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Research Article

Heat Transfer Analysis on the Hiemenz Flow of a Non-Newtonian Fluid: A Homotopy Method Solution

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The mathematical model for the incompressible two-dimensional/axisymmetric non-Newtonian fluid flows and heat transfer analysis in the region of stagnation point over a stretching/shrinking sheet and axisymmetric shrinking sheet is presented. The governing equations are transformed into dimensionless nonlinear ordinary differential equations by similarity transformation. Analytical technique, namely, the homotopy perturbation method (HPM) with general form of linear operator is used to solve dimensionless nonlinear ordinary differential equations. The series solution is obtained without using the diagonal Padé approximants to handle the boundary condition at infinity which can be considered as a clear advantage of homotopy perturbation technique over the decomposition method. The effects of the pertinent parameters on the velocity and temperature field are discussed through graphs. To the best of authors' knowledge, HPM solution with general form of linear operator for two-dimensional/axisymmetric non-Newtonian fluid flows and heat transfer analysis in the region of stagnation point is presented for the first time in the literature.

1. Introduction

Stagnation point flow is of great importance in the prediction of skin friction as well as heat/mass transfer near stagnation regions of bodies in high speed flows and also in the design of thrust bearings and radial diffusers, drag reduction, transpiration cooling, and thermal oil. In 1911, Hiemenz [1] revealed that stagnation point flow can be examined by the Navier-Stokes (NS) equations. He used the similarity of the solution to reduce number of variables by means of a coordinate transformation. Later Howann [2] discovered the stagnation point flow in case of axisymmetric situation. Recently, a number of researchers studied the stagnation point flow considering different fluids models, geometries, and assumptions that were proposed in the literature. The literature on the topic is quite extensive and hence cannot be described here in detail. However some most recent works of eminent researchers regarding the analytical/numerical solution of stagnation point for different geometries may be mentioned in [3–5]. Attia [6], Massoudi and Ramezan [7], and Garg [8] extended the stagnation point flow for heat transfer.

The main aim of this paper is to extend the HPM [9–17] for solving non-Newtonian fluid flow and heat transfer analysis in the region of stagnation point flow towards a stretching/shrinking and axisymmetric shrinking sheet. Also the main motivation is to perform such analysis [3] (shrinking/axisymmetric shrinking sheet) for a non-Newtonian fluid in the presence of heat transfer. Heat transfer plays very important role in nuclear energy because nuclear chain reaction creates heat, and it is used to boil water, produce steam, and drive a steam turbine. The steady Navier-Stokes equations are reduced to the nonlinear ordinary differential equations by using similarity solutions. Graphical results explicitly reveal the complete reliability and efficiency of the suggested algorithm.

2. Governing Equations

The flow and heat characteristics are governed by the following equations [3]:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0, \tag{1}$$

$$\begin{aligned} u\frac{\partial u}{\partial x} + w\frac{\partial u}{\partial z} &= -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\frac{\partial^2 u}{\partial z^2} \\ &\quad + \frac{\alpha_1}{\rho}\left(u\frac{\partial^3 u}{\partial x \partial z^2} + \frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial z^2} + \frac{\partial u}{\partial z}\frac{\partial^2 u}{\partial x \partial z} + w\frac{\partial^3 u}{\partial z^3}\right), \end{aligned} \tag{2}$$

$$\rho c_p\left(u\frac{\partial T}{\partial x} + w\frac{\partial T}{\partial z}\right) = k\nabla^2 T. \tag{3}$$

The similarity transformations for two-dimensional stagnation flow case are as follows [3]:

$$\begin{gathered} \eta = \sqrt{\frac{a}{\nu}}z, \quad u = axf'(\eta) + bch(\eta), \quad v = 0, \\ w = -\sqrt{a\nu}f(\eta), \quad \theta = \frac{T - T_\infty}{T_0 - T_\infty}. \end{gathered} \tag{4}$$

The steady Navier-Stokes equations yield a system of nonlinear ordinary differential equations in the form

$$\begin{gathered} f''' + ff'' - f'^2 + 1 + \beta\left(2f'f''' + f''^2 - ff^{(IV)}\right) = 0, \\ h'' + fh' - f'h + \beta\left(hf''' + f'h'' + f''h' - fh'''\right) = 0, \\ \theta'' + \Pr f\theta' = 0, \end{gathered} \tag{5}$$

and corresponding boundary conditions take the form

$$\begin{gathered} f(0) = 0, \quad f'(0) = \frac{b}{a} = \alpha, \quad \theta(0) = 1, \\ f'(\infty) = 1, \quad h(0) = 1, \quad h(\infty) = 0, \quad \theta(\infty) = 0. \end{gathered} \tag{6}$$

The similarity transformations for axisymmetric stagnation flow towards an axisymmetric shrinking surface are as follows [3]:

$$\begin{gathered} \eta(x, y) = \sqrt{\frac{a}{\nu}}z, \quad u = axg'(\eta) + bcl(\eta), \\ v = ayg'(\eta), \quad w = -2\sqrt{a\nu}g(\eta), \quad \theta = \frac{T - T_\infty}{T_0 - T_\infty}. \end{gathered} \tag{7}$$

Upon making use of the above substitutions in (2) and (3), the resulting nonlinear system has the following form:

$$\begin{gathered} g''' + 2gg'' - g'^2 + 1 + \beta\left(2g'g''' + g''^2 - 2gg^{(IV)}\right) = 0, \\ l'' + 2gl' - g'l + \beta\left(\lg''' + g'l'' + g''l' - 2gl'''\right) = 0, \\ \theta'' + 2\Pr g\theta' = 0, \\ g(0) = 0, \quad g'(0) = \frac{b}{a} = \alpha, \quad \theta(0) = 1, \\ g'(\infty) = 1, \quad l(0) = 1, \quad l(\infty) = 0, \quad \theta(\infty) = 0. \end{gathered} \tag{8}$$

3. Analytical Solution

For the HPM [9] solution, we select

$$f_0(\eta) = (1 - \alpha)\left(e^{-\eta} - 1\right) + \eta, \qquad h_0(\eta) = e^{-\eta}, \tag{9}$$

$$\theta_0(\eta) = e^{-\eta}, \tag{10}$$

as initial approximations of f, h, and θ. We further choose the following auxiliary linear operators:

$$L_1 = \frac{\partial^3}{\partial^3\eta} + \frac{\partial^2}{\partial^2\eta}, \quad L_2 = \frac{\partial^2}{\partial^2\eta} + \frac{\partial}{\partial\eta}, \quad L_3 = \frac{\partial^2}{\partial^2\eta} + \frac{\partial}{\partial\eta}. \tag{11}$$

In view of the basic idea of the HPM [9], (5) is expressed as

$$\begin{gathered} (1 - p)L_1(f - f_0) \\ + p\left(f''' + ff'' - f'^2 + 1 + \beta\left(2f'f''' + f''^2 - ff^{(IV)}\right)\right) \\ = 0, \\ (1 - p)L_2(h - h_0) \\ + p\left(h'' + fh' - f'h + \beta\left(hf''' + f'h'' + f''h' - fh'''\right)\right) \\ = 0, \\ (1 - p)L_3(\theta - \theta_0) + p\left(\theta'' + \Pr f\theta'\right) = 0, \end{gathered} \tag{12}$$

$$\begin{gathered} f = f_0 + pf_1 + p^2 f_2 + \cdots, \\ h = h_0 + ph_1 + p^2 h_2 + \cdots, \\ \theta = \theta_0 + p\theta_1 + p^2\theta_2 + \cdots. \end{gathered} \tag{13}$$

Assuming $L_1 f = 0$, $L_2 h = 0$, and $L_3\theta = 0$ and substituting f, h, and θ from (13) into (12) and some simplification and rearrangement based on powers of p-terms, we have

$$\begin{gathered} p^1 : L_1 f_1 + f_0''' + f_0 f_0'' - f_0'^2 + 1 \\ + \beta\left(2f_0' f_0''' + f_0''^2 - f_0 f_0^{(IV)}\right) = 0, \\ f_1(0) = f_1'(0) = f_1'(\infty) = 0, \\ \vdots \\ p^j : L_1 f_j - L_1 f_{j-1} + f_{j-1}''' + \sum_{k=0}^{j-1} f_k f_{j-1-k}'' \\ - \sum_{k=0}^{j-1} f_k' f_{j-1-k}' + 1 \\ + \beta\left(2\sum_{k=0}^{j-1} f_k' f_{j-1-k}''' + \sum_{k=0}^{j-1} f_k'' f_{j-1-k}'' - \sum_{k=0}^{j-1} f_k f_{j-1-k}^{(IV)}\right) \\ = 0, \\ f_j(0) = f_j'(0) = f_j'(\infty) = 0, \quad j \geq 2; \end{gathered}$$

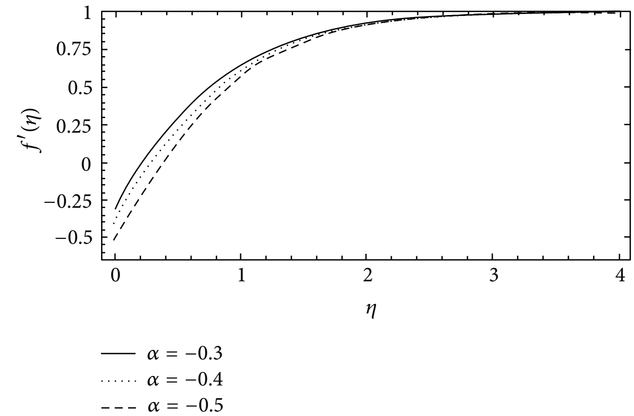

Figure 1: Effect of α on f' for two-dimensional case.

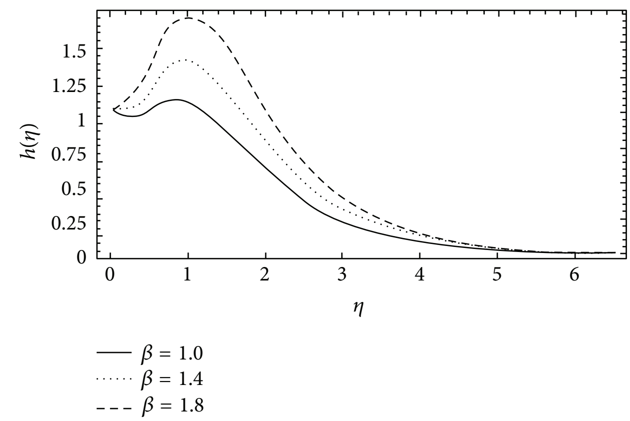

Figure 2: Effect of β on h for two-dimensional case.

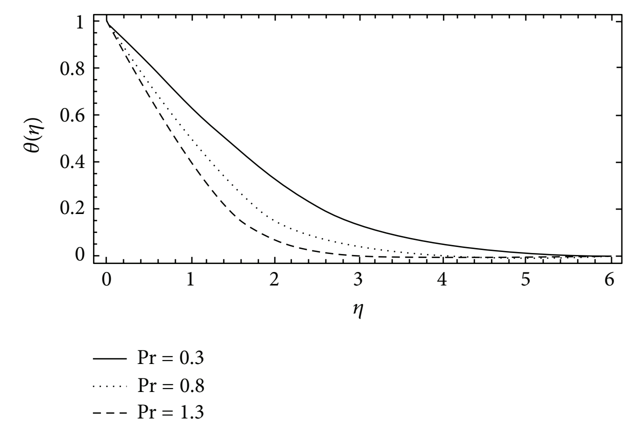

Figure 3: Effect of Pr on θ for two-dimensional case.

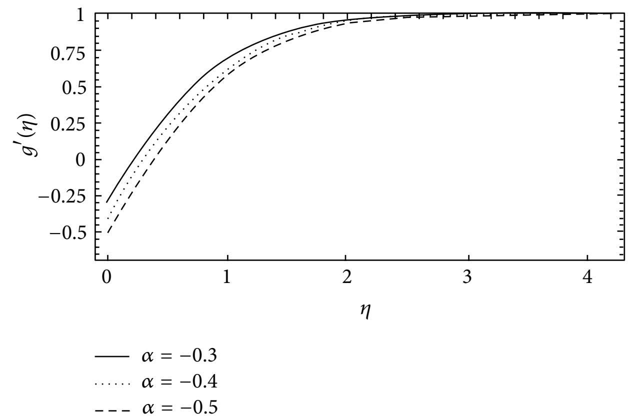

Figure 4: Effect of α on g' for axisymmetric case.

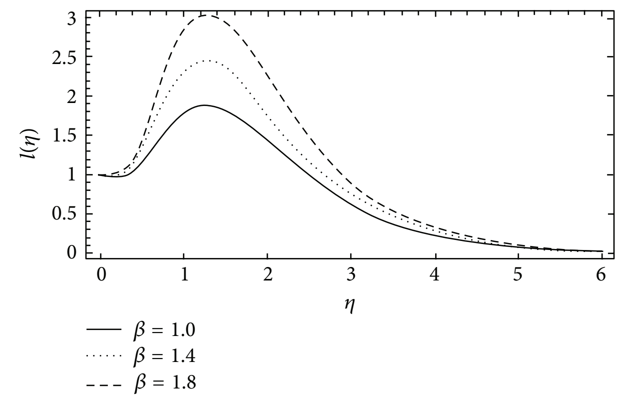

Figure 5: Effect of β on l for axisymmetric case.

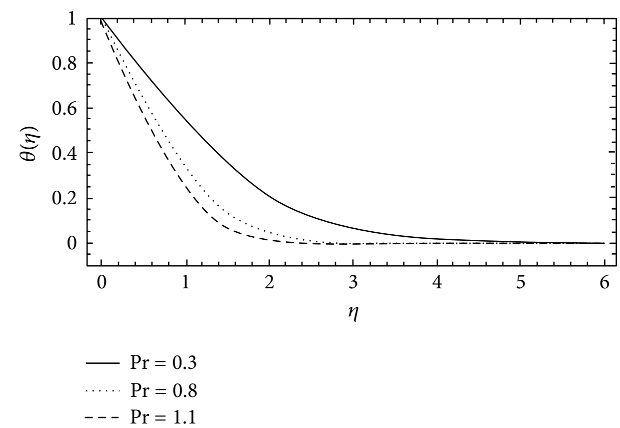

Figure 6: Effect of Pr on θ for axisymmetric case.

$$p^1: L_2 h_1'' + h_0''' + f_0 h_0' - f_0' h_0 + \beta\left(h_0 f_0''' + f_0' h_0'' + f_0'' h_0' - f_0 h_0'''\right) = 0,$$

$$h_1(0) = h_1(\infty) = 0,$$

$$\vdots$$

$$p^j: L_2 h_j - L_2 h_{j-1} + h_{j-1}'' + \sum_{k=0}^{j-1} f_k h_{j-1-k}' - \sum_{k=0}^{j-1} f_k' h_{j-1-k} + \beta\left(\sum_{k=0}^{j-1} h_k f_{j-1-k}''' + \sum_{k=0}^{j-1} f_k' h_{j-1-k}'' + \sum_{k=0}^{j-1} f_k'' h_{j-1-k}' - \sum_{k=0}^{j-1} f_k h_{j-1-k}'''\right) = 0,$$

$$h_j(0) = h_j(\infty) = 0, \quad j \geq 2;$$

$$p^1: L_3\theta_1'' + \theta_0'' + \Pr f_0 \theta_0' = 0,$$

$$\theta_1(0) = \theta_1(\infty) = 0,$$

$$\vdots$$

$$p^j: L_3\theta_j - L_3\theta_{j-1} + \theta_{j-1}'' + \Pr\sum_{k=0}^{j-1} f_k \theta_{j-1-k}' = 0,$$

$$\theta_j(0) = \theta_j(\infty) = 0, \quad j \geq 2. \tag{14}$$

On solving (14) in any software like Mathematica, Maple or MATLAB we can get any order of approximation.

Adopting the same procedure for axisymmetric stagnation flow towards an axisymmetric shrinking surface (8), we can get the required solution for (8)

$$\begin{aligned} g &= g_0 + g_1 + g_2 + \cdots, \\ l &= l_0 + l_1 + l_2 + \cdots, \\ \theta &= \theta_0 + \theta_1 + \theta_2 + \cdots. \end{aligned} \tag{15}$$

4. Concluding Remarks

In this paper, we have studied non-Newtonian Stagnation point flow in the presence of heat transfer by using HPM. The HPM is used in a direct way without using linearization, discretization, or restrictive assumption. The variations of various emerging parameters on the velocities (f', h, g', l) and temperature field (θ) are discussed through Figures 1, 2, 3, 4, 5, and 6. The main results of the present analysis are as follows:

(i) for two-dimensional case, the velocity f' decreases for shrinking parameter α while for axisymmetric shrinking surface, the velocity g' shows opposite behavior for α;

(ii) for two dimensional case and axisymmetric shrinking surface, the velocity profiles h and l increase with increasing value of β;

(iii) the effects of Prandtl number Pr are same on the temperature field for both cases.

Notations

ρ:	Density of fluid
v:	Kinematic viscosity
α_1:	Second grade parameter
T:	Temperature
α:	Stretching and shrinking parameter
k:	Thermal conductivity
c_p:	Specific heat
T_0 and T_∞:	The temperatures at and far away from the plate
Pr:	Prandtl number
β:	Dimensionless second grade parameter
f, g, h, l:	Dimensionless velocity profiles
θ:	Dimensionless temperature profile
u:	Velocity component in x direction
v:	Velocity component in y direction
w:	Velocity component in z direction
η:	Independent dimensionless parameter.

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