Hindawi Publishing Corporation
Journal of Applied Mathematics
Volume 2013, Article ID 396484, 6 pages
http://dx.doi.org/10.1155/2013/396484

Research Article

New Criteria for Meromorphic Multivalent Alpha-Convex Functions

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Received 9 March 2013; Revised 14 July 2013; Accepted 15 July 2013

Academic Editor: Zhihua Zhang

The aim of the present paper is to obtain sufficient condition for the class of meromorphic p-valent alpha convex functions of order ξ and then to study mapping properties of the newly defined integral operators. Many known results appeared as special consequences of our work.

1. Introduction

Let $\Sigma_p(n)$ denote the class of meromorphic functions $f(z)$ normalized by

$$f(z) = z^{-p} + \sum_{k=n}^{\infty} a_k z^{k-p+1}, \quad (p \in \mathbb{N}), \tag{1}$$

which are analytic and p-valent in the punctured unit disk $\mathbb{U}^* = \{z : 0 < |z| < 1\}$. In particular, $\Sigma_1(n) = \Sigma(n)$, $\Sigma_p(1) = \Sigma_p$, and $\Sigma_1(1) = \Sigma$. For λ which is real with $|\lambda| < \pi/2$, $\alpha \geq 0$, $0 \leq \xi < p$, and n, $p \in \mathbb{N}$, we denote by $\Sigma\mathcal{S}_p(\lambda, n, \xi)$, $\Sigma\mathcal{C}_p(\lambda, n, \xi)$, and $\Sigma\mathcal{M}_p(\lambda, n, \alpha, \xi)$, the subclasses of $\Sigma_p(n)$ consisting of all meromorphic p-valent functions of the form (1) which are defined, respectively, by

$$\begin{gathered} -\operatorname{Re} e^{i\lambda} \frac{zf'(z)}{f(z)} > \xi \cos\lambda, \quad (z \in \mathbb{U}^*), \\ -\operatorname{Re} e^{i\lambda} \frac{\left(zf'(z)\right)'}{f'(z)} > \xi \cos\lambda, \quad (z \in \mathbb{U}^*), \\ -\operatorname{Re} e^{i\lambda} \left\{(1-\alpha)\frac{zf'(z)}{f(z)} + \alpha \frac{\left(zf'(z)\right)'}{f'(z)}\right\} \\ > \xi\cos\lambda, \quad (z \in \mathbb{U}^*). \end{gathered} \tag{2}$$

Making $\lambda = 0$, $n = 1$ in (2), we get the well-known subclasses of Σ_p consisting of meromorphic p-valent functions which are starlike, convex, and alpha convex of order ξ $(0 \leq \xi < p)$, respectively. For details of the classes defined by (2) and related topics, we refer the raeder to the work of Aouf and Hossen [1], Aouf and Srivastava [2], Ali and Ravichandran [3], Goyal and Prajapat [4], Joshi and Srivastava [5], Liu and Srivastava [6], Raina and Srivastava [7], Xu and Yang [8], and Owa et al. [9].

For $f(z) \in \Sigma$, Wang et al. [10] and Nehari and Netanyahu [11] introduced and studied the subclass $\Sigma_N(\tau)$ of Σ consisting of functions $f(z)$ satisfying

$$-\operatorname{Re}\frac{\left(zf'(z)\right)'}{f'(z)} < \tau, \quad (\tau > 1, z \in \mathbb{U}^*). \tag{3}$$

We now extended this concept to define a subclass $\Sigma\mathcal{N}_p(\lambda, n, \alpha, \tau)$ of $\Sigma_p(n)$ consisting of functions $f(z)$ of the form (1) satisfying

$$\begin{gathered} -\operatorname{Re} e^{i\lambda}\left((1-\alpha)\frac{zf'(z)}{f(z)} + \alpha\frac{\left(zf'(z)\right)'}{f'(z)}\right) \\ < \tau\cos\lambda, \quad (\tau > p, z \in \mathbb{U}^*). \end{gathered} \tag{4}$$

For $\alpha = 0$ and $\alpha = 1$ in (4), we obtain the classes $\Sigma\mathcal{NC}_p(\lambda, n, \tau)$ and $\Sigma\mathcal{NS}_p(\lambda, n, \tau)$ of $\Sigma_p(n)$, respectively, studied by Arif [12]; also see [13, 14].

Integral operators for different classes of analytic, univalent, and multivalent functions in the open unit disk are studied by various authors; see [15–21]. We now define the following general integral operator of meromorphic p-valent functions:

$$G_m(z) = I_{m,p}\left(\delta, \alpha_j; f_j(z)\right) = \left\{\frac{\delta}{z^{p+1}}\int_0^z t^{\delta-1}\prod_{j=1}^{m}\left(t^p\left(f_j(t)\right)\right)^{\alpha_j}dt\right\}^{1/\delta}. \quad (5)$$

For $\delta = 1$, we obtain the integral operator $I_{m,p}(f_j(z))$ studied recently in [22, 23], and, further for $p = 1$, we obtain the integral operator introduced and studied by Mohammed and Darus [24].

Sufficient conditions were studied by various authors for different subclasses of analytic and multivalent functions; for some of the related work see [25–27]. The object of the present paper is to obtain sufficient conditions for the class $\Sigma\mathcal{M}_p(\lambda, n, \alpha, \xi)$ and then study mapping properties of the integral operator given by (5). We also consider some special cases of our results which lead to various interesting corollaries and relevance of some of these results with other known results which are also mentioned.

We will assume throughout our discussion, unless otherwise stated, that λ is real with $|\lambda| < \pi/2$, $0 \le \xi < p$, $\tau > p$, p, $n \in \mathbb{N}$, $\alpha_j > 0$ for $j \in \{1, \ldots m\}$, $\delta > 0$, $\alpha \ge 0$, and

$$J_\alpha(f) = (1-\alpha)\frac{zf'(z)}{f(z)} + \alpha\frac{\left(zf'(z)\right)'}{f'(z)}. \quad (6)$$

To obtain our main results, we need the following lemmas.

Lemma 1 (see [27]). *If $q(z) \in \Sigma(n)$ with $n \ge 1$ and satisfies the condition*

$$\left|z^2q'(z) + 1\right| < \frac{n}{\sqrt{n^2+1}} \quad (z \in \mathbb{U}^*), \quad (7)$$

then

$$q(z) \in \Sigma\mathcal{S}(n). \quad (8)$$

Lemma 2 (see [28]). *Let $\psi : \mathbb{C}^2 \to \mathbb{C}$ satisfy the following condition:*

$$\operatorname{Re}\{\psi(is, t)\} \le 0, \quad \left(s, t \in \mathbb{R}; t \le -\frac{|a+is|^2}{2}, \operatorname{Re}(a) > 0,\right). \quad (9)$$

If the function $h(z) = a + h_1z + h_2z^2 + \cdots$ is analytic in $\mathbb{U} = \mathbb{U}^* \setminus \{0\}$ and

$$\operatorname{Re}\left\{\psi\left(h(z), zh'(z)\right)\right\} > 0, \quad (z \in \mathbb{U}), \quad (10)$$

then

$$\operatorname{Re} h(z) > 0, \quad (z \in \mathbb{U}). \quad (11)$$

2. Sufficiency Criteria for the Class $\Sigma\mathcal{M}_p(\lambda,n,\alpha,\xi)$

In this section we establish a new sufficiency criteria for the subclass $\Sigma\mathcal{M}_p(\lambda, n, \alpha, \xi)$ of $\Sigma_p(n)$.

Theorem 3. *If $f(z) \in \Sigma_p(n)$ satisfies*

$$\left|\left\{\left(z^pf(z)\left(\frac{-zf'(z)}{pf(z)}\right)^\alpha\right)^{e^{i\lambda}/(p-\xi)\cos\lambda} e^{i\lambda}J_\alpha(f) + \xi\cos\lambda + ip\sin\lambda\right\} + (p-\xi)\cos\lambda\right| < \frac{n}{\sqrt{n^2+1}}(p-\xi)\cos\lambda \quad (z \in \mathbb{U}^*), \quad (12)$$

then $f(z) \in \Sigma\mathcal{M}_p(\lambda, n, \alpha, \xi)$, where $J_\alpha(f)$ is given by (6).

Proof. Let us set a function $q(z)$ by

$$q(z) = \frac{1}{z}\left(z^pf(z)\left(\frac{-zf'(z)}{pf(z)}\right)^\alpha\right)^{e^{i\lambda}/(p-\xi)\cos\lambda} = \frac{1}{z} + \frac{\alpha e^{i\lambda}a_nb_n}{(p-\xi)\cos\lambda}z^n + \cdots \quad (13)$$

for $f(z) \in \Sigma_p(n)$. Then clearly (13) shows that $q(z) \in \Sigma(n)$.

Logarithmic differentiating of (13) gives

$$\frac{q'(z)}{q(z)} = \frac{e^{i\lambda}}{(p-\xi)\cos\lambda} \times \left[(1-\alpha)\frac{f'(z)}{f(z)} + \alpha\frac{\left(zf'(z)\right)'}{zf'(z)} + \frac{p}{z}\right] - \frac{1}{z} \quad (14)$$

which further implies

$$\left|z^2q'(z) + 1\right| = \left|\left(z^pf(z)\left(\frac{-zf'(z)}{pf(z)}\right)^\alpha\right)^{e^{i\lambda}/(p-\xi)\cos\lambda}\frac{e^{i\lambda}}{(p-\xi)\cos\lambda} \times \left[J_\alpha(f) + \xi\cos\lambda + ip\sin\lambda\right] + 1\right|. \quad (15)$$

Thus using (12), we get

$$\left|z^2q'(z) + 1\right| \le \frac{n}{\sqrt{n^2+1}}, \quad (z \in \mathbb{U}^*). \quad (16)$$

Therefore by Lemma 1, we have $q(z) \in \Sigma\mathcal{S}(n)$.

From (14), we can write

$$\frac{zq'(z)}{q(z)} = \frac{1}{(p-\xi)\cos\lambda}\left[e^{i\lambda}J_\alpha(f) + \xi\cos\lambda + ip\sin\lambda\right]. \tag{17}$$

Since $q(z) \in \Sigma\mathcal{S}(n)$, it implies that $\mathrm{Re}(-zq'(z)/q(z)) > 0$. Therefore, we get

$$\begin{aligned}\frac{1}{(p-\xi)\cos\lambda}&\left[-\mathrm{Re}\, e^{i\lambda}J_\alpha(f) - \xi\cos\lambda\right]\\ &= \mathrm{Re}\left(-\frac{zq'(z)}{q(z)}\right) > 0\end{aligned} \tag{18}$$

or

$$-\mathrm{Re}\, e^{i\lambda}J_\alpha(f) > \xi\cos\lambda. \tag{19}$$

And therefore $f(z) \in \Sigma\mathcal{M}_p(\lambda, n, \alpha, \xi)$.

By taking $\alpha = 0$ and $\alpha = 1$ in Theorem 3, we obtain Corollaries 4 and 5, respectively, proved by Arif [12]. □

Corollary 4. *If $f(z) \in \Sigma_p(n)$ satisfies*

$$\begin{aligned}&\left|\left(z^p f(z)\right)^{e^{i\lambda}/(p-\xi)\cos\lambda}\left\{e^{i\lambda}\frac{zf'(z)}{f(z)} + \xi\cos\lambda + ip\sin\lambda\right\}\right.\\ &\qquad\left. + (p-\xi)\cos\lambda\right| < \frac{n(p-\xi)\cos\lambda}{\sqrt{n^2+1}} \quad (z \in \mathbb{U}^*),\end{aligned} \tag{20}$$

then $f(z) \in \Sigma\mathcal{S}_p(\lambda, n, \xi)$.

Corollary 5. *If $f(z) \in \Sigma_p(n)$ satisfies*

$$\begin{aligned}&\left|\left(\frac{z^{p+1}f'(z)}{-p}\right)^{e^{i\lambda}/(p-\xi)\cos\lambda}\right.\\ &\quad\left.\times\left\{e^{i\lambda}\left(\frac{zf''(z)}{f'(z)}+1\right)+\xi\cos\lambda+ip\sin\lambda\right\}+(p-\xi)\cos\lambda\right|\\ &\quad < \frac{n}{\sqrt{n^2+1}}(p-\xi)\cos\lambda, \quad (z \in \mathbb{U}^*),\end{aligned} \tag{21}$$

then $f(z) \in \Sigma\mathcal{C}_p(\lambda, n, \xi)$.

Remarks. We note that by simple computation (13) gives

$$f(z) = \left[-\frac{p}{\alpha}\int_0^z \frac{(tq(t))^{(p-\xi)\cos\lambda/\alpha e^{i\lambda}}}{t^{p/\alpha+1}}\right]^\alpha. \tag{22}$$

By taking suitable meromorphic starlike function for $q(z)$ in (22) such as $q(z) = (1-z^n)^2/z$, which satisfies the inequality of Lemma 1, we can conclude that the function f of (22) is the subclass of a meromorphic function.

3. Some Properties of the Integral Operator $G_m(z)$

In this section, we discuss some mapping properties of the integral operator $G_m(z)$.

Theorem 6. *For $j \in \{1, \ldots m\}$, let $f_j(z) \in \Sigma_p(n)$ and satisfy* (20). *If*

$$\sum_{j=1}^m \alpha_j < \frac{(p+1)(2-\delta)}{(p-\xi)}, \quad 0 < \delta < 2, \tag{23}$$

then $G_m(z) \in \Sigma\mathcal{N}_p(\lambda, n, \delta, \eta)$ with $\eta > p$, and $G_m(z)$ is given by (5).

Proof. From (5), we obtain

$$\begin{aligned}\delta z^{p+1}G_m^{\delta-1}(z)\,G_m'(z) &+ (p+1)z^pG_m^\delta(z)\\ &= \delta z^{\delta-1}\prod_{j=1}^m\left(z^pf_j(z)\right)^{\alpha_j}.\end{aligned} \tag{24}$$

Dividing both sides by $z^pG_m^{\delta-1}(z)$, we have

$$\begin{aligned}\delta zG_m'(z) &+ (p+1)G_m(z)\\ &= \delta z^{\delta-p-1}G_m^{1-\delta}(z)\prod_{j=1}^m\left(z^pf_j(z)\right)^{\alpha_j}.\end{aligned} \tag{25}$$

Differentiating again logarithmically, we have

$$\begin{aligned}&\frac{\delta zG_m''(z) + (p+\delta+1)G_m'(z)}{\delta zG_m'(z) + (p+1)G_m(z)}\\ &\qquad = (\delta-p-1)\frac{1}{z} + (1-\delta)\frac{G_m'(z)}{G_m(z)}\\ &\qquad\quad + \sum_{j=1}^m\alpha_j\left(\frac{f_j'(z)}{f_j(z)} + \frac{p}{z}\right).\end{aligned} \tag{26}$$

Now by simple computation, we get

$$\begin{aligned}&\left(1-\frac{1}{\delta}\right)\frac{zG_m'(z)}{G_m(z)} + \frac{1}{\delta}\frac{\left(zG_m'(z)\right)'}{G_m'(z)}\\ &\quad = \frac{1}{\delta}\sum_{j=1}^m\alpha_j\left(\frac{zf_j'(z)}{f_j(z)} + p\right)\\ &\qquad - \frac{1}{\delta}(2+2p-\delta) + \frac{G_m(z)}{zG_m'(z)}\\ &\qquad \times\left[(p+1)\sum_{j=1}^m\alpha_j\left(\frac{zf_j'(z)}{f_j(z)} + p\right) + (p+1)(\delta-p-1)\right],\end{aligned} \tag{27}$$

or equivalently we have

$$-e^{i\lambda}\left\{\left(1-\frac{1}{\delta}\right)\frac{zG_m'(z)}{G_m(z)}+\frac{1}{\delta}\frac{\left(zG_m'(z)\right)'}{G_m'(z)}\right\}$$
$$=\frac{1}{\delta}\sum_{j=1}^{m}\alpha_j\left(-e^{i\lambda}\frac{zf_j'(z)}{f_j(z)}-pe^{i\lambda}\right)$$
$$+\frac{1}{\delta}(2+2p-\delta)e^{i\lambda}+\frac{G_m(z)}{zG_m'(z)}$$
$$\times\left[(p+1-\delta)-\sum_{j=1}^{m}\alpha_j\left(\frac{zf_j'(z)}{f_j(z)}+p\right)\right](p+1)e^{i\lambda}. \quad (28)$$

By taking real part on both sides, we obtain

$$-\operatorname{Re}e^{i\lambda}\left\{\left(1-\frac{1}{\delta}\right)\frac{zG_m'(z)}{G_m(z)}+\frac{1}{\delta}\frac{\left(zG_m'(z)\right)'}{G_m'(z)}\right\}$$
$$=\frac{1}{\delta}\sum_{j=1}^{m}\alpha_j\left(-\operatorname{Re}e^{i\lambda}\frac{zf_j'(z)}{f_j(z)}-p\cos\lambda\right)$$
$$+\frac{1}{\delta}(2+2p-\delta)\cos\lambda+\operatorname{Re}\frac{G_m(z)}{zG_m'(z)}$$
$$\times\left[(p+1-\delta)-\sum_{j=1}^{m}\alpha_j\left(\frac{zf_j'(z)}{f_j(z)}+p\right)\right](p+1)e^{i\lambda}, \quad (29)$$

which further implies that

$$-\operatorname{Re}e^{i\lambda}\left\{\left(1-\frac{1}{\delta}\right)\frac{zG_m'(z)}{G_m(z)}+\frac{1}{\delta}\frac{\left(zG_m'(z)\right)'}{G_m'(z)}\right\}$$
$$\leq\frac{1}{\delta}\sum_{j=1}^{m}\alpha_j\left(-\operatorname{Re}e^{i\lambda}\frac{zf_j'(z)}{f_j(z)}-p\cos\lambda\right)$$
$$+\frac{1}{\delta}(2+2p-\delta)\cos\lambda$$
$$+\left|\frac{G_m(z)}{zG_m'(z)}\left[(p+1-\delta)-\sum_{j=1}^{m}\alpha_j\left(\frac{zf_j'(z)}{f_j(z)}+p\right)\right](p+1)e^{i\lambda}\right|. \quad (30)$$

Let

$$\eta=\left|\frac{G_m(z)}{zG_m'(z)}\left[(p+1-\delta)-\sum_{j=1}^{m}\alpha_j\left(\frac{zf_j'(z)}{f_j(z)}+p\right)\right](p+1)e^{i\lambda}\right|$$
$$+\frac{1}{\delta}(2+2p-\delta)$$
$$+\frac{1}{\delta}\sum_{j=1}^{m}\alpha_j\left(-\frac{1}{\cos\lambda}\operatorname{Re}e^{i\lambda}\frac{zf_j'(z)}{f_j(z)}-p\right). \quad (31)$$

Clearly we have

$$\eta>\frac{1}{\delta}(2+2p-\delta)$$
$$+\frac{1}{\delta\cos\lambda}\sum_{j=1}^{m}\alpha_j\left(-\operatorname{Re}e^{i\lambda}\frac{zf_j'(z)}{f_j(z)}-p\cos\lambda\right). \quad (32)$$

Then by using (23) and Theorem 3 with $\alpha=0$, we obtain

$$\eta>\frac{1}{\delta}\left[\sum_{j=1}^{m}\alpha_j(\xi-p)+(2+2p-\delta)\right]>p. \quad (33)$$

Therefore $G_m(z)\in\Sigma\mathcal{N}_p(\lambda,n,\delta,\eta)$ with $\eta>p$.

Making $\delta=1$ in Theorem 6, we have the following. □

Corollary 7. *For $j\in\{1,\dots m\}$, let $f_j(z)\in\Sigma_p(n)$ and satisfy (20). If*

$$\sum_{j=1}^{m}\alpha_j<\frac{(p+1)}{(p-\xi)}, \quad (34)$$

then $I_{m,p}(f_j(z))\in\Sigma\mathcal{NC}_p(\lambda,n,\eta)$ with $\eta>p$.

Theorem 8. *For $j\in\{1,\dots m\}$, let $f_j(z)\in\Sigma\mathcal{S}_p(0,n,\xi)$. If*

$$\sum_{j=1}^{m}\alpha_j<\frac{p+1-\delta}{p-\xi}, \quad (35)$$

then $G_m(z)\in\Sigma\mathcal{S}_p(0,n,\rho)$, where $0<\rho\leq p$ with $\rho=p+1-\delta+(\xi-p)\sum_{j=1}^{m}\alpha_j$.

Proof. From (26), we obtain

$$\frac{\delta\left(zG_m''(z)/G_m'(z)+1\right)+(p+1)}{\delta\left(zG_m'(z)/G_m(z)\right)+(p+1)}\frac{zG_m'(z)}{G_m(z)}$$
$$=(\delta-p-1)+(1-\delta)\frac{zG_m'(z)}{G_m(z)} \quad (36)$$
$$+\sum_{j=1}^{m}\alpha_j\left(\frac{zf_j'(z)}{f_j(z)}+p\right).$$

Let

$$-e^{i\lambda}\frac{zG_m'(z)}{G_m(z)} = q(z) = \left(\left(p-\rho\right)p(z)+\rho\right)\cos\lambda + i\sin\lambda, \tag{37}$$

such that $q(z)$ is analytic in $\mathbb{U}^*$ with $q(0) = p$. Then (36) can be written as

$$\begin{aligned} q(z) &+ \frac{zq'(z)e^{i\lambda}}{(p+1)e^{i\lambda}-\delta q(z)} \\ &= \frac{(p+1-\delta)}{\delta}e^{i\lambda} + \frac{1}{\delta}\sum_{j=1}^{m}\alpha_j\left(-e^{i\lambda}\frac{zf_j'(z)}{f_j(z)} - pe^{i\lambda}\right). \end{aligned} \tag{38}$$

Taking real part on both sides, we have

$$\begin{aligned} \operatorname{Re}&\left(q(z) + \frac{zq'(z)e^{i\lambda}}{(p+1)e^{i\lambda}-\delta q(z)}\right) \\ &= \frac{p+1-\delta}{\delta}\cos\lambda \\ &\quad + \frac{1}{\delta}\sum_{j=1}^{m}\alpha_j\left(-\operatorname{Re}e^{i\lambda}\frac{zf_j'(z)}{f_j(z)} - p\cos\lambda\right) \\ &> \frac{1}{\delta}\left(p+1-\delta-(p-\xi)\sum_{j=1}^{m}\alpha_j\right)\cos\lambda > 0, \end{aligned} \tag{39}$$

where we have used (35) and the assumption that $f_j(z) \in \Sigma\mathcal{S}_p(\lambda, n, \xi)$. Let us put

$$\psi(u,v) = u + \frac{ve^{i\lambda}}{(p+1)e^{i\lambda}-\delta u}. \tag{40}$$

Then, for $s, t \in \mathbb{R}$ such that $t \le -(p^2+s^2)/2$, we have

$$\begin{aligned} \operatorname{Re}\left(\psi(is,t)\right) &= \operatorname{Re}\left\{is + \frac{te^{i\lambda}}{(p+1)e^{i\lambda}-i\delta s}\right\} \\ &= -\frac{\left[(p+1)-\delta s\sin\lambda\right]\left(p^2+s^2\right)}{2\left[(p+1)^2\cos^2\lambda + \delta^2 s^2\left(1-(p+1)\sin\lambda\right)^2\right]} < 0. \end{aligned} \tag{41}$$

Thus using Lemma 2, we conclude that $\operatorname{Re}q(z) > 0$, which is equivalent to

$$-\operatorname{Re}e^{i\lambda}\frac{zG_m'(z)}{G_m(z)} > \rho\cos\lambda. \tag{42}$$

That is, $G_m(z) \in \Sigma\mathcal{S}_p(\lambda, n, \rho)$. □

Acknowledgment

The work presented here was partially supported by LRGS/TD/2011/UKM/ICT/03/02.

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