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Research Article

Analysis of Stochastic Delay Predator-Prey System with Impulsive Toxicant Input in Polluted Environments

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A stochastic delay predator-prey model in a polluted environment with impulsive toxicant input is proposed and studied. The thresholds between stability in time average and extinction of each population are obtained. Some recent results are extended and improved greatly. Several simulation figures are introduced to support the conclusions.

1. Introduction

Environmental pollution by industries, agriculture, and other human activities is one of the most important socio-ecological problems in the world today. Due to toxins in the environment, lots of species have gone extinct, and many are on the verge of extinction. Thus, controlling the environmental pollution and the conservation of biodiversity are the major focus areas of all the countries around the world. This motivates scholars to study the effects of toxins on populations and to find out a theoretical persistence-extinction threshold.

Recently, a lot of population models in a polluted environment have been proposed and investigated; here, we may mention, among many others, [1–23]. Particularly, Yang et al. [15] pointed out that in many cases toxicants should be emitted in regular pulses, for example, the use of pesticides and the pollution by heavy metals (see, e.g., [24]). Thus, they proposed the following two-species Lotka-Volterra predator-prey system in a polluted environment with impulsive toxicant input:

$$\begin{aligned}
\frac{dx_1(t)}{dt} &= x_1(t)\left[r_{10} - r_{11}C_{10}(t) - a_{11}x_1(t) - a_{12}x_2(t)\right],\\
\frac{dx_2(t)}{dt} &= x_2(t)\left[-r_{20} - r_{21}C_{20}(t) + a_{21}x_1(t) - a_{22}x_2(t)\right],\\
\frac{dC_{10}(t)}{dt} &= k_1C_e(t) - (g_1 + m_1)C_{10}(t),\\
\frac{dC_{20}(t)}{dt} &= k_2C_e(t) - (g_2 + m_2)C_{20}(t),\\
\frac{dC_e(t)}{dt} &= -hC_e(t),\\
& t \neq n\gamma, \quad n \in Z^+,\\
\Delta x_i(t) &= 0, \quad \Delta C_{i0}(t) = 0, \quad \Delta C_e(t) = b,\\
& t = n\gamma, \quad n \in Z^+, \quad i = 1, 2,
\end{aligned} \tag{1}$$

where all the parameters are positive constants and $\Delta f(t) = f(t^+) - f(t)$, $Z^+ = \{1, 2, \ldots\}$; $x_1(t)$ and $x_2(t)$: the size of prey population and the predator population, respectively; r_{i0}: the intrinsic growth rate of the ith population without toxicant; r_{i1}: the ith population response to the pollutant present in the organism; $C_{i0}(t)$: the concentration of toxicant in the ith organism; $C_e(t)$: the concentration of toxicant in the environment; $kC_e(t)$: the organism's net uptake of toxicant from the environment; $gC_{i0}(t) + mC_{i0}(t)$: the egestion and depuration rates of the toxicant in the ith organism; $hC_e(t)$: the toxicant loss from the environment itself by volatilization and so on; γ: the period of the impulsive effect about the exogenous input of toxicant; b: the toxicant input amount at every time.

Yang et al. [15] showed that in the following Lemma holds.

Lemma 1. *For system* (1), *define*

$$\Delta_2 = r_{10}a_{21} - r_{20}a_{11}, \qquad \overline{\Delta}_2 := \frac{a_{21}r_{11}K_1}{\gamma} + \frac{a_{11}r_{21}K_2}{\gamma}, \qquad K_i = \frac{k_i b}{h(g_i + m_i)}. \tag{2}$$

(a) *If* $r_{10} < r_{11}K_1/\gamma$, *then* $\lim_{t\to+\infty} x_i(t) = 0$, $i = 1, 2$.

(b) *If* $r_{10} > r_{11}K_1/\gamma$ *and* $\Delta_2 < \overline{\Delta}_2$, *then* $\limsup_{t\to+\infty} t^{-1}\int_0^t x_1(s)ds > 0$ *and* $x_2(t)$ *goes to extinction.*

(c) *If* $\Delta_2 > \overline{\Delta}_2$, *then* $\limsup_{t\to+\infty} t^{-1}\int_0^t x_i(s)ds > 0$, $i = 1, 2$.

Some interesting and important problems arise naturally.

(Q1) In the real world, the growth of species depends on various environmental factors, such as temperature, humidity and parasites and so forth. Therefore population models should be stochastic rather than deterministic (May [25]). Thus, what happens if model (1) is subject to stochastic noises?

(Q2) In addition, time delays occur in almost every situation. Kuang [26] has pointed out that ignoring time delays means ignoring reality. Therefore, what happens if model (1) takes time delays into account?

(Q3) Can we improve the results given in Lemma 1?

The aim of this paper is to study the above problems. Suppose that stochastic noises mainly affect the growth rates, with $r_{i0} \to r_{i0} + \alpha_i \dot{B}_i(t)$ (see, e.g., [27–39]), where $\dot{B}_i(t)$ is a white noise and α_i^2 is the intensity of the noise. Moreover, taking time delays into account, we obtain the following model:

$$\begin{aligned}
dx_1(t) &= x_1(t)\left[r_{10} - r_{11}C_{10}(t) - a_{11}x_1(t) - a_{12}x_2(t-\tau_1)\right]dt + \alpha_1 x_1(t)\,dB_1(t),\\
dx_2(t) &= x_2(t)\left[-r_{20} - r_{21}C_{20}(t) - a_{22}x_2(t) + a_{21}x_1(t-\tau_2)\right]dt + \alpha_2 x_2(t)\,dB_2(t),\\
\frac{dC_{10}(t)}{dt} &= k_1C_e(t) - (g_1 + m_1)C_{10}(t),\\
\frac{dC_{20}(t)}{dt} &= k_2C_e(t) - (g_2 + m_2)C_{20}(t),\\
\frac{dC_e(t)}{dt} &= -hC_e(t),\\
& t \neq n\gamma, \quad n \in Z^+,\\
\Delta x_i(t) &= 0, \quad \Delta C_{i0}(t) = 0,\\
\Delta C_e(t) &= b, \quad t = n\gamma, \ n \in Z^+, \ i = 1, 2,
\end{aligned} \tag{3}$$

with initial condition

$$x_i(t) = \phi_i(t) > 0, \quad t \in [-\tau, 0]; \ \phi_i(0) > 0, \ i = 1, 2, \tag{4}$$

where $\tau_i \geq 0$, $\tau = \max\{\tau_1, \tau_2\}$, $\phi_i(t)$ is continuous on $[-\tau, 0]$. Our main result is the following theorem.

Theorem 2. *For system* (3), *define*

$$\begin{gathered}
\theta_1 = r_{10} - 0.5\alpha_1^2, \quad \theta_2 = r_{20} + 0.5\alpha_2^2, \quad \Delta = a_{11}a_{22} + a_{12}a_{21},\\
\widetilde{\Delta}_1 := \theta_1 a_{22} + \theta_2 a_{12}, \quad \widetilde{\Delta}_2 := \theta_1 a_{21} - \theta_2 a_{11},\\
\overline{\Delta}_1 := \frac{a_{22}r_{11}K_1}{\gamma} - \frac{a_{12}r_{21}K_2}{\gamma}.
\end{gathered} \tag{5}$$

(i) *If* $\theta_1 < r_{11}K_1/\gamma$, *then both* x_1 *and* x_2 *go to extinction almost surely (a.s.); that is,* $\lim_{t\to+\infty} x_i(t) = 0$ *a.s.,* $i = 1, 2$.

(ii) *If* $\theta_1 > r_{11}K_1/\gamma$ *and* $\widetilde{\Delta}_2 < \overline{\Delta}_2$, *then* $x_2(t)$ *goes to extinction and* x_1 *is stable in time average a.s.; that is,*

$$\lim_{t\to+\infty} t^{-1}\int_0^t x_1(s)\,ds = \frac{\theta_1 - r_{11}K_1/\gamma}{a_{11}} > 0, \ \text{a.s.} \tag{6}$$

(iii) *If* $\widetilde{\Delta}_2 > \overline{\Delta}_2$, *then both* x_1 *and* x_2 *are stable in time average a.s.*

$$\begin{aligned}
\lim_{t\to+\infty} t^{-1}\int_0^t x_1(s)\,ds &= \frac{\widetilde{\Delta}_1 - \overline{\Delta}_1}{\Delta} > 0, \ \text{a.s.}\\
\lim_{t\to+\infty} t^{-1}\int_0^t x_2(s)\,ds &= \frac{\widetilde{\Delta}_2 - \overline{\Delta}_2}{\Delta} > 0, \ \text{a.s.}
\end{aligned} \tag{7}$$

Remark 3. By comparing Lemma 1 with our Theorem 2, we can see that on the one hand, if $\alpha_1 = \alpha_2 = 0$ and $\tau_1 = \tau_2 = 0$, then $\theta_i = r_{i0}$, $\widetilde{\Delta}_i = \Delta_i$, $i = 1, 2$, and our stochastic delay system (3) becomes model (1); on the other hand, our results in Theorem 2 improve that in Lemma 1. Lemma 1 shows that the superior limit is positive, while Theorem 2 reveals that the limit exists and gives the explicit form of the limit. The contribution of this paper is therefore clear.

2. Proof

For the sake of simplicity, we introduce some notations:

$$\begin{gathered}
R_+^2 = \left\{a = (a_1, a_2) \in R^2 \mid a_i > 0, \ i = 1, 2\right\},\\
\langle f(t)\rangle = t^{-1}\int_0^t f(s)\,ds;\\
\langle f(t)\rangle^* = \limsup_{t\to+\infty} t^{-1}\int_0^t f(s)\,ds,\\
\langle f(t)\rangle_* = \liminf_{t\to+\infty} t^{-1}\int_0^t f(s)\,ds.
\end{gathered} \tag{8}$$

Lemma 4. *For any given initial value* $\phi(t) = (\phi_1(t), \phi_2(t)) \in C([-\tau, 0], R_+^2)$, *there is a unique global positive solution* $x(t) = (x_1(t), x_2(t))^T$ *to the first two equations of system* (3) *a.s.*

Proof. The proof is similar to Hung [29] by defining

$$V(x) = [x_1 - 1 - \ln x_1] + \frac{a_{11}a_{22}}{a_{21}^2}[x_2 - 1 - \ln x_2] + 0.5a_{11}\int_{t-\tau_2}^{t} x_1^2(s)\,ds \tag{9}$$

and hence is omitted. □

To begin with, let us consider the following subsystem of (3):

$$\begin{gathered} \frac{dC_{10}(t)}{dt} = k_1C_e(t) - (g_1 + m_1)C_{10}(t), \\ \frac{dC_{20}(t)}{dt} = k_2C_e(t) - (g_2 + m_2)C_{20}(t), \\ \frac{dC_e(t)}{dt} = -hC_e(t), \\ t \neq n\gamma, \quad n \in Z^+, \\ \Delta C_{10}(t) = 0, \quad \Delta C_{20}(t) = 0, \quad \Delta C_e(t) = b, \\ t = n\gamma, \quad n \in Z^+, \\ 0 \le C_0(0) \le 1, \quad 0 \le C_e(0) \le 1. \end{gathered} \tag{10}$$

Lemma 5 (see [13, 15]). *System* (10) *has a unique positive* γ*-periodic solution* $(\widetilde{C}_{10}(t), \widetilde{C}_{20}(t), \widetilde{C}_e(t))^T$*, and for each solution* $(C_{10}(t), C_{20}(t), C_e(t))^T$ *of* (10)*,* $C_{10}(t) \to \widetilde{C}_{10}(t)$*,* $C_{20}(t) \to \widetilde{C}_{20}(t)$*, and* $C_e(t) \to \widetilde{C}_e(t)$ *as* $t \to \infty$*. Moreover,* $C_{i0}(t) > \widetilde{C}_{i0}(t)$ *and* $C_e(t) > \widetilde{C}_e(t)$ *for all* $t \ge 0$ *if* $C_{i0}(0) > \widetilde{C}_{i0}(0)$ *and* $C_e(0) > \widetilde{C}_e(0)$*,* $i = 1, 2$*, where*

$$\begin{aligned} \widetilde{C}_{10}(t) &= \widetilde{C}_{10}(0)e^{-(g_1+m_1)(t-n\gamma)} + \frac{k_1b\left(e^{-(g_1+m_1)(t-n\gamma)} - e^{-h(t-n\gamma)}\right)}{(h - g_1 - m_1)(1 - e^{-h\gamma})}, \\ \widetilde{C}_{20}(t) &= \widetilde{C}_{20}(0)e^{-(g_2+m_2)(t-n\gamma)} + \frac{k_2b\left(e^{-(g_2+m_2)(t-n\gamma)} - e^{-h(t-n\gamma)}\right)}{(h - g_2 - m_2)(1 - e^{-h\gamma})}, \\ \widetilde{C}_e(t) &= \frac{be^{-h(t-n\gamma)}}{1 - e^{-h\gamma}}, \\ \widetilde{C}_{10}(0) &= \frac{k_1b\left(e^{-(g_1+m_1)\gamma} - e^{-h\gamma}\right)}{(h - g_1 - m_1)\left(1 - e^{-(g_1+m_1)\gamma}\right)(1 - e^{-h\gamma})}, \\ \widetilde{C}_{20}(0) &= \frac{k_2b\left(e^{-(g_2+m_2)\gamma} - e^{-h\gamma}\right)}{(h - g_2 - m_2)\left(1 - e^{-(g_2+m_2)\gamma}\right)(1 - e^{-h\gamma})}, \\ \widetilde{C}_e(0) &= \frac{b}{1 - e^{-h\gamma}} \end{aligned} \tag{11}$$

for $t \in (n\gamma, (n+1)\gamma]$ *and* $n \in Z^+$*. In addition,*

$$\lim_{t\to+\infty} t^{-1}\int_0^t \widetilde{C}_{i0}(s)\,ds = \frac{k_ib}{h(g_i + m_i)\gamma} = \frac{K_i}{\gamma}, \quad i = 1, 2. \tag{12}$$

Lemma 6 (see [34]). *Suppose that* $x(t) \in C[\Omega \times [0, +\infty), R_+]$*.*

(I) If there exist σ *and positive constants* σ_0*,* T *such that*

$$\ln x(t) \le \sigma t - \sigma_0\int_0^t x(s)\,ds + \sum_{i=1}^{n}\beta_iB_i(t) \tag{13}$$

for $t \ge T$*, where* $B_i(t)$ *are independent standard Brownian motions and* β_i *are constants,* $1 \le i \le n$*, then one has the following: if* $\sigma \ge 0$*, then* $\langle x\rangle^* \le \sigma/\sigma_0$ *a.s.; if* $\sigma < 0$*, then* $\lim_{t\to+\infty}x(t) = 0$ *a.s.*

(II) If there exist positive constants σ_0*,* T *and* σ *such that*

$$\ln x(t) \ge \sigma t - \sigma_0\int_0^t x(s)\,ds + \sum_{i=1}^{n}\beta_iB_i(t) \tag{14}$$

for $t \ge T$*, then* $\langle x\rangle_* \ge \sigma/\sigma_0$ *a.s.*

Now, let us consider the following auxiliary system:

$$\begin{aligned} dy_1(t) &= y_1(t)\left[r_{10} - r_{11}C_{10}(t) - a_{11}y_1(t)\right]dt + \alpha_1y_1(t)\,dB_1(t), \\ dy_2(t) &= y_2(t)\left[-r_{20} - r_{21}C_{20}(t) + a_{21}y_1(t - \tau_2) - a_{22}y_2(t)\right]dt + \alpha_2y_2(t)\,dB_2(t), \end{aligned} \tag{15}$$

with initial value $\phi(t) \in C([-\tau, 0], R_+^2)$.

Lemma 7. *If* $\theta_1 = r_{10} - 0.5\alpha_1^2 > r_{11}K_1/\gamma$*, then the solution* $y(t)$ *of system* (15) *obeys*

$$\begin{gathered} \lim_{t\to+\infty}\langle y_1(t)\rangle = \frac{\theta_1 - r_{11}K_1/\gamma}{a_{11}}; \\ \lim_{t\to+\infty}y_2(t) = 0 \text{ a.s.}, \quad \text{if } \widetilde{\Delta}_2 < \overline{\Delta}_2; \\ \lim_{t\to+\infty}\langle y_2(t)\rangle = \frac{\widetilde{\Delta}_2 - \overline{\Delta}_2}{a_{11}a_{22}} \text{ a.s.}, \quad \text{if } \widetilde{\Delta}_2 > \overline{\Delta}_2. \end{gathered} \tag{16}$$

Proof. By Lemma 5,

$$\lim_{t\to+\infty} t^{-1}\int_0^t C_{i0}(s)\,ds = \lim_{t\to+\infty} t^{-1}\int_0^t \widetilde{C}_{i0}(s)\,ds = \frac{K_i}{\gamma}, \quad i = 1, 2. \tag{17}$$

Then, for all $\varepsilon > 0$, there exists $T > 0$ such that

$$\frac{K_i}{\gamma} - \varepsilon \le \langle C_{i0}(t)\rangle \le \frac{K_i}{\gamma} + \varepsilon, \quad t > T, \; i = 1, 2. \tag{18}$$

An application of Itô's formula to (15) yields

$$\begin{aligned}
&\ln y_1(t) - \ln y_1(0) \\
&\quad = \theta_1 t - r_{11}\int_0^t C_{10}(s)\,ds - a_{11}\int_0^t y_1(s)\,ds + \alpha_1 B_1(t), \\
&\ln y_2(t) - \ln y_2(0) \\
&\quad = -\theta_2 t - r_{21}\int_0^t C_{20}(s)\,ds \\
&\qquad + a_{21}\int_0^t y_1(s-\tau_2)\,ds - a_{22}\int_0^t y_2(s)\,ds + \alpha_2 B_2(t) \\
&\quad = -\theta_2 t - r_{21}\int_0^t C_{20}(s)\,ds + a_{21}\int_0^t y_1(s)\,ds \\
&\qquad - a_{21}\left[\int_{t-\tau_2}^t y_1(s)\,ds - \int_{-\tau_2}^0 y_1(s)\,ds\right] \\
&\qquad - a_{22}\int_0^t y_2(s)\,ds + \alpha_2 B_2(t).
\end{aligned} \tag{19}$$

That is to say, we have shown that

$$t^{-1}\ln\frac{y_1(t)}{y_1(0)} = \theta_1 - r_{11}\langle C_{10}(t)\rangle - a_{11}\langle y_1(t)\rangle + t^{-1}\alpha_1 B_1(t), \tag{20}$$

$$\begin{aligned}
&t^{-1}\ln\frac{y_2(t)}{y_2(0)} + t^{-1}a_{21}\left[\int_{t-\tau_2}^t y_1(s)\,ds - \int_{-\tau_2}^0 y_1(s)\,ds\right] \\
&\quad = -\theta_2 - r_{21}\langle C_{20}(t)\rangle + a_{21}\langle y_1(t)\rangle \\
&\qquad - a_{22}\langle y_2(t)\rangle + t^{-1}\alpha_2 B_2(t).
\end{aligned} \tag{21}$$

When (18) is used in (20), we can see that for $t > T$,

$$\begin{aligned}
t^{-1}\ln\frac{y_1(t)}{y_1(0)} \le \theta_1 - \frac{r_{11}K_1}{\gamma} + r_{11}\varepsilon \\
- a_{11}\langle y_1(t)\rangle + t^{-1}\alpha_1 B_1(t),
\end{aligned} \tag{22}$$

$$\begin{aligned}
t^{-1}\ln\frac{y_1(t)}{y_1(0)} \ge \theta_1 - \frac{r_{11}K_1}{\gamma} - r_{11}\varepsilon \\
- a_{11}\langle y_1(t)\rangle + t^{-1}\alpha_1 B_1(t).
\end{aligned} \tag{23}$$

Let ε be sufficiently small such that $\theta_1 - r_{11}K_1/\gamma - r_{11}\varepsilon > 0$. Making use of (I) and (II) in Lemma 6 to (22) and (23), respectively, we have

$$\begin{aligned}
\langle y_1(t)\rangle^* \le \frac{\theta_1 - r_{11}K_1/\gamma + r_{11}\varepsilon}{a_{11}}, \\
\langle y_1(t)\rangle_* \ge \frac{\theta_1 - r_{11}K_1/\gamma - r_{11}\varepsilon}{a_{11}}.
\end{aligned} \tag{24}$$

It then follows from the arbitrariness of ε that

$$\lim_{t\to+\infty}\langle y_1(t)\rangle = \frac{\theta_1 - r_{11}K_1/\gamma}{a_{11}}. \tag{25}$$

Substituting (17) and (25) into (20) and noting that $\lim_{t\to+\infty} t^{-1}B_1(t) = 0$, one can derive that

$$\lim_{t\to+\infty} t^{-1}\ln y_1(t) = 0, \quad \text{a.s.} \tag{26}$$

Employing (20) and (21) in the expression $a_{21}\ln(y_1(t)/y_1(0)) + a_{11}\ln(y_2(t)/y_2(0))$ yields

$$\begin{aligned}
&a_{11}t^{-1}\ln\frac{y_2(t)}{y_2(0)} + a_{21}t^{-1}\ln\frac{y_1(t)}{y_1(0)} \\
&\quad = \widetilde{\Delta}_2 - r_{11}a_{21}\langle C_{10}(t)\rangle - r_{21}a_{11}\langle C_{20}(t)\rangle \\
&\qquad - a_{11}a_{22}\langle y_2(t)\rangle \\
&\qquad - t^{-1}a_{11}a_{21}\left[\int_{t-\tau_2}^t y_1(s)\,ds - \int_{-\tau_2}^0 y_1(s)\,ds\right] \\
&\qquad + t^{-1}\left[a_{21}\alpha_1 B_1(t) + a_{11}\alpha_2 B_2(t)\right].
\end{aligned} \tag{27}$$

In view of (25), we get

$$\begin{aligned}
&\lim_{t\to+\infty} t^{-1}\int_{t-\tau_2}^t y_1(s)\,ds \\
&\quad = \lim_{t\to+\infty} t^{-1}\left(\int_0^t y_1(s)\,ds - \int_0^{t-\tau_2} y_1(s)\,ds\right) = 0, \quad \text{a.s.}
\end{aligned} \tag{28}$$

By (17), (26), (27), and (28), for all $\varepsilon > 0$, there exists $T > 0$ such that, for $t \ge T$,

$$\begin{aligned}
a_{11}t^{-1}\ln\frac{y_2(t)}{y_2(0)} \le \widetilde{\Delta}_2 - \overline{\Delta}_2 + \varepsilon - a_{11}a_{22}\langle y_2(t)\rangle \\
+ t^{-1}\left[a_{21}\sigma_1 B_1(t) + a_{11}\sigma_2 B_2(t)\right],
\end{aligned} \tag{29}$$

$$\begin{aligned}
a_{11}t^{-1}\ln\frac{y_2(t)}{y_2(0)} \ge \widetilde{\Delta}_2 - \overline{\Delta}_2 - \varepsilon - a_{11}a_{22}\langle y_2(t)\rangle \\
+ t^{-1}\left[a_{21}\sigma_1 B_1(t) + a_{11}\sigma_2 B_2(t)\right].
\end{aligned} \tag{30}$$

If $\widetilde{\Delta}_2 < \overline{\Delta}_2$, then we can choose ε sufficiently small such that $\widetilde{\Delta}_2 - \overline{\Delta}_2 + \varepsilon < 0$. Then, by (29) and (I) in Lemma 6, we obtain $\lim_{t\to+\infty} y_2(t) = 0$ a.s. If $\widetilde{\Delta}_2 > \overline{\Delta}_2$, then we can choose ε sufficiently small such that $\widetilde{\Delta}_2 - \overline{\Delta}_2 - \varepsilon > 0$. An application of (I) and (II) in Lemma 6 to (29) and (30), respectively, makes one observe that

$$\frac{\widetilde{\Delta}_2 - \overline{\Delta}_2 - \varepsilon}{a_{11}a_{22}} \le \langle y_2(t)\rangle_* \le \langle y_2(t)\rangle^* \le \frac{\widetilde{\Delta}_2 - \overline{\Delta}_2 + \varepsilon}{a_{11}a_{22}}, \quad \text{a.s.} \tag{31}$$

Therefore, using the arbitrariness of ε results in

$$\lim_{t\to+\infty}\langle y_2(t)\rangle = \frac{\widetilde{\Delta}_2 - \overline{\Delta}_2}{a_{11}a_{22}} \quad \text{a.s.} \tag{32}$$

This completes the proof. □

We are now in the position to prove our main results.

Proof of Theorem 2. Applying Itô's formula to (3) leads to

$$\begin{aligned}\ln x_1(t)&-\ln x_1(0)\\ &=\theta_1 t-r_{11}\int_0^t C_{10}(s)\,ds-a_{11}\int_0^t x_1(s)\,ds\\ &\quad-a_{12}\int_0^t x_2(s-\tau_1)\,ds+\alpha_1B_1(t)\\ &=\theta_1 t-r_{11}\int_0^t C_{10}(s)\,ds-a_{12}\int_0^t x_2(s)\,ds\\ &\quad+a_{12}\left[\int_{t-\tau_1}^t x_2(s)\,ds-\int_{-\tau_1}^0 x_2(s)\,ds\right]\\ &\quad-a_{11}\int_0^t x_1(s)\,ds+\alpha_1B_1(t).\end{aligned} \tag{33}$$

$$\begin{aligned}\ln x_2(t)&-\ln x_2(0)\\ &=-\theta_2 t-r_{21}\int_0^t C_{20}(s)\,ds+a_{21}\int_0^t x_1(s)\,ds\\ &\quad-a_{21}\left[\int_{t-\tau_2}^t x_1(s)\,ds-\int_{-\tau_2}^0 x_1(s)\,ds\right]\\ &\quad-a_{22}\int_0^t x_2(s)\,ds+\alpha_2B_2(t).\end{aligned} \tag{34}$$

(i) It follows from (17) and (33) that

$$\begin{aligned}t^{-1}\ln x_1(t)&-t^{-1}\ln x_1(0)\\ &\le\theta_1-r_{11}\langle C_{10}(t)\rangle-a_{11}\langle x_1(t)\rangle+\frac{\alpha_1B_1(t)}{t}\\ &\le\theta_1-\frac{r_{11}K_1}{\lambda}+\varepsilon-a_{11}\langle x_1(t)\rangle+\frac{\alpha_1B_1(t)}{t}\end{aligned} \tag{35}$$

for sufficiently large t. Since $\theta_1-r_{11}K_1/\lambda<0$, then we can choose ε sufficiently small such that $\theta_1-r_{11}K_1/\lambda+\varepsilon<0$. Then, by (I) in Lemma 6,

$$\lim_{t\to+\infty}x_1(t)=0,\ \text{a.s.} \tag{36}$$

When (36) is used in (34), one can see that

$$t^{-1}\ln x_2(t)-\ln x_2(0)\le-\theta_2+\varepsilon-a_{22}\langle x_2(t)\rangle+\frac{\alpha_2B_2(t)}{t} \tag{37}$$

for sufficiently large t, where $\varepsilon>0$ obeys $-\theta_2+\varepsilon<0$. In view of Lemma 6 again, $\lim_{t\to+\infty}x_2(t)=0$, a.s.

(ii) By the stochastic comparison theorem [40], one can observe that

$$x_1(t)\le y_1(t),\qquad x_2(t)\le y_2(t). \tag{38}$$

Note that $\theta_1>r_{11}K_1/\gamma$ and $\widetilde{\Delta}_2<\overline{\Delta}_2$; it then follows from Lemma 7 that $\lim_{t\to+\infty}y_2(t)=0$, a.s. Making use of (38) gives $\lim_{t\to+\infty}x_2(t)=0$, a.s. Thus, for all $\varepsilon>0$, there exists $T>0$ such that, for $t\ge T$,

$$\frac{\varepsilon}{2}\le a_{12}x_2(t)\le\frac{\varepsilon}{2}. \tag{39}$$

Substituting the above inequalities into (33) and then using (18), we obtain

$$\begin{aligned}t^{-1}\ln x_1(t)&\le t^{-1}\ln x_1(0)+\theta_1-r_{11}\langle C_{10}(t)\rangle\\ &\quad-a_{11}\langle x_1(t)\rangle+\frac{\varepsilon}{2}+\frac{\alpha_1B_1(t)}{t}\\ &\le\theta_1-\frac{r_{11}K_1}{\gamma}+2\varepsilon-a_{11}\langle x_1(t)\rangle+\frac{\alpha_1B_1(t)}{t},\end{aligned} \tag{40}$$

$$\begin{aligned}t^{-1}\ln x_1(t)&\ge t^{-1}\ln x_1(0)+\theta_1-r_{11}\langle C_{10}(t)\rangle\\ &\quad-a_{11}\langle x_1(t)\rangle-\frac{\varepsilon}{2}+\frac{\alpha_1B_1(t)}{t}\\ &\ge\theta_1-\frac{r_{11}K_1}{\gamma}-2\varepsilon-a_{11}\langle x_1(t)\rangle+\frac{\alpha_1B_1(t)}{t}.\end{aligned} \tag{41}$$

Let ε be sufficiently small such that $\theta_1-r_{11}K_1/\gamma-\varepsilon>0$, and then, applying (I) and (II) in Lemma 6 to (40) and (41), respectively, one can see that

$$\begin{aligned}\frac{\theta_1-r_{11}K_1/\gamma-2\varepsilon}{a_{11}}&\le\langle x_1(t)\rangle_*\le\langle x_1(t)\rangle^*\\ &\le\frac{\theta_1-r_{11}K_1/\gamma+2\varepsilon}{a_{11}}\ \text{a.s.}\end{aligned} \tag{42}$$

An application of the arbitrariness of ε gives

$$\lim_{t\to+\infty}\langle x_1(t)\rangle=\frac{\theta_1-r_{11}K_1/\gamma}{a_{11}},\ \text{a.s.} \tag{43}$$

(iii) Clearly, $\widetilde{\Delta}_2>\overline{\Delta}_2$ implies $\theta_1>r_{11}K_1/\gamma$, and then, by Lemma 7,

$$\lim_{t\to+\infty}\langle y_2(t)\rangle=\frac{\widetilde{\Delta}_2-\overline{\Delta}_2}{a_{11}a_{22}}. \tag{44}$$

Thus, similar to the proof of (28), we get

$$\lim_{t\to+\infty}t^{-1}\int_{t-\tau_1}^t y_2(s)\,ds=0\ \text{a.s.} \tag{45}$$

Therefore, by (26), (28), and (38), we can observe that

$$\limsup_{t\to+\infty}t^{-1}\ln x_1(t)\le\lim_{t\to+\infty}t^{-1}\ln y_1(t)=0, \tag{46}$$

$$\begin{aligned}&\lim_{t\to+\infty}t^{-1}\int_{t-\tau_2}^t x_1(s)\,ds=0,\\ &\lim_{t\to+\infty}t^{-1}\int_{t-\tau_1}^t x_2(s)\,ds=0,\ \text{a.s.}\end{aligned} \tag{47}$$

Employing (33) and (34) in the expression $a_{21}\ln(x_1(t)/x_1(0)) + a_{11}\ln(x_2(t)/x_2(0))$ yields

$$\begin{aligned} t^{-1}a_{21}\ln\frac{x_1(t)}{x_1(0)} + t^{-1}a_{11}\ln\frac{x_2(t)}{x_2(0)} &= a_{12}a_{21}t^{-1}\left[\int_{t-\tau_1}^{t}x_2(s)\,ds - \int_{-\tau_1}^{0}x_2(s)\,ds\right] \\ &\quad - a_{11}a_{21}t^{-1}\left[\int_{t-\tau_2}^{t}x_1(s)\,ds - \int_{-\tau_2}^{0}x_1(s)\,ds\right] \\ &\quad + \widetilde{\Delta}_2 - a_{21}r_{11}\langle C_{10}(t)\rangle - a_{11}r_{21}\langle C_{20}(t)\rangle \\ &\quad - \Delta\langle x_2(t)\rangle + t^{-1}a_{21}\alpha_1B_1(t) + t^{-1}a_{11}\alpha_2B_2(t). \end{aligned} \tag{48}$$

When (18), (46) and (47), are used in (48), one can obtain

$$\begin{aligned} t^{-1}a_{11}\ln\frac{x_2(t)}{x_2(0)} &\geq \widetilde{\Delta}_2 - \overline{\Delta}_2 - \varepsilon - \Delta\langle x_2(t)\rangle \\ &\quad + t^{-1}a_{21}\alpha_1B_1(t) + t^{-1}a_{11}\alpha_2B_2(t) \end{aligned} \tag{49}$$

for sufficiently large t, where $\varepsilon > 0$ obeys $\widetilde{\Delta}_2 - \overline{\Delta}_2 - \varepsilon > 0$. It then follows from (II) in Lemma 6 that

$$\langle x_2(t)\rangle_* \geq \frac{\widetilde{\Delta}_2 - \overline{\Delta}_2 - \varepsilon}{\Delta}. \tag{50}$$

By virtue of the arbitrariness of ε, we can see that

$$\langle x_2(t)\rangle_* \geq \frac{\widetilde{\Delta}_2 - \overline{\Delta}_2}{\Delta}. \tag{51}$$

Consequently, for every $0 < \varepsilon < a_{12}(\widetilde{\Delta}_2 - \overline{\Delta}_2)/\Delta$, there is $T > 0$ such that

$$a_{12}\langle x_2(t)\rangle \geq a_{12}\langle x_2\rangle_* - \varepsilon \geq \frac{a_{12}\left(\widetilde{\Delta}_2 - \overline{\Delta}_2\right)}{\Delta} - \varepsilon, \quad t > T. \tag{52}$$

Substituting the above inequality into (33) and then using (18) and (47), one can see that

$$\begin{aligned} t^{-1}\ln\frac{x_1(t)}{x_1(0)} &\leq \theta_1 - \frac{a_{12}\left(\widetilde{\Delta}_2 - \overline{\Delta}_2\right)}{\Delta} + 3\varepsilon \\ &\quad - a_{11}t^{-1}\int_0^t x_1(s)\,ds + t^{-1}\alpha_1B_1(t) \\ &= \frac{a_{11}\left(\widetilde{\Delta}_1 - \overline{\Delta}_1\right)}{\Delta} + 3\varepsilon - a_{11}t^{-1}\int_0^t x_1(s)\,ds \\ &\quad + t^{-1}\alpha_1B_1(t) \end{aligned} \tag{53}$$

for sufficiently large t. Since $\widetilde{\Delta}_1 - \overline{\Delta}_1 > 0$, and then, by Lemma 6 and the arbitrariness of ε, one can observe that

$$\langle x_1(t)\rangle^* \leq \frac{\widetilde{\Delta}_1 - \overline{\Delta}_1}{\Delta}. \tag{54}$$

When this inequality, (18) and (47), are used in (34), we can see that

$$\begin{aligned} t^{-1}\ln\frac{x_2(t)}{x_2(0)} &\leq -\theta_2 + a_{21}\frac{\widetilde{\Delta}_1 - \overline{\Delta}_1}{\Delta} + 3\varepsilon \\ &\quad - a_{22}t^{-1}\int_0^t x_2(s)\,ds + t^{-1}\alpha_2B_2(t) \\ &= \frac{a_{22}\left(\widetilde{\Delta}_2 - \overline{\Delta}_2\right)}{\Delta} + 3\varepsilon - a_{22}t^{-1}\int_0^t x_2(s)\,ds \\ &\quad + t^{-1}\alpha_2B_2(t) \end{aligned} \tag{55}$$

for sufficiently large t. Then, it follows from Lemma 6 and the arbitrariness of ε that

$$\langle x_2(t)\rangle^* \leq \frac{\widetilde{\Delta}_2 - \overline{\Delta}_2}{\Delta}. \tag{56}$$

Substituting the above inequality and (18) into (33), we get

$$\begin{aligned} t^{-1}\ln\frac{x_1(t)}{x_1(0)} &\geq \theta_1 - a_{12}\frac{\widetilde{\Delta}_2 - \overline{\Delta}_2}{\Delta} - 3\varepsilon \\ &\quad - a_{11}t^{-1}\int_0^t x_1(s)\,ds + t^{-1}\alpha_1B_1(t) \\ &= \frac{a_{11}\left(\widetilde{\Delta}_1 - \overline{\Delta}_1\right)}{\Delta} - 3\varepsilon - a_{11}t^{-1}\int_0^t x_1(s)\,ds \\ &\quad + t^{-1}\alpha_1B_1(t) \end{aligned} \tag{57}$$

for sufficiently large t. By (II) in Lemma 6 and the arbitrariness of ε again, we obtain

$$\langle x_1(t)\rangle_* \geq \frac{\widetilde{\Delta}_1 - \overline{\Delta}_1}{\Delta}. \tag{58}$$

Then, the required assertion follows from (51), (54), (56), and (58). □

3. Numerical Simulations

Let us use the famous Milstein method (see, e.g., [41]) to illustrate the analytical results.

To begin with, we choose $r_{10} = 0.85$, $r_{20} = 0.05$, $r_{11} = r_{21} = 1$, $a_{11} = 0.4$, $a_{12} = 0.4$, $a_{21} = 0.3$, $a_{22} = 0.3$, $\tau_1 = 3$, $\tau_2 = 8$, $\alpha_2^2 = 0.1$, $k_i = g_i = m_i = 0.1$, $i = 1, 2$, $h = 0.5$, $b = 0.6$, and $\gamma = 12$. Then,

$$\begin{aligned} K_i &= \frac{k_ib}{h\left(g_i + m_i\right)} = 0.6, \\ \Delta_2 &= r_{10}a_{21} - r_{20}a_{11} = 0.235 > \overline{\Delta}_2 \\ &= \frac{a_{21}r_{11}K_1}{\gamma} + \frac{a_{11}r_{21}K_2}{\gamma} = 0.035. \end{aligned} \tag{59}$$

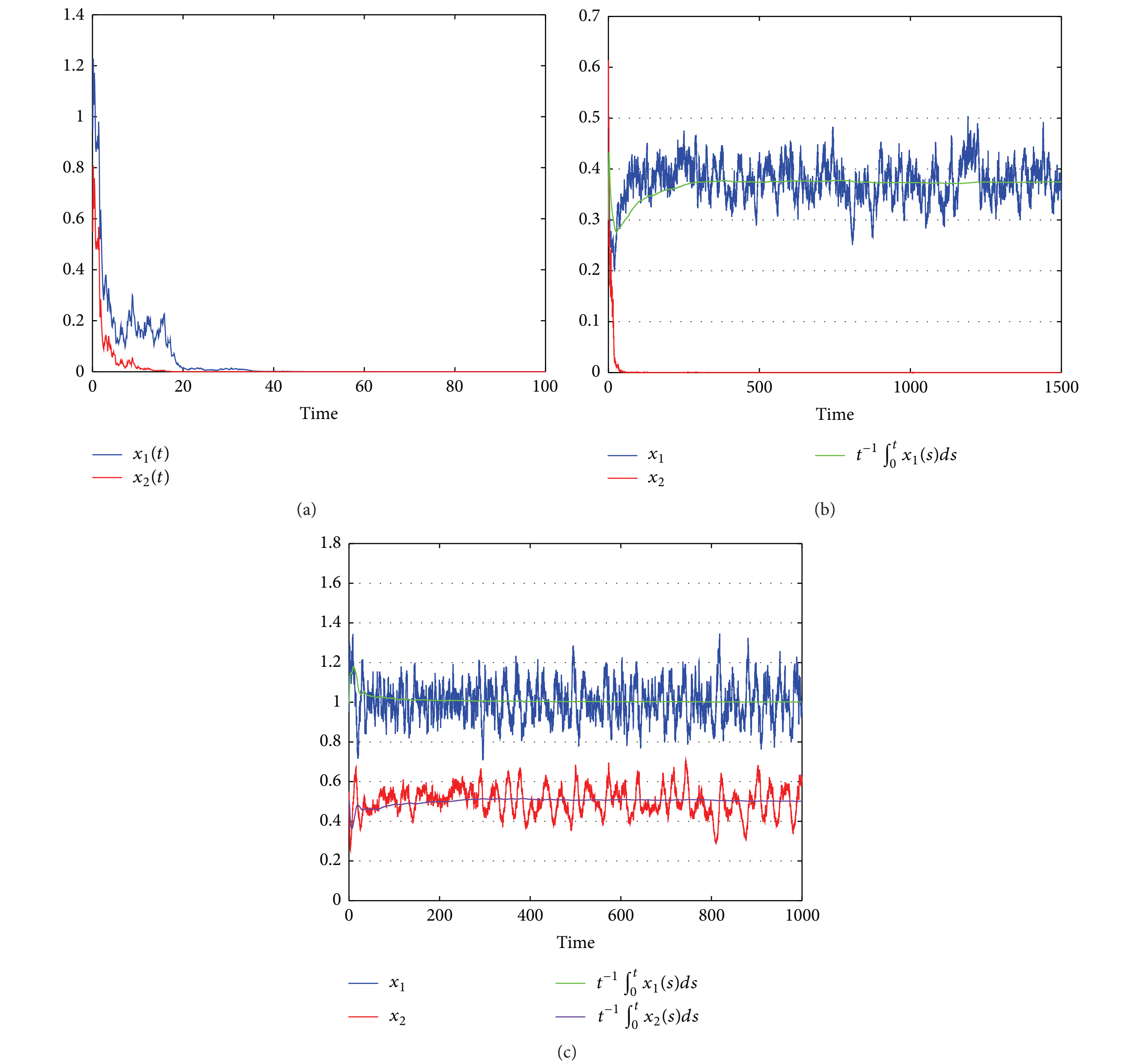

Figure 1: Solutions of system (3) for $r_{10} = 0.85$, $r_{20} = 0.05$, $r_{11} = r_{21} = 1$, $a_{11} = 0.4$, $a_{12} = 0.4$, $a_{21} = 0.3$, $a_{22} = 0.3$, $\tau_1 = 3$, $\tau_2 = 8$, $\alpha_2^2 = 0.1$, $k_i = g_i = m_i = 0.1$, $i = 1, 2$, $h = 0.5$, $b = 0.6$, $\gamma = 12$, $x_1(0) = 0.9$, $x_2(0) = 0.5$, $C_0(0) = C_e(0) = 0.1$, and step size $\Delta t = 0.001$. (a) is with $\alpha_1^2/2 = 0.82$; (b) is with $\alpha_1^2/2 = 0.65$; (c) is with $\alpha_1^2/2 = 0.2$.

By (c) in Lemma 1, the solution of model (1) obeys

$$\limsup_{t\to+\infty} t^{-1}\int_0^t x_1(s)\,ds > 0, \qquad \limsup_{t\to+\infty} t^{-1}\int_0^t x_2(s)\,ds > 0. \tag{60}$$

However, when the white noises are taken into account, the properties of the system may be changed greatly. In Figure 1, we let the coefficients be same with the above. The only difference between conditions of Figures 1(a), 1(b), and 1(c) is that the value of α_1^2 is different. In Figure 1(a), we choose $\alpha_1^2/2 = 0.82$. Therefore,

$$\theta_1 = r_{10} - \frac{\alpha_1^2}{2} = 0.03 < \frac{r_{11}K_1}{\gamma} = 0.05. \tag{61}$$

Then, by (i) in Theorem 2, both x_1 and x_2 are extinctive. Figure 1(a) confirms these. In Figure 1(b), we choose $\alpha_1^2/2 = 0.65$. That is to say $\theta_1 = 0.2 > r_{11}K_1/\gamma = 0.05$ and $\tilde{\Delta}_2 = 0.02 < \overline{\Delta}_2 = 0.035$. It then follows from (ii) in

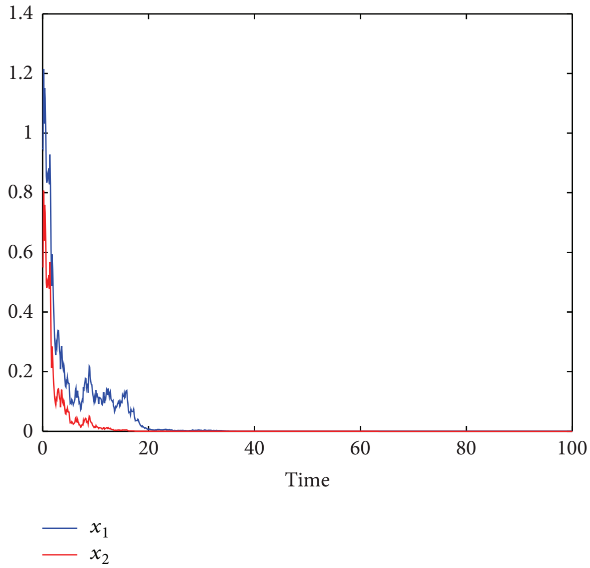

Figure 2: Solutions of system (3) for $r_{10} = 0.85$, $r_{20} = 0.05$, $r_{11} = r_{21} = 1$, $a_{11} = 0.4$, $a_{12} = 0.4$, $a_{21} = 0.3$, $a_{22} = 0.3$, $\tau_1 = 3$, $\tau_2 = 8$, $\alpha_1^2 = 0.4$, $\alpha_2^2 = 0.1$, $k_i = g_i = m_i = 0.1$, $i = 1, 2$, $h = 0.5$, $b = 0.6$, $\gamma = 0.8$, $x_1(0) = 0.9$, $x_2(0) = 0.5$, $C_0(0) = C_e(0) = 0.1$, and step size $\Delta t = 0.001$.

Theorem 2 that x_2 is extinctive and x_1 is stable in time average:

$$\lim_{t \to +\infty} \langle x_1(t) \rangle = \frac{\theta_1 - r_{11} K_1 / \gamma}{a_{11}} = 0.375. \tag{62}$$

See Figure 1(b). In Figure 1(c), we choose $\alpha_1^2/2 = 0.2$. Then, $\widetilde{\Delta}_2 = 0.155 > \overline{\Delta}_2 = 0.035$. In view of (iii) in Theorem 2, we can obtain that both x_1 and x_2 are stable in time average:

$$\begin{aligned} \lim_{t \to +\infty} \langle x_1(t) \rangle &= \frac{\widetilde{\Delta}_1 - \overline{\Delta}_1}{\Delta} = \frac{0.24}{0.24} = 1, \\ \lim_{t \to +\infty} \langle x_2(t) \rangle &= \frac{\widetilde{\Delta}_2 - \overline{\Delta}_2}{\Delta} = \frac{0.12}{0.24} = 0.5. \end{aligned} \tag{63}$$

Figure 1(c) confirms these.

In Figure 2, we choose $r_{10} = 0.85$, $r_{20} = 0.05$, $r_{11} = r_{21} = 1$, $a_{11} = 0.4$, $a_{12} = 0.4$, $a_{21} = 0.3$, $a_{22} = 0.3$, $\tau_1 = 3$, $\tau_2 = 8$, $\alpha_1^2 = 0.4$, $\alpha_2^2 = 0.1$, $k_i = g_i = m_i = 0.1$, $i = 1, 2$, $h = 0.5$, and $b = 0.6$. The only difference between conditions of Figures 1(c) and 2 is that the value of γ is different. In Figure 2, we choose $\gamma = 0.8$. Then, $\theta_1 = 0.65 < r_{11}K_1/\gamma = 0.75$. It follows from (i) in Theorem 2 that both x_1 and x_2 are extinctive. Figure 2 confirms these. By comparing Figure 1(c) with Figure 2, one can see that the impulsive period γ plays a key role in determining the stability in time average and the extinction of the species.

4. Conclusions and Future Directions

This paper is concerned with stochastic delay predator-prey model in a polluted environment with impulsive toxicant input. For each species, the threshold between stability in time average and extinction is established. Some recent results are improved and extended. Our Theorem 2 reveals some interesting and important results.

(A) Firstly, time delay is harmless for stability in time average and extinction of the stochastic system (3).

(B) The white noise $\alpha_1 dB_1(t)$ and $\alpha_2 dB_2(t)$ can change the properties of the system greatly.

(C) The impulsive period γ plays an important role in determining the stability in time average and the extinction of the species.

Some interesting questions deserve further investigations. One may consider some more realistic but more complex systems, for example, stochastic delay model with Markov switching (see, e.g., [30, 32, 39]). It is also interesting to investigate what happens if a_{ij} is stochastic.

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