^{3}q_{1,j})}. \tag{23}$$

For the second and third equations of (8), we also have

$$\left(\sum_{k=a}^{b-1}\left|\Delta^m x(k)\right|^{p_1}\right)^{q_{2,1}/p_1}\left(\sum_{k=a}^{b-1}\left|\Delta^m y(k)\right|^{p_2}\right)^{(q_{2,2}+1)/p_2-1}\times\left(\sum_{k=a}^{b-1}\left|\Delta^m z(k)\right|^{p_3}\right)^{q_{2,3}/p_3}\left(\sum_{n=a}^{b-1}\left|f_2(n)\right|^{p_2'}\right)^{1/p_2'}\geq(b-a)^{\sum_{j=1}^{3}(q_{2,j}/p_j)}\left(\frac{2}{b-a}\right)^{m(\sum_{j=1}^{3}q_{2,j})}, \tag{24}$$

$$\left(\sum_{k=a}^{b-1}\left|\Delta^m x(k)\right|^{p_1}\right)^{q_{3,1}/p_1}\left(\sum_{k=a}^{b-1}\left|\Delta^m y(k)\right|^{p_2}\right)^{q_{3,2}/p_2}\times\left(\sum_{k=a}^{b-1}\left|\Delta^m z(k)\right|^{p_3}\right)^{(q_{3,3}+1)/p_3-1}\left(\sum_{n=a}^{b-1}\left|f_3(n)\right|^{p_3'}\right)^{1/p_3'}\geq(b-a)^{\sum_{j=1}^{3}(q_{3,j}/p_j)}\left(\frac{2}{b-a}\right)^{m(\sum_{j=1}^{3}q_{3,j})}. \tag{25}$$

Raising both sides of inequalities (23)–(25) to the powers $e_1, e_2,$ and e_3, respectively, and multiplying the resulting inequalities give

$$\left(\sum_{k=a}^{b-1}\left|\Delta^m x(k)\right|^{p_1}\right)^{(\sum_{i=1}^{3}q_{i,1}e_i)/p_1+(1-p_1)e_1/p_1}\times\left(\sum_{k=a}^{b-1}\left|\Delta^m y(k)\right|^{p_2}\right)^{(\sum_{i=1}^{3}q_{i,2}e_i)/p_2+(1-p_2)e_2/p_2}\times\left(\sum_{k=a}^{b-1}\left|\Delta^m z(k)\right|^{p_3}\right)^{(\sum_{i=1}^{3}q_{i,3}e_i)/p_3+(1-p_3)e_3/p_3}\times\prod_{k=1}^{3}\left(\sum_{n=a}^{b-1}\left|f_k(n)\right|^{p_k'}\right)^{e_k/p_k'}\geq(b-a)^{\sum_{i=1}^{3}\sum_{j=1}^{3}((q_{i,j}/p_j)e_i)}\left(\frac{2}{b-a}\right)^{m(\sum_{i=1}^{3}\sum_{j=1}^{3}q_{i,j}e_i)}. \tag{26}$$

Since (e_1, e_2, e_3) is a positive solution of the linear homogeneous system (10), then

$$\prod_{k=1}^{3}\left(\sum_{n=a}^{b-1}\left|f_k(n)\right|^{p_k'}\right)^{e_k/p_k'}\geq(b-a)^{\sum_{i=1}^{3}\sum_{j=1}^{3}((q_{i,j}/p_j)e_i)}\left(\frac{2}{b-a}\right)^{m(\sum_{i=1}^{3}\sum_{j=1}^{3}q_{i,j}e_i)}. \tag{27}$$

Summing both sides of linear homogeneous system (10) yields

$$\sum_{i=1}^{3}\sum_{j=1}^{3}q_{i,j}e_i=\sum_{i=1}^{3}(p_i-1)e_i. \tag{28}$$

Noting that $1/p_k + 1/p_k' = 1, k = 1, 2, 3$, we have

$$\prod_{k=1}^{3}\left(\sum_{n=a}^{b-1}\left|f_k(n)\right|^{p_k/(p_k-1)}\right)^{(1-1/p_k)e_k}\geq(b-a)^{\sum_{i=1}^{3}\sum_{j=1}^{3}((q_{i,j}/p_j)e_i)}\left(\frac{2}{b-a}\right)^{m\sum_{i=1}^{3}((p_i-1)e_i)}. \tag{29}$$

□

Corollary 2. *Let $a < b$ and assume*

$$\begin{aligned}(q_{1,1}+1-p_1)+q_{2,1}+q_{3,1}&=0,\\ q_{1,2}+(q_{2,2}+1-p_2)+q_{3,2}&=0,\\ q_{1,3}+q_{2,3}+(q_{3,3}+1-p_3)&=0.\end{aligned} \tag{30}$$

If $(x(n), y(n), z(n))$ is a nonzero solution of (8) *satisfying the antiperiodic boundary conditions* (9)*, then*

$$\prod_{k=1}^{3}\left(\sum_{n=a}^{b-1}\left|f_k(n)\right|^{p_k/(p_k-1)}\right)^{(1-1/p_k)}\geq(b-a)^{\sum_{i=1}^{3}\sum_{j=1}^{3}(q_{i,j}/p_j)}\left(\frac{2}{b-a}\right)^{m\sum_{i=1}^{3}(p_i-1)}. \tag{31}$$

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