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Research Article

LMI-Based Stability Criterion of Impulsive T-S Fuzzy Dynamic Equations via Fixed Point Theory

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By formulating a contraction mapping and the matrix exponential function, the authors apply linear matrix inequality (LMI) technique to investigate and obtain the LMI-based stability criterion of a class of time-delay Takagi-Sugeno (T-S) fuzzy differential equations. To the best of our knowledge, it is the first time to obtain the LMI-based stability criterion derived by a fixed point theory. It is worth mentioning that LMI methods have high efficiency and other advantages in largescale engineering calculations. And the feasibility of LMI-based stability criterion can efficiently be computed and confirmed by computer Matlab LMI toolbox. At the end of this paper, a numerical example is presented to illustrate the effectiveness of the proposed methods.

1. Introduction

In this paper, we consider a class of delayed impulsive differential equations, which admits some biomathematics, physics, and engineering backgrounds, including the famous cellular neural networks proposed by Chua and Yang in 1988 [1, 2]. In practice, both time delays and impulse are unavoidable and may cause undesirable dynamic network behaviors such as oscillation and instability. So the stability analysis for delayed impulsive neural networks has become a topic of great theoretic and practical importance in recent years [3–25]. However, the research skill of the above literature is mainly based on Lyapunov theory. And there are many difficulties in applications of corresponding theory to the specific problems [26–32]. Recently, Burton and other authors have applied fixed point theory to investigate the stability of deterministic systems and obtained some more applicable results [11, 26–42]. For example, in [11], the authors used Leray-Schauders fixed point theorem to obtain the stability criteria of neural networks. Besides, the contraction-mapping theory is also an important fixed point theory in studying the stability of dynamics equations (see, e.g., [11, 33, 39–42]). On the other hand, fuzzy logic theory has shown to be an appealing and efficient approach to dealing with the analysis and synthesis problems for complex nonlinear system [20–25]. In practice, the fuzzy model is far more important than stochastic model. Among various kinds of fuzzy methods, Takagi-Sugeno (T-S) fuzzy models provide a successful method to describe certain complex nonlinear systems using some local linear subsystems. To the best of our knowledge, few authors have used the fixed point theorem to study the stability of Takagi-Sugeno fuzzy differential equations with impulses. In addition, the LMI-based stability criterion of neural networks has never been investigated or obtained via any of the fixed point theories. Such a situation motivates our present study. Motivated by the above related literature [3–9, 11, 26–42], we will not only apply the fixed point theory to study the impulsive Takagi-Sugeno fuzzy dynamics equations but also try to obtain the LMI-based stability criterion by applying the contraction-mapping theory. To the best of our knowledge, it is the first time to obtain the LMI-based stability criterion derived by a fixed point theory. It is worth mentioning that LMI methods have high efficiency and other advantages in large-scale engineering calculations. And the feasibility of LMI-based stability criterion can efficiently be computed and confirmed by computer Matlab LMI toolbox. In the end of this paper, a numerical example is presented to illustrate the effectiveness of the proposed methods. Finally, a conclusion is given in the final chapter.

2. Preliminaries

Let us consider the following delayed differential equations:

$$\frac{dx(t)}{dt} = -Bx(t) + Cf(x(t)) + Dg(x(t-\tau(t))), \quad t \geqslant 0,\ t \neq t_k, \tag{1}$$

equipped with the impulsive condition

$$x(t_k^+) - x(t_k) = \rho(x(t_k)), \quad k = 1, 2, \ldots, \tag{2}$$

and the initial condition

$$x(\theta) = \phi(\theta), \quad \theta \in [-\tau, 0], \tag{3}$$

where $x(t) = (x_1(t), x_2(t), \ldots, x_n(t))^T \in R^n$, $\phi(\theta) \in C[[-\tau, 0], R^n]$. Functions $f(x) = (f_1(x_1(t)), f_2(x_2(t)), \ldots, f_n(x_n(t)))^T \in R^n$, $g(x(t-\tau(t))) = (g_1(x_1(t-\tau(t))), g_2(x_2(t-\tau(t))), \ldots, g_n(x_n(t-\tau(t))))^T \in R^n$, $\rho(x(t)) = (\rho_1(x_1(t)), \rho_2(x_2(t)), \ldots, \rho_n(x_n(t)))^T \in R^n$, and time delays $0 \leqslant \tau(t) \leqslant \tau$ for all $i = 1, 2, \ldots, n$. The fixed impulsive moments t_k $(k = 1, 2, \ldots)$ satisfy $0 < t_1 < t_2 < \cdots$ with $\lim_{k\to\infty} t_k = \infty$. $x(t_k^+)$, and $x(t_k^-)$ stand for the right-hand and left-hand limits of $x(t)$ at time t_k, respectively. We always assume $x(t_k^-) = x(t_k)$, for all $k = 1, 2, \ldots$. Similarly as in [33], we assume in this paper that $f(0) = g(0) = \rho(0) = 0 \in R^n$.

Constant matrix $B = \text{diag}(b_1, b_2, \ldots, b_n)$ is a positive definite diagonal matrix, and both $C = (c_{ij})_{n\times n}$ and $D = (d_{ij})_{n\times n}$ are matrices with $n \times n$ dimension. It is well known that the above equation admits its practical implications. For example, it can serve as a model of impulsive cellular neural networks with time-varying delays. The parameter c_{ij} denotes the connection weight of the jth neuron on the ith neuron at time t. And the parameter d_{ij} represents the connection strength of the jth neuron on the ith neuron at time $t - \tau(t)$. The constant b_i represents the rate with which the ith neuron will reset its potential to the resting state when disconnected from the network and external inputs. $f_j(x_j(t))$ is the activation function of the jth neuron at time t, and $g_j(x_j(t-\tau(t)))$ represents the activation function of the jth neuron at time $t - \tau(t)$.

Below, we describe the T-S fuzzy mathematical model with time delay as follows.

Fuzzy Rule j:

IF $\omega_1(t)$ is μ_{j1} and $\ldots \omega_s(t)$ is μ_{js}, THEN

$$\begin{aligned} \frac{dx(t)}{dt} &= -Bx(t) + C_j f(x(t)) + D_j g(x(t-\tau(t))), \\ &\qquad t \geqslant 0,\ t \neq t_k,\ k = 1, 2, \ldots \\ x(t_k^+) - x(t_k) &= \rho(x(t_k)), \quad k = 1, 2, \ldots \\ x(\theta) &= \phi(\theta), \quad \theta \in [-\tau, 0], \end{aligned} \tag{4}$$

where $\omega_k(t)$ $(k = 1, 2, \ldots, s)$ is the premise variable, μ_{jk} $(j = 1, 2, \ldots, r; k = 1, 2, \ldots, s)$ is the fuzzy set that is characterized by membership function, r is the number of the IF-THEN rules, and s is the number of the premise variables. By the way of a standard fuzzy inference method, the system (4) is inferred as follows:

$$\begin{aligned} \frac{dx(t)}{dt} &= -Bx(t) + \sum_{j=1}^{r} h_j(\omega(t)) \left[C_j f(x(t)) + D_j g(x(t-\tau(t)))\right], \\ &\qquad t \geqslant 0,\ t \neq t_k,\ k = 1, 2, \ldots, \\ x(t_k^+) - x(t_k) &= \rho(x(t_k)), \quad k = 1, 2, \ldots, \\ x(\theta) &= \phi(\theta), \quad \theta \in [-\tau, 0], \end{aligned} \tag{5}$$

where $\omega(t) = [\omega_1(t), \omega_2(t), \ldots, \omega_s(t)]$, $h_j(\omega(t)) = w_j / \sum_{k=1}^{r} wk(w(t)), w_j(\omega(t)) : R^s \to [0, 1]$ $(j = 1, 2, \ldots, r)$ is the membership function of the system with respect to the fuzzy rule j. h_j can be regarded as the normalized weight of each IF-THEN rule, satisfying $h_j(\omega(t)) \geqslant 0$ and $\sum_{j=1}^{r} h_j(\omega(t)) = 1$.

For convenience's sake, we introduce the following standard notations similarly as [10, (iii)–(X)]:

$$\begin{gathered} Q = (q_{ij})_{n\times n} > 0 \ (< 0), \\ Q = (q_{ij})_{n\times n} \geqslant 0 \ (\leqslant 0), \\ Q_1 \geqslant Q_2 \ (Q_1 \leqslant Q_2), \\ Q_1 > Q_2 \ (Q_1 < Q_2), \\ \lambda_{\min}\Phi, \text{the identity matrix } I \text{ and the symmetric terms } *. \end{gathered} \tag{6}$$

In addition, we denote $|C| = (|c_{ij}|)_{n\times n}$ for any matrix $C = (c_{ij})_{n\times n}$ and $|v| = (|v_1|, |v_2|, \ldots, |v_n|)^T$ for any $v = (v_1, v_2, \ldots, v_n)^T \in R^n$. Denote the finite set $\mathcal{N} = \{1, 2, \ldots, n\}$.

Throughout this paper, we assume

(A1) f_j is locally Lipschitz continuous, and there exists a positive constant $F_j > 0$ such that $|f_j'(r)| \leqslant F_j$ for all $r \in R$ at which f_j is differentiable;

(A2) g_j is locally Lipschitz continuous, and there exists a positive constant $G_j > 0$ such that $|g_j'(r)| \leqslant G_j$ for all $r \in R$ at which g_j is differentiable;

(A3) ρ_j is locally Lipschitz continuous, and there exists a positive constant $H_j > 0$ such that $|\rho_j'(r)| \leqslant H_j$ for all $r \in R$ at which ρ_j is differentiable.

Lemma 1 (see [25]). *Let $f : R^n \to R^n$ be locally Lipschitz continuous. For any given $x, y \in R^n$, there exists an element $\mathfrak{w}$ in the union $\cup_{z\in[x,y]}\partial f(z)$ such that*

$$f(y) - f(x) = \mathfrak{w}(y - x), \tag{7}$$

where $[x, y]$ denotes the segment connecting x and y.

Remark 2. From Lemma 1, (A1)–(A3), and [10, equation (27)], we can similarly derive

$$\begin{aligned} |f(x)-f(y)| &\leqslant F|x-y|, \quad x,y\in R^n, \\ |g(x)-g(y)| &\leqslant G|x-y|, \quad x,y\in R^n, \\ |\rho(x)-\rho(y)| &\leqslant H|x-y|, \quad x,y\in R^n, \end{aligned} \tag{8}$$

where matrices $F = \operatorname{diag}(F_1, F_2, \ldots, F_n)$, $G = \operatorname{diag}(G_1, G_2, \ldots, G_n)$, and $H = \operatorname{diag}(H_1, H_2, \ldots, H_n)$.

Remark 3. In many previous literature, f_i, g_i $(i \in \mathcal{N})$ are always assumed to be globally Lipschtiz continuous. However, the most common function $f_i(r) = r^2$ is not globally Lipschtiz continuous in R^1. Note that we extend the functions f, g from global Lipschtiz continuous functions to locally Lipschtiz continuous functions. Obviously, $f_i(r) = r^2$ is a local Lipschitz continuous function.

Similarly as is [11, Definition 2.2], the exponential stability is defined as follows.

Definition 4. Dynamic equation (5) is said to be exponentially stable if, for any initial condition $\phi(s) \in C[[-\tau, 0], R^n]$, there exists a pair of positive constants a and b such that

$$\|x(t;s,\phi)\| \leqslant be^{-at}, \quad \forall t > 0, \tag{9}$$

where the norm $\|x(t)\| = (\sum_{i=1}^{n} |x_i(t)|^2)^{1/2}$.

3. Main Result

Before giving the main result of this paper, we need to define the matrix exponential function as follows.

Definition 5. For a diagonal constants matrix $B = \operatorname{diag}(b_1, b_2, \ldots, b_n)$, we denote the matrix exponential function $e^{Bt} = \operatorname{diag}(e^{b_1 t}, e^{b_2 t}, \ldots, e^{b_n t})$ for all $t \in R$.

From the above definition of the matrix exponential function, we are not difficult to obtain the following lemma.

Lemma 6. *Let B be a diagonal constants matrix, and let e^{Bt} be the matrix exponential function of B. Then, we have*

(1) $(d/dt)e^{Bt} = Be^{Bt}$, $t \in R$,

(2) $(d/dt)(e^{Bt}\eta) = Be^{Bt}\eta$, $t \in R$,

where $\eta = (\eta_1, \eta_2, \ldots, \eta_n)^T \in R^n$, and each $\eta_i \in R$ $(i = 1, 2, \ldots, n)$ is a constant.

In addition, we need to define the rule on vectors in R^n as follows.

Definition 7. $v \leqslant w$ if $v_i - w_i \leqslant 0$ for all $i \in \mathcal{N}$, where $v = (v_1, v_2, \ldots, v_n)^T \in R^n$, $w = (w_1, w_2, \ldots, w_n)^T \in R^n$.

Now, we present the main result of this paper as follows.

Theorem 8. *Assume that there exists a positive constant δ such that $\inf_{k=1,2,\ldots}(t_{k+1} - t_k) \geqslant \delta$. In addition, there exists a constant $0 < \lambda < 1$ such that*

$$B^{-1}\sum_{j=1}^{r}\left(\left|C_j\right|F + \left|D_j\right|G\right) + \frac{1}{\delta}B^{-1}H + H < \lambda I, \tag{10}$$

where B^{-1} is the inverse matrix of B. Then, the impulsive fuzzy dynamic equation (5) is exponentially stable in the mean square.

Proof. To apply the fixed point theory, we firstly define a complete metric space Ω as follows.

Let Ω be the space consisting of functions $q(t) : [-\tau, \infty) \to R^n$, satisfying that

(a) $q(t)$ is continuous on $t \neq t_k$ $(k = 1, 2, \ldots)$,

(b) $\lim_{t\to t_k^-} q(t)$ and $\lim_{t\to t_k^+} q(t)$ exist, and $q(t_k^-) = q(t_k)$ for all $k = 1, 2, \ldots$,

(c) $q(t) = \phi(t)$ for $t \in [-\tau, 0]$,

(d) $e^{\beta t}q(t) \to 0 \in R^n$ as $t \to \infty$, where $\beta > 0$ is a positive constant, satisfying $\beta < \lambda_{\min} B$.

It is not difficult to verify that the above-mentioned space Ω is a complete metric space if it is equipped with the following metric:

$$\operatorname{dist}(\overline{q}, \widetilde{q}) = \max_{i=1,2,\ldots,n}\left(\sup_{t\geqslant-\tau}\left|\overline{q}_i(t) - \widetilde{q}_i(t)\right|\right), \tag{11}$$

where $\overline{q} = \overline{q}(t) = (\overline{q}_1(t), \overline{q}_2(t), \ldots, \overline{q}_n(t))^T \in \Omega$, and $\widetilde{q} = \widetilde{q}(t) = (\widetilde{q}_1(t), \widetilde{q}_2(t), \ldots, \widetilde{q}_n(t))^T \in \Omega$.

Remark 9. Here, we consider the above-defined metric, which is different from those of some previous related literature (see, e.g., [33]) so that the LMI-based stability criterion in this paper may be obtained expediently.

Next, we formulate and define a contraction mapping $P : \Omega \to \Omega$, which may be divided into three steps.

Step 1. Formulating the mapping.

Let $x(t)$ be a solution of the fuzzy equation (5).

Then, for $t \geqslant 0$, $t \neq t_k$, we have

$$\begin{aligned} \frac{dx(t)}{dt}\left(e^{Bt}x(t)\right) &= Be^{Bt}x(t) + e^{Bt}\frac{dx(t)}{dt} \\ &= e^{Bt}\sum_{j=1}^{r}h_j(\omega(t))\left[C_j f(x(t)) \right. \\ &\qquad \left. + D_j g(x(t-\tau(t)))\right]. \end{aligned} \tag{12}$$

Further, we get by the integral nature

$$x(t) = e^{-Bt}\left\{\int_0^t e^{Bs}\sum_{j=1}^r h_j(\omega(s))\left[C_j f(x(s)) + D_j g(x(s-\tau(s)))\right]ds + \eta\right\}, \quad t \geqslant 0, \tag{13}$$

where $\eta \in R^n$ is the vector to be determined.

From (2) and (3) or $x(0) = \phi(0)$, it is not difficult to conclude $\eta = \phi(0) + \sum_{0<t_i<t} e^{Bt_i}\rho(x(t_i))$. And, hence,

$$x(t) = e^{-Bt}\left\{\int_0^t e^{Bs}\sum_{j=1}^r h_j(\omega(s)) \times \left[C_j f(x(s)) + D_j g(x(s-\tau(s)))\right]ds + \phi(0) + \sum_{0<t_i<t} e^{Bt_i}\rho(x(t_i))\right\}, \quad t \geqslant 0. \tag{14}$$

On the other hand, it follows from (14) that

$$\begin{aligned} x(t_k) &= e^{-Bt_k}\left\{\int_0^{t_k} e^{Bs}\sum_{j=1}^r h_j(\omega(s)) \times \left[C_j f(x(s)) + D_j g(x(s-\tau(s)))\right]ds + \phi(0) + \sum_{0<t_i<t_k} e^{Bt_i}\rho(x(t_i))\right\}, \\ x(t_{k+\varepsilon}) &= e^{-Bt_{k+\varepsilon}}\left\{\int_0^{t_{k+\varepsilon}} e^{Bs}\sum_{j=1}^r h_j(\omega(s)) \times \left[C_j f(x(s)) + D_j g(x(s-\tau(s)))\right]ds + \phi(0) + \sum_{0<t_i<t_{k+\varepsilon}} e^{Bt_i}\rho(x(t_i))\right\}. \end{aligned} \tag{15}$$

Let $\varepsilon \to 0^+$; then we can derive from the two equations above that

$$\lim_{\varepsilon\to 0^+} x(t_{k+\varepsilon}) - x(t_k) = x(t_k^+) - x(t_k) = \rho(x(t_k)). \tag{16}$$

So, we may define the mapping P on the space Ω as follows:

$$Px(t) = e^{-Bt}\phi(0) + e^{-Bt}\int_0^t e^{Bs}\sum_{j=1}^r h_j(\omega(s)) \times \left[C_j f(x(s)) + D_j g(x(s-\tau(s)))\right]ds + e^{-Bt}\sum_{0<t_i<t} e^{Bt_i}\rho(x(t_i)), \tag{17}$$

on $t \geqslant 0$, and $Px(s) = \phi(s)$ on $s \in [-\tau, 0]$.

Step 2. We claim that $Px(t) \in \Omega$ for any $x(t) \in \Omega$. That is, $Px(t)$ satisfies the conditions (a)–(d) of Ω.

Indeed, since $Px(s) = \phi(s)$ on $s \in [-\tau, 0]$, the condition (c) is satisfied. It is obvious from (17) that $Px(t)$ is continuous on $t \neq t_k$ and $t \geqslant 0$. And then the condition (a) is satisfied.

Next, we verify the condition (b).

Indeed, for any given t_k, we can get from (17)

$$Px(t_k + \varepsilon) - Px(t_k) = \pi_1 + \pi_2 + \pi_3, \tag{18}$$

where

$$\begin{aligned} \pi_1 &= e^{-B(t_k+\varepsilon)}\phi(0) - e^{-B(t_k)}\phi(0), \\ \pi_2 &= e^{-B(t_k+\varepsilon)}\int_0^{t_k+\varepsilon} e^{Bs}\sum_{j=1}^r h_j(\omega(s)) \times \left[C_j f(x(s)) + D_j g(x(s-\tau(s)))\right]ds \\ &\quad - e^{-Bt_k}\int_0^{t_k} e^{Bs}\sum_{j=1}^r h_j(\omega(s)) \times \left[C_j f(x(s)) + D_j g(x(s-\tau(s)))\right]ds \\ \pi_3 &= e^{-B(t_k+\varepsilon)}\sum_{0<t_i<t_k+\varepsilon} e^{Bt_i}\rho(x(t_i)) - e^{-Bt_k}\sum_{0<t_i<t_k} e^{Bt_i}\rho(x(t_i)). \end{aligned} \tag{19}$$

Obviously, $\pi_1 \to 0$ and $\pi_2 \to 0$ as $\varepsilon \to 0$. In addition, letting $\varepsilon \to 0^-$, we have $\pi_3 \to 0$. Letting $\varepsilon \to 0^+$, we get by (18)

$$Px(t_k^+) - Px(t_k) = \lim_{\varepsilon\to 0^+} \pi_3 = \rho(x(t_k)). \tag{20}$$

So the condition (b) is satisfied.

Finally, it is followed from (17) that

$$\begin{aligned} e^{\beta t}Px(t) &= e^{\beta t}e^{-Bt}\phi(0) \\ &\quad + e^{\beta t}e^{-Bt}\int_0^t e^{Bs}\sum_{j=1}^r h_j(\omega(s)) \\ &\qquad \times \left[C_j f(x(s)) + D_j g(x(s-\tau(s)))\right] ds \\ &\quad + e^{\beta t}e^{-Bt}\sum_{0<t_i<t} e^{Bt_i}\rho(x(t_i)). \end{aligned} \tag{21}$$

Obviously, $e^{\beta t}e^{-Bt}\phi(0) \to 0 \in R^n$ as $t \to \infty$.

On the other hand, it follows from $e^{\beta t}x(t) \to 0$ that, for any given $\varepsilon > 0$, there exists a corresponding constant t_* such that $|e^{\beta t}x(t)| < \varepsilon u$ for all $t \geqslant t_*$, where

$$u = (1, 1, \ldots, 1)^T \in R^n. \tag{22}$$

In addition, (A3) derives

$$\begin{aligned} \left| e^{\beta t}e^{-Bt}\sum_{0<t_i<t} e^{Bt_i}\rho(x(t_i)) \right| &\leqslant e^{\beta t}e^{-Bt}\sum_{0<t_i<t} e^{Bt_i}H|x(t_i)| \\ &\leqslant e^{\beta t}e^{-Bt}\left(\sum_{0<t_i\leqslant t_*} e^{Bt_i}H|x(t_i)| + \sum_{t_*<t_i<t} e^{Bt_i}H|x(t_i)|\right). \end{aligned} \tag{23}$$

Obviously,

$$\begin{aligned} &e^{\beta t}e^{-Bt}\sum_{0<t_i\leqslant t_*} e^{Bt_i}H|x(t_i)| \\ &\quad = e^{(\beta I-B)t}\sum_{0<t_i\leqslant t_*} e^{Bt_i}H|x(t_i)| \longrightarrow 0 \in R^n, \quad t \longrightarrow \infty. \end{aligned} \tag{24}$$

Besides,

$$\begin{aligned} &e^{\beta t}e^{-Bt}\sum_{t_*<t_i<t} e^{Bt_i}H|x(t_i)| \\ &\quad = e^{(\beta I-B)t}\sum_{t_*<t_i<t} e^{(B-\beta I)t_i}H\left(e^{\beta t_i}|x(t_i)|\right) \\ &\quad \leqslant \varepsilon e^{(\beta I-B)t}\sum_{t_*<t_i<t} e^{(B-\beta I)t_i}Hu \\ &\quad = \begin{pmatrix} \varepsilon e^{(\beta-b_1)t}\sum_{t_*<t_i<t} e^{(b_1-\beta)t_i}H_1 \\ \varepsilon e^{(\beta-b_2)t}\sum_{t_*<t_i<t} e^{(b_2-\beta)t_i}H_2 \\ \vdots \\ \varepsilon e^{(\beta-b_n)t}\sum_{t_*<t_i<t} e^{(b_n-\beta)t_i}H_n \end{pmatrix}. \end{aligned} \tag{25}$$

To prove $e^{\beta t}e^{-Bt}\sum_{t_*<t_i<t} e^{Bt_i}H|x(t_i)| \to 0 \in R^n$, we only need to prove

$$\varepsilon e^{(\beta-b_1)t}\sum_{t_*<t_i<t} e^{(b_1-\beta)t_i}H_1 \longrightarrow 0, \quad t \longrightarrow \infty, \tag{26}$$

for the other cases can be similarly proved.

Indeed, we may as well assume $t_k < t \leqslant t_{k+1}$ and $t_{m-1} < t_* \leqslant t_m$. Then, we have

$$\begin{aligned} &\varepsilon e^{(\beta-b_1)t}\sum_{t_*<t_i<t} e^{(b_1-\beta)t_i}H_1 \\ &\quad = \frac{\varepsilon}{\delta}H_1 e^{(\beta-b_1)t}\sum_{t_m\leqslant t_i\leqslant t_k} \delta e^{(b_1-\beta)t_i} \\ &\quad \leqslant \frac{\varepsilon}{\delta}H_1 e^{(\beta-b_1)t}\left(\delta e^{(b_1-\beta)t_k} + \sum_{t_m\leqslant t_i\leqslant t_{k-1}} (t_{i+1}-t_i)e^{(b_1-\beta)t_i}\right) \\ &\quad \leqslant \frac{\varepsilon}{\delta}H_1 e^{(\beta-b_1)t}\left(\delta e^{(b_1-\beta)t_k} + \int_{t_*}^t e^{(b_1-\beta)s}ds\right) \\ &\quad \leqslant \varepsilon H_1 e^{-(b_1-\beta)(t-t_k)} + \frac{H_1}{\delta(b_1-\beta)}\varepsilon, \end{aligned} \tag{27}$$

which implies that (26) holds. And, hence, $e^{\beta t}e^{-Bt}\sum_{t_*<t_i<t} e^{Bt_i}H|x(t_i)| \to 0 \in R^n$, $t \to \infty$.

Below, we only need to prove

$$\begin{aligned} &e^{\beta t}e^{-Bt}\int_0^t e^{Bs}\sum_{j=1}^r h_j(\omega(s)) \\ &\qquad \times \left[C_j f(x(s)) + D_j g(x(s-\tau(s)))\right] ds \longrightarrow 0, \\ &\qquad t \longrightarrow \infty. \end{aligned} \tag{28}$$

In fact, it follows from $e^{\beta t}x(t) \to 0$ that, for any given $\varepsilon > 0$, there exists a corresponding constant t^* such that $|e^{\beta t}x(t)| < \varepsilon u$ for all $t \geqslant t^*$. Then, we get by (A1)

$$\begin{aligned} &\left| e^{\beta t}e^{-Bt}\int_0^t e^{Bs}\sum_{j=1}^r h_j(\omega(s))C_j f(x(s))\,ds \right| \\ &\quad \leqslant e^{-(B-\beta I)t}\int_0^{t^*} e^{Bs}\sum_{j=1}^r \left|C_j\right| F|x(s)|\,ds \\ &\qquad + e^{-(B-\beta I)t}\int_{t^*}^t e^{Bs}\sum_{j=1}^r \left|C_j\right| F|x(s)|\,ds. \end{aligned} \tag{29}$$

On the one hand,

$$\begin{aligned} & e^{-(B-\beta I)t}\int_0^{t^*} e^{Bs}\sum_{j=1}^{r}\left|C_j\right| F\left|x(s)\right| ds \\ & \quad\leqslant e^{-(B-\beta I)t}\int_0^{t^*} e^{Bs}\sum_{j=1}^{r}\left|C_j\right| F\left(\sum_{i\in\mathcal{N}}\sup_{s\in[0,t^*]}\left|x_i(s)\right|\right) u ds \\ & \quad\leqslant\left(\sum_{i=1}^{m} w_i\right) e^{-(B-\beta I)t}\int_0^{t^*} e^{Bs} u ds \longrightarrow 0\in R^n, \quad t\longrightarrow\infty, \end{aligned} \tag{30}$$

where $w=\sum_{j=1}^{r}|C_j|F(\sum_{i\in\mathcal{N}}\sup_{s\in[0,t^*]}|x_i(s)|)u=(w_1,w_2,\ldots,w_n)^T\in R^n$.

On the other hand,

$$\begin{aligned} & e^{-(B-\beta I)t}\int_{t^*}^{t} e^{Bs}\sum_{j=1}^{r}\left|C_j\right| F\left|x(s)\right| ds \\ & \quad\leqslant \varepsilon e^{-(B-\beta I)t}\int_{t^*}^{t} e^{(B-\beta I)s} w^* ds \\ & \quad\leqslant \varepsilon\left(\sum_{i=1}^{n} w_i^*\right)\left(\frac{1}{b_1-\beta},\frac{1}{b_2-\beta},\ldots,\frac{1}{b_n-\beta}\right)^T, \end{aligned} \tag{31}$$

where $w^*=\sum_{j=1}^{r}|C_j|Fu=(w_1^*,w_2^*,\ldots,w_n^*)^T\in R^n$.

Now we can conclude from (29)–(31) that

$$\begin{aligned} & e^{\beta t}e^{-Bt}\int_0^{t} e^{Bs}\sum_{j=1}^{r} h_j(\omega(s)) \\ & \quad\times\left[C_j f(x(s))\right] ds \longrightarrow 0\in R^n, \quad t\longrightarrow\infty. \end{aligned} \tag{32}$$

Similarly, we can prove

$$\begin{aligned} & e^{\beta t}e^{-Bt}\int_0^{t} e^{Bs}\sum_{j=1}^{r} h_j(\omega(s)) \\ & \quad\times\left[D_j g(x(s-\tau(s)))\right] ds \longrightarrow 0\in R^n, \quad t\longrightarrow\infty. \end{aligned} \tag{33}$$

Indeed, we can similarly define the corresponding constant T_* for any given $\varepsilon>0$, satisfying $|e^{\beta t}x(t)|<\varepsilon u$ for all $t\geqslant T_*$. Then, we have

$$\begin{aligned} & \left|e^{\beta t}e^{-Bt}\int_0^{t} e^{Bs}\sum_{j=1}^{r} h_j(\omega(s))\left[D_j g(x(s-\tau(s)))\right] ds\right| \\ & \quad\leqslant e^{\beta t}e^{-Bt}\left(\int_0^{T_*} e^{Bs}\sum_{j=1}^{r}\left|D_j\right| G\left|x(s-\tau(s))\right| ds\right. \\ & \qquad\left.+\int_{T_*}^{t} e^{Bs}\sum_{j=1}^{r}\left|D_j\right| G\left|x(s-\tau(s))\right| ds\right). \end{aligned} \tag{34}$$

Similarly, we can prove

$$\begin{aligned} & e^{\beta t}e^{-Bt}\int_0^{T_*} e^{Bs}\sum_{j=1}^{r}\left|D_j\right| G\left|x(s-\tau(s))\right| ds \\ & \quad\leqslant\left(\sum_{i=1}^{m}\overline{w}_i\right) e^{-(B-\beta I)t}\int_0^{T_*} e^{Bs} u ds \longrightarrow 0\in R^n, \quad t\longrightarrow\infty, \end{aligned} \tag{35}$$

where $\overline{w}=\sum_{j=1}^{r}|D_j|G(\sum_{i\in\mathcal{N}}\sup_{s\in[-\tau,T_*]}|x_i(s)|)u=(\overline{w}_1,\overline{w}_2,\ldots,\overline{w}_n)^T\in R^n$.

On the other hand,

$$\begin{aligned} & e^{\beta t}e^{-Bt}\int_{T_*}^{t} e^{Bs}\sum_{j=1}^{r}\left|D_j\right| G\left|x(s-\tau(s))\right| ds \\ & \quad\leqslant \varepsilon e^{\beta\tau}e^{-(B-\beta I)t}\int_{T_*}^{t} e^{(B-\beta I)s}\overline{w}^* ds \\ & \quad\leqslant \varepsilon\left(\sum_{i\in\mathcal{N}}\overline{w}_i^*\right) e^{\beta\tau}\left(\frac{1}{b_1-\beta},\frac{1}{b_2-\beta},\ldots,\frac{1}{b_n-\beta}\right)^T, \end{aligned} \tag{36}$$

where $\overline{w}^*=\sum_{j=1}^{r}|D_j|Gu=(\overline{w}_1^*,\overline{w}_2^*,\ldots,\overline{w}_n^*)^T\in R^n$.

From (34)–(36), we can conclude that (33) holds. Hence, the condition (d) is satisfied.

Step 3. Below, we only need to prove that P is a contraction mapping.

Indeed, for any $x=x(t)=(x_1(t),x_2(t),\ldots,x_n(t))^T$, $y=y(t)=(y_1(t),y_2(t),\ldots,y_n(t))^T\in\Omega$, we estimate $|Px(t)-Py(t)|\leqslant K_1+K_2+K_3$, where

$$\begin{aligned} K_1 &= e^{-Bt}\int_0^{t} e^{Bs}\sum_{j=1}^{r}\left|C_j\right|\left|f(x(s))-f(y(s))\right| ds \\ &\leqslant e^{-Bt}\int_0^{t} e^{Bs}\sum_{j=1}^{r}\left|C_j\right| F\left|x(s)-y(s)\right| ds, \\ K_2 &= e^{-Bt}\int_0^{t} e^{Bs}\sum_{j=1}^{r} h_j(\omega(s))\left|D_j\right| \\ &\quad\times\left|g(x(s-\tau(s)))-g(y(s-\tau(s)))\right| ds, \\ K_3 &= e^{-Bt}\sum_{0<t_i<t} e^{Bt_i}\left|\rho(x(t_i))-\rho(y(t_i))\right|. \end{aligned} \tag{37}$$

From mathematical analysis and computation, we can derive

$$\begin{aligned} K_1 &= e^{-Bt}\int_0^t e^{Bs}\sum_{j=1}^{r}\left|C_j\right|\left|f\left(x\left(s\right)\right)-f\left(y\left(s\right)\right)\right|ds \\ &\leqslant e^{-Bt}\int_0^t e^{Bs}\sum_{j=1}^{r}\left|C_j\right|F\left|x\left(s\right)-y\left(s\right)\right|ds \\ &\leqslant \operatorname{dist}\left(x,y\right)e^{-Bt}\int_0^t e^{Bs}\sum_{j=1}^{r}\left|C_j\right|Fuds \\ &\leqslant \left(B^{-1}\sum_{j=1}^{r}\left|C_j\right|F\right)\operatorname{dist}\left(x,y\right)u. \end{aligned} \tag{38}$$

Similarly, we have

$$\begin{aligned} K_2 &= e^{-Bt}\int_0^t e^{Bs}\sum_{j=1}^{r}h_j\left(\omega\left(s\right)\right)\left|D_j\right|\left|g\left(x\left(s-\tau\left(s\right)\right)\right)\right. \\ &\qquad \left.-g\left(y\left(s-\tau\left(s\right)\right)\right)\right|ds \\ &\leqslant \left(B^{-1}\sum_{j=1}^{r}\left|D_j\right|G\right)\operatorname{dist}\left(x,y\right)u \\ K_3 &\leqslant e^{-Bt}\sum_{0<t_i<t}e^{Bt_i}H\left|\left(x\left(t_i\right)\right)-\left(y\left(t_i\right)\right)\right| \\ &\leqslant \frac{1}{\delta}\operatorname{dist}\left(x,y\right)e^{-Bt}\left(\sum_{t_1\leqslant t_i\leqslant t_{k-1}}e^{Bt_i}\left(t_{i+1}-t_i\right)Hu\right. \\ &\qquad \left.+e^{Bt_k}\delta Hu\right) \\ &\leqslant \frac{1}{\delta}\operatorname{dist}\left(x,y\right)e^{-Bt}\left(\int_0^t e^{Bs}Hu\,ds+e^{Bt}\delta Hu\right) \\ &\leqslant \operatorname{dist}\left(x,y\right)\left(\frac{1}{\delta}B^{-1}H+H\right)u. \end{aligned} \tag{39}$$

Combining the above three inequalities results in

$$\begin{aligned} &\left|Px\left(t\right)-Py\left(t\right)\right| \\ &\leqslant \left(B^{-1}\sum_{j=1}^{r}\left(\left|C_j\right|F+\left|D_j\right|G\right)+\frac{1}{\delta}B^{-1}H+H\right)u\operatorname{dist}\left(x,y\right) \\ &< \lambda Iu\operatorname{dist}\left(x,y\right), \end{aligned} \tag{40}$$

and hence

$$\operatorname{dist}\left(P\left(x\right),P\left(y\right)\right)\leqslant\lambda\operatorname{dist}\left(x,y\right). \tag{41}$$

Therefore, $P:\Omega\rightarrow\Omega$ is a contraction mapping such that there exists the fixed point $x(t)$ of P in Ω, which implies that $x(t)$ is the solution for the the impulsive fuzzy dynamic equation (5), satisfying $e^{\beta t}\parallel x(t)\parallel\rightarrow 0$ as $t\rightarrow\infty$. So the proof is completed. □

4. Numerical Example

Example 1. Consider the T-S fuzzy impulsive dynamic equations as follows.

Fuzzy Rule 1:

IF $\omega_1(t)$ is $1/e^{-2\omega_1(t)}$, THEN

$$\begin{aligned} \frac{dx\left(t\right)}{dt} &= -Bx\left(t\right)+C_1f\left(x\left(t\right)\right)+D_1g\left(x\left(t-\tau\left(t\right)\right)\right), \\ &\qquad t\geqslant 0,\ t\neq t_k,\ k=1,2,\ldots \\ x\left(t_k^+\right)-x\left(t_k\right) &= \rho\left(x\left(t_k\right)\right),\quad k=1,2,\ldots \\ x\left(\theta\right) &= \phi\left(\theta\right),\quad \theta\in\left[-\tau,0\right], \end{aligned} \tag{42}$$

Fuzzy Rule 2:

IF $\omega_2(t)$ is $1-1/e^{-2\omega_1(t)}$, THEN

$$\begin{aligned} \frac{dx\left(t\right)}{dt} &= -Bx\left(t\right)+C_2f\left(x\left(t\right)\right)+D_2g\left(x\left(t-\tau\left(t\right)\right)\right), \\ &\qquad t\geqslant 0,\ t\neq t_k,\ k=1,2,\ldots \\ x\left(t_k^+\right)-x\left(t_k\right) &= \rho\left(x\left(t_k\right)\right),\quad k=1,2,\ldots \\ x\left(\theta\right) &= \phi\left(\theta\right),\quad \theta\in\left[-\tau,0\right], \end{aligned} \tag{43}$$

where

$$\begin{aligned} &B=\begin{pmatrix}2 & 0\\ 0 & 1.9\end{pmatrix},\qquad C_1=\begin{pmatrix}-0.2 & 0\\ 0 & 0.3\end{pmatrix}=D_1, \\ &C_2=\begin{pmatrix}0.3 & 0\\ 0 & -0.1\end{pmatrix}=D_2, \\ &F=\begin{pmatrix}0.1 & 0\\ 0 & 0.2\end{pmatrix}=G,\qquad H=\begin{pmatrix}0.7 & 0\\ 0 & 0.6\end{pmatrix}. \end{aligned} \tag{44}$$

Let $\delta=1.5$. Then we can use Matlab LMI toolbox to solve the LMI condition (10) and obtain $t\min=-0.0046<0$ which implies it is feasible (see [10, Remark 29(3)] for details). Further, extracting the datum shows

$$\lambda=0.9883, \tag{45}$$

which means $0<\lambda<1$. Thereby, we can conclude from Theorem 8 that this impulsive fuzzy dynamic equation is exponentially stable in the mean square.

5. Conclusion

By formulating a contraction mapping and the matrix exponential function, the author applies linear matrix inequality (LMI) technique to investigate and obtain the LMI-based stability criterion of a class of time-delay Takagi-Sugeno (T-S) fuzzy differential equations. It is the first time to obtain the LMI-based stability criterion derived by a fixed point theory. The LMI methods have high efficiency and other advantages in large-scale engineering calculations. And the feasibility of LMI-based stability criterion can efficiently be computed and confirmed by computer Matlab LMI toolbox. A numerical example is presented to illustrate the effectiveness of the proposed methods. In the end of this paper, we have to point out that there are still many difficulties in obtaining the LMI-based stability criteria for some other dynamics equations, such as Cohen-Grossberg neural networks (see, e.g., [11, 19]) and other neural networks. These problems remain open and challenging.

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