Hindawi Publishing Corporation
Abstract and Applied Analysis
Volume 2013, Article ID 270937, 5 pages
http://dx.doi.org/10.1155/2013/270937

Research Article

A Kind of Infinite-Dimensional Novikov Algebras and Its Realizations

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Received 24 May 2013; Accepted 16 July 2013

Academic Editor: T. Raja Sekhar

We construct a kind of infinite-dimensional Novikov algebras and give its realization by hyperbolic sine functions and hyperbolic cosine functions.

1. Introduction

Novikov algebras were introduced in connection with Hamiltonian operators in the formal variational calculus and the Poisson brackets of hydrodynamic type. They were used to construct the Virasoro-type Lie algebras. So the study of Novikov algebras is interesting in both mathematics and mathematical physics.

When Gel'fand and Diki [1, 2] and Gel'fand and Dorfman [3] studied the following operator:

$$H_{ij} = \sum_k c_{ijk} u_k^{(1)} + d_{ijk} u_k^{(0)} \frac{\mathrm{d}}{\mathrm{d}x}, \quad c_{ijk} \in \mathbb{C},\ d_{ijk} = c_{ijk} + c_{jik}, \tag{1}$$

they gave the definition of Novikov algebras. Concretely, let c_{ijk} be the structural coefficients, and let a product of $L = L(e_0, e_1, \ldots)$ be $\circ$ such that

$$e_i \circ e_j = \sum c_{ijk} e_k. \tag{2}$$

For any $a, b, c \in L$, the product is Hamilton operator if and only if $\circ$ satisfies

$$\begin{gathered} (a \circ b) \circ c = (a \circ c) \circ b, \\ (a \circ b) \circ c + c \circ (a \circ b) = (c \circ b) \circ a + a \circ (c \circ b). \end{gathered} \tag{3}$$

Ma presented many new soliton hierarchies of commuting bi-Hamiltonian evolution equations from the so-called Novikov algebras [4–6]. In 1987, Zel'manov [7] began to study Novikov algebras and proved that the dimension of finite-dimensional simple Novikov algebras over a field of characteristic zero is one. In algebras, what are paid attention to by mathematician are classifications and structures, but so far we have not got the systematic theory for general Novikov algebras. In 1992, Osborn [8–10] had finished the classification of infinite simple Novikov algebras with nilpotent elements over a field of characteristic zero and finite simple Novikov algebras with nilpotent elements over a field of characteristic $p > 0$. In 1995, Xu [10–13] developed his theory and got the classification of simple Novikov algebras over an algebraically closed field of characteristic zero. Bai and Meng [14–16] did a series of researches on low dimensional Novikov algebras, such as the structure and classification. We construct two kinds of Novikov algebras [17]. Recently, people obtained some properties in Novikov superalgebras [18, 19]. In this paper, we construct an infinite-dimensional Novikov algebra and give its realization by hyperbolic sine functions and hyperbolic cosine functions.

Definition 1 (see [17]). Let $(\mathscr{A}, \circ)$ be an algebra over $\mathbf{F}$ such that

$$a \circ (b \circ c) - (a \circ b) \circ c = b \circ (a \circ c) - (b \circ a) \circ c, \tag{4}$$

$$(a \circ b) \circ c = (a \circ c) \circ b, \quad \forall a, b, c \in \mathscr{A}, \tag{5}$$

and then $\mathscr{A}$ is called a Novikov algebra over $\mathbf{F}$.

Remark 2. An algebra $\mathscr{A}$ is called a left symmetric algebra if it only satisfies (4). It is clear that left symmetric algebras contain Novikov algebras.

Remark 3. (1) If $(\mathscr{A}, \circ)$ is a left symmetric algebra satisfying

$$[a, b] = a \circ b - b \circ a, \quad \forall a, b \in \mathscr{A}, \tag{6}$$

then $(\mathscr{A}, [,])$ is a Lie algebra. Usually, it is called an adjoining Lie algebra.

(2) Let $(\mathscr{A}, \cdot)$ be a commutative algebra, and then $(\mathscr{A}, d_0, \circ)$ is a Novikov algebra if d_0 is a derivation of $\mathscr{A}$ with a bilinear operator $\circ$ such that

$$a \circ b = a \cdot d_0(b), \quad \forall a, b \in \mathscr{A}. \tag{7}$$

2. Main Results

Lemma 4. *Let $\{b_0, a_1, b_1, a_2, b_2, \ldots a_n, b_n, \ldots\}$ be a basis of the linear space $\mathscr{A}$ over a field $\mathbf{F}$ of characteristic $p \neq 2$ satisfying*

$$\begin{aligned} a_m a_n &= \frac{1}{2}(b_{m+n} - b_{m-n}), \\ b_m b_n &= \frac{1}{2}(b_{m+n} + b_{m-n}), \\ a_m b_n = b_n a_m &= \frac{1}{2}(a_{m+n} + a_{m-n}), \end{aligned} \tag{8}$$

where $b_{-m} = b_m$, $a_{-m} = -a_m$. Then $\mathscr{A}$ is a commutative and associative algebra.

Proof. It is clear that $\mathscr{A}$ is a commutative algebra over $\mathbf{F}$:

$$\begin{aligned} &(a_k, a_n, a_m) \\ &= a_k(a_n a_m) - (a_k a_n) a_m \\ &= \frac{1}{2} a_k (b_{m+n} - b_{n-m}) - \frac{1}{2}(b_{k+n} - b_{k-n}) a_m \\ &= \frac{1}{4}(a_{k+m+n} + a_{k-m-n} - a_{k+n-m} - a_{k-n+m} \\ &\qquad - a_{m+k+n} - a_{m-k-n} + a_{m+k-n} + a_{m-k+n}) \\ &= 0. \end{aligned} \tag{9}$$

Similarly, we have that $(b_k, b_n, b_m) = (a_k, a_n, b_m) = (a_k, b_n, a_m) = (b_k, a_n, a_m) = (b_k, b_n, a_m) = (b_k, a_n, b_m) = (a_k, b_n, b_m) = 0$. Then $(a, b, c) = 0$, $\forall a, b, c \in \mathscr{A}$. The result follows. □

Corollary 5. *b_0 of Lemma 4 is a unity of $\mathscr{A}$.*

Lemma 6. *Let $\mathscr{A}$ be a commutative and associative algebra satisfying Lemma 4. Then the following statements hold:*

(1) *If D_0 is a linear transformation of $\mathscr{A}$ such that*

$$\begin{aligned} D_0(a_n) &= n b_n, \quad n = 1, 2, 3, \ldots, \\ D_0(b_n) &= n a_n, \quad n = 0, 1, 2, \ldots, \end{aligned} \tag{10}$$

then D_0 is a derivation of $\mathscr{A}$.

(2) *If aD_0 is a linear transformation of $\mathscr{A}$ such that*

$$(aD_0)(b) = aD_0(b), \quad \forall a, b \in \mathscr{A}, \tag{11}$$

then aD_0 is a derivation of $\mathscr{A}$.

(3) *$\mathscr{D}_1 = \{aD_0 \mid a \in \mathscr{A}\}$ is a subalgebra of Lie algebra $\mathrm{Der}\mathscr{A}$.*

Proof. (1) We have

$$\begin{aligned} D_0(a_n a_m) &= D_0\left(\frac{1}{2}(b_{n+m} - b_{n-m})\right) \\ &= \frac{1}{2}((m+n) a_{n+m} - (n-m) a_{n-m}), \\ D_0(a_n) a_m + a_n D_0(a_m) &= n b_n a_m + m a_n b_m \\ &= \frac{n}{2}(a_{n+m} - a_{n-m}) \\ &\quad + \frac{m}{2}(a_{n+m} - a_{m-n}) \\ &= \frac{1}{2}((m+n) a_{m+n} - (n-m) a_{n-m}). \end{aligned} \tag{12}$$

So D_0 is a derivation of $\mathscr{A}$.

(2) For $\forall a, b, c \in \mathscr{A}$, we have

$$\begin{aligned} (aD_0)(bc) &= aD_0(bc) = aD_0(b)c + abD_0(c) \\ &= (aD_0)(b)c + b(aD_0)(c), \end{aligned} \tag{13}$$

so aD_0 is a derivation of $\mathscr{A}$.

(3) For $\forall a, b, c \in \mathscr{A}$, we have

$$\begin{aligned} [aD_0, bD_0](c) &= (aD_0)(bD_0)(c) - (bD_0)(aD_0)(c) \\ &= aD_0(b) D_0(c) - bD_0(a) D_0(c) \\ &= (aD_0(b) - bD_0(a)) D_0(c). \end{aligned} \tag{14}$$

Then $[aD_0, bD_0] = (aD_0(b) - bD_0(a))D_0 \in \mathscr{D}_1$, and so (3) holds. □

Theorem 7. *Let $\mathscr{A}$ be a commutative and associative algebra satisfying Lemma 4, and let a be an element of $\mathscr{A}$. If D_0 satisfies Lemma 6 and $\circ$ satisfies*

$$b \circ c = baD_0(c), \quad \forall b, c \in \mathscr{A}, \tag{15}$$

then the following statements hold:

(1) *$(\mathscr{A}, aD_0, \circ)$ is a Novikov algebra.*

(2) *$(\mathscr{A}, aD_0, [,])$ is an adjoining Lie algebra of $(\mathscr{A}, aD_0, \circ)$ and $[,]$ such that*

$$[b, c] = a(bD_0(c) - cD_0(b)), \quad \forall b, c \in \mathscr{A}. \tag{16}$$

Proof. (1) By Lemma 6, aD_0 is a derivation of the commutative algebra $\mathscr{A}$. So $(\mathscr{A}, aD_0, \circ)$ is a Novikov algebra by Remark 3(2).

(2) $(\mathscr{A}, aD_0, [,])$ is an adjoining Lie algebra of $(\mathscr{A}, aD_0, \circ)$ by Remark 3(1). For $\forall b, c \in \mathscr{A}$, $\exists a \in \mathscr{A}$, we have

$$\begin{aligned} [b, c] &= b \circ c - c \circ b \\ &= baD_0(c) - caD_0(b) = a(bD_0(c) - cD_0(b)) \end{aligned} \tag{17}$$

since $\mathscr{A}$ is commutative. Hence we obtain the desired result. □

Let b_0 be a unity of $\mathscr{A}$. If we set $a = b_0$ in Theorem 7, then $a_n \circ a_m = a_n b_0 D_0(a_m) = a_n(mb_m) = (m/2)(a_{m+n} + a_{n-m})$. Similarly, we obtain the following corollary.

Corollary 8. *Let $\mathscr{A}$ be a commutative and associative algebra satisfying Lemma 4. Then the following statements hold:*

$$\begin{aligned}
a_n \circ a_m &= \frac{m}{2}(a_{n+m} + a_{n-m}), \\
b_n \circ b_m &= \frac{m}{2}(a_{n+m} + a_{m-n}), \\
a_n \circ b_m &= \frac{m}{2}(b_{n+m} - b_{n-m}), \\
b_n \circ a_m &= \frac{m}{2}(b_{n+m} + b_{n-m}), \\
[a_n, a_m] &= \frac{1}{2}(m-n)a_{n+m} + \frac{1}{2}(m+n)a_{n-m}, \\
[b_n, b_m] &= \frac{1}{2}(m-n)a_{n+m} - \frac{1}{2}(m+n)a_{n-m}, \\
[a_n, b_m] &= \frac{1}{2}(m-n)b_{n+m} - \frac{1}{2}(n+m)b_{n-m}, \\
[b_n, a_m] &= \frac{1}{2}(m-n)b_{n+m} + \frac{1}{2}(m+n)b_{n-m}.
\end{aligned} \tag{18}$$

We have the following: let $\sinh x = (e^x - e^{-x})/2$, $\cosh x = (e^x + e^{-x})/2$, and let the field $\mathbf{F}$ be assumed $\mathbf{R}$ or $\mathbf{C}$. We will construct Novikov algebras over the linear space which is generated by $\sinh x$ and $\cosh x$.

First, let $\mathscr{T}$ be a linear space generated by $\{\sinh mx, \cosh nx \mid m, n \in \mathbf{N}\}$ over $\mathbf{F}$.

Lemma 9. *$\mathscr{T}$ satisfying the above product is a commutative associative algebra.*

Proof. Since the above product is commutative and associative, we only need $\mathscr{T}$ to be closed for the product. In fact,

$$\begin{aligned}
\sinh mx \sinh nx &= \frac{1}{2}[\cosh(m+n)x - \cosh(m-n)x], \\
\cosh mx \cosh nx &= \frac{1}{2}[\cosh(m+n)x + \cosh(m-n)x], \\
\sinh mx \cosh nx &= \frac{1}{2}[\sinh(m+n)x + \sinh(m-n)x].
\end{aligned} \tag{19}$$

So $\mathscr{T}$ is a commutative and associative algebra. □

Lemma 10. *Let $\mathscr{T}$ be a linear space generated by $\{\sinh mx, \cosh nx \mid m, n \in \mathbf{N}\}$ over $\mathbf{F}$, and then $\{1, \sinh mx, \cosh nx \mid m, n \in \mathbf{N_0}\}$ is a basis of $\mathscr{T}$.*

Proof. For $\forall n \in \mathbf{N_0}$, suppose that there are $c_0, a_i, b_j \in \mathbf{F}$, $i, j \in \mathbf{N_0}$ such that

$$\begin{aligned}
&c_0 + a_1 \sinh x + b_1 \cosh x + \cdots + a_n \sinh nx + b_n \cosh nx \\
&\quad = 0.
\end{aligned} \tag{20}$$

We take derivative for (20) such that its derivative order is $2k-1$ ($k \in \mathbf{N_0}$), and put $x = 0$. Then we have

$$a_1 + 2^{2k-1}a_2 + \cdots + n^{2k-1}a_n = 0. \tag{21}$$

Let $k = 1, 2, \ldots, n$, and then we obtain the following system of n linear equations:

$$\begin{gathered}
a_1 + 2a_2 + \cdots + na_n = 0 \\
a_1 + 2^3a_2 + \cdots + n^3a_n = 0 \\
\vdots \\
a_1 + 2^{2n-1}a_2 + \cdots + n^{2n-1}a_n = 0.
\end{gathered} \tag{22}$$

If $a_1, \ldots, a_n$ are seen to be unknown, then the coefficient matrix of (22) is the Vandermonde matrix whose determinant is not 0, so $a_i = 0$, $i = 1, \ldots, n$.

We take derivative for (20) such that its derivative order is $2k$ ($k \in \mathbf{N_0}$), and put $x = 0$. Then we have

$$b_1 + 2^{2k}b_2 + \cdots + n^{2k}b_n = 0. \tag{23}$$

Let $k = 1, 2, \ldots, n$, and then we obtain the following system of n linear equations:

$$\begin{gathered}
b_1 + 2^2b_2 + \cdots + n^2b_n = 0 \\
b_1 + 2^4b_2 + \cdots + n^4b_n = 0 \\
\vdots \\
b_1 + 2^{2n}b_2 + \cdots + n^{2n}b_n = 0.
\end{gathered} \tag{24}$$

If $b_1, \ldots, b_n$ are seen to be unknown, then the coefficient matrix of (24) is the Vandermonde matrix whose determinant is not 0, so $b_i = 0$, $i = 1, \ldots, n$. Since, for any $i \in \mathbf{N_0}$, $a_i = 0$ and $b_i = 0$ satisfy (20), we have $c_0 = 0$. Hence $\{1, \sinh x, \cosh x, \ldots, \sinh nx, \cosh nx\}$ are linearly independent for any $n \in \mathbf{N_0}$, and then $\{1, \sinh nx, \cosh mx \mid n, m \in \mathbf{N_0}\}$ are linearly independent and so they form a basis of $\mathscr{T}$ as desired. □

Theorem 11. *Let $\mathscr{A}_1$, $\mathscr{A}_2$ be commutative and associative algebras over $\mathbf{F}$. If $\varphi: \mathscr{A}_1 \to \mathscr{A}_2$ is an isomorphism and $D_1 \in \mathrm{Der}\mathscr{A}_1$, then the following statements hold:*

(1) $D_2 := \varphi D_1 \varphi^{-1} \in \mathrm{Der}\mathscr{A}_2$,

(2) $\varphi: (\mathscr{A}_1, D_1, \circ) \to (\mathscr{A}_2, D_2, \circ)$ *is also an isomorphism of Novikov algebras.*

Proof. (1) For any $a, b \in \mathscr{A}_1$, we have

$$\begin{aligned}&\left(\varphi D_1 \varphi^{-1}\right)(\varphi(a)\varphi(b))\\&=\left(\varphi D_1\varphi^{-1}\right)(\varphi(ab))\\&=\varphi D_1(ab)=\varphi\left(D_1(a)b+aD_1(b)\right)\\&=\varphi\left(D_1(a)\right)\varphi(b)+\varphi(a)\varphi\left(D_1(b)\right)\\&=\left(\varphi D_1\varphi^{-1}\right)(\varphi(a))\varphi(b)+\varphi(a)\left(\varphi D_1\varphi^{-1}\right)(\varphi(b)).\end{aligned} \quad (25)$$

So (1) holds.

(2) For any $a, b \in \mathscr{A}_1$, we have

$$\begin{aligned}\varphi(a\circ b)&=\varphi\left(aD_1(b)\right)=\varphi(a)\varphi\left(D_1(b)\right)\\&=\varphi(a)\left(\varphi D_1\varphi^{-1}\right)(\varphi(b))=\varphi(a)D_2(\varphi(b))\\&=\varphi(a)\circ\varphi(b).\end{aligned} \quad (26)$$

So (2) holds. □

Theorem 12. *Let $\mathscr{A}$ be a commutative and associative algebra over* **F** *satisfying Lemma 4, let D_0 be its derivation satisfying* (10)*, and let $\mathscr{T}$ be a commutative and associative algebra over* **F** *satisfying Lemmas 9 and 10. If $\varphi : \mathscr{A} \rightarrow \mathscr{T}$ satisfies*

$$\begin{aligned}\varphi\left(b_m\right)&=\cosh mx, \quad m=0,1,2,\ldots,\\ \varphi\left(a_n\right)&=\sinh nx, \quad n=1,2,\ldots,\end{aligned} \quad (27)$$

then the following statements hold:

(1) *φ is an isomorphism of commutative and associative algebras,*

(2) $\varphi D_0 \varphi^{-1} = \mathrm{d}/\mathrm{d}x$,

(3) *$\varphi : (\mathscr{A}, aD_0, \circ) \rightarrow (\mathscr{T}, \varphi(a)(\mathrm{d}/\mathrm{d}x), \circ)$ is an isomorphism of Novikov algebras.*

Proof. It is clear by Lemma 10, (8), and (19).

(2) By Lemma 6, we have

$$\begin{aligned}\varphi D_0\varphi^{-1}(\sinh nx)&=\varphi D_0\left(a_n\right)\\&=\varphi\left(nb_n\right)=n\cosh nx\\&=\frac{\mathrm{d}\sinh nx}{\mathrm{d}x},\\ \varphi D_0\varphi^{-1}(\cosh nx)&=\varphi D_0\left(b_n\right)\\&=\varphi\left(na_n\right)=n\sinh nx\\&=\frac{\mathrm{d}\cosh nx}{\mathrm{d}x}.\end{aligned} \quad (28)$$

So (2) holds.

(3) It is clear that $\varphi(aD_0)\varphi^{-1} = \varphi(a)\mathrm{d}/\mathrm{d}x$. By (27) and (10), we have

$$\begin{aligned}\varphi\left(aD_0\right)\varphi^{-1}(\sinh nx)&=\varphi\left(aD_0\right)\left(a_n\right)\\&=\varphi\left(aD_0\left(a_n\right)\right)=\varphi\left(anb_n\right)\\&=\varphi(a)\varphi\left(nb_n\right)=\varphi(a)n\cosh nx\\&=\frac{\varphi(a)\,\mathrm{d}(\sinh nx)}{\mathrm{d}x}.\end{aligned} \quad (29)$$

Similarly, we have $\varphi(aD_0)\varphi^{-1}(\cosh nx) = \varphi(a)\mathrm{d}(\cosh nx)/\mathrm{d}x$. So $\varphi(aD_0)\varphi^{-1} = \varphi(a)\mathrm{d}/\mathrm{d}x$.

By Theorems 7 and 11 and Remark 3(2), we have

$$\begin{aligned}\varphi(b\circ c)&=\varphi\left(baD_0(c)\right)\\&=\varphi(b)\varphi\left(aD_0(c)\right)\\&=\varphi(b)\left[\varphi\left(aD_0\right)\varphi^{-1}(\varphi(c))\right]\\&=\frac{\varphi(b)\varphi(a)\,\mathrm{d}}{\mathrm{d}x(\varphi(c))}\\&=\varphi(b)\circ\varphi(c), \quad \forall b,c\in\mathscr{A}.\end{aligned} \quad (30)$$

So $\varphi : (\mathscr{A}_0, aD_0, \circ) \rightarrow (\mathscr{T}, \varphi(a)(\mathrm{d}/\mathrm{d}x), \circ)$ is an isomorphism of Novikov algebras. □

Acknowledgments

The authors would like to thank the referee for valuable comments and suggestions on this paper. This paper supported by NNSF of China (no. 11171055), NSF of Jilin province (No. 201115006), Scientific Research Foundation for Returned Scholars Ministry of Education of China, and the Fundamental Research Funds for the Central Universities.

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