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Research Article

Robust Stability of a Class of Uncertain Lur'e Systems of Neutral Type

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This paper deals with the problem of stability for a class of Lur'e systems with interval time-varying delay and sector-bounded nonlinearity. The interval time-varying delay function is not assumed to be differentiable. We analyze the global exponential stability for uncertain neutral and Lur'e dynamical systems with some sector conditions. By constructing a set of improved Lyapunov-Krasovskii functional combined with Leibniz-Newton's formula, we establish some stability criteria in terms of linear matrix inequalities. Numerical examples are given to illustrate the effectiveness of the results.

1. Introduction

In many practical systems, models of system are described by neutral differential equations, in which the models depend on the delays of state and state derivatives. Heat exchanges, distributed networks containing lossless transmission lines, and population ecology are examples of neutral systems. Because of its wider application, therefore, several researchers have studied neutral systems and provided sufficient conditions to guarantee the asymptotic stability of neutral time delay systems, see [1–6] and references cited therein.

It is well known that nonlinearities may cause instability and poor performance of practical systems, which have driven many researchers to study [3–9]. Many nonlinear control systems can be modeled as a feedback connection of a linear neutral system and a nonlinear element. One of the important classes of nonlinear systems is the Lur'e system whose nonlinear element satisfies certain sector constraints. Lur'e systems with sector bound have been widely interested in the control system such as Kalman-YaKubovih-Popov lemma, Popov criterion, and Circle criterion [10–12]. On the other hand, it is well known that the existence of time delay in a system may cause instability and oscillations. Example of time delay systems are chemical engineering systems, biological modeling, electrical networks, physical networks, and many others, [3–5].

The stability criteria for system with time delays can be classified into two categories: delay-independent and delay-dependent. Delay-independent criteria does not employ any information on the size of the delay; while delay-dependent criteria makes use of such information at different levels. Delay-dependent stability conditions are generally less conservative than delay-independent ones especially when the delay is small. In most of the existing results, the range of time-varying delay considered in paper is form 0 to an upper bound. In practice, the range of delay may vary in a range for which the lower bound is not restricted to be 0, that is, interval time-varying delay. A typical example with interval time delay is the networked control system, which has been widely studied in the recent literature (see, e.g., [3, 13, 14]). However there are fewer results for removing restriction to the derivative of interval time-varying delays. Therefore their methods have a conservatism which can be improved upon.

It is known that exponential stability is more favorite property than asymptotic stability since it gives a faster convergence rate to the equilibrium point, the decay rates (i.e., convergent rates) are important indices of practical systems, and the exponential stability analysis of time-delay systems has been a popular topic in the past decades; see, for example, [3, 13, 15] and their references. In [13], delay-dependent exponential stability for uncertain linear systems with interval time-varying delays, in [15], global exponential periodicity and global exponential stability of a class of recurrent neural networks with time-varying delays.

Recently, there are many research study on the asymptotic stability of a class of neutral and Lur'e dynamical systems with time delay, see, for example, [8, 9, 16, 17]. The problems have been dealt with stability analysis for neutral systems with mixed delays and sector-bounded nonlinearity [8], robust absolute stability criteria for uncertain Lur'e systems of neutral type [16], and robust stability criteria for a class of Lur'e systems with interval time-varying delay [9]. However, it is worth pointing out that, even though these results above were elegant, there still exist some points waiting for the improvement. Firstly, most of the work above the time-varying delays are required to be differentiable. In fact, the constraint on the derivative of the time-varying delay is not required which allows the time-delay to be a fast time-varying function. Secondly, in most studies on the asymptotic stability of Lur'e dynamical systems still need to be improved to the exponential stability.

Based on the above discussions, we consider the problem of robust stability for a class of uncertain neutral and Lur'e dynamical systems with sector-bounded nonlinearity. The time delay is a continuous function belonging to a given interval, which means that the lower and upper bounds for the time varying delay are available, but the delay function is not necessary to be differentiable. To the best of the authors knowledge, there were no global stability results for uncertain neutral and Lur'e dynamical systems with some sector conditions [8, 9, 16, 17]. Based on the construction of improved Lyapunov-Krasovskii functionals combined with Liebniz-Newton's formula and the integral terms, new delay-dependent sufficient conditions for the uncertain neutral and Lur'e dynamical of system are established of LMIs. The new stability condition is much less conservative and more general than some existing results. Numerical examples are given to illustrate the effectiveness of our theoretical results.

The rest of this paper is organized as follows. In Section 2, we give notations, definition, propositions, and lemma to be used in the proof of the main results. Delay-dependent sufficient conditions for uncertain neutral and Lur'e dynamical systems with sector-bounded nonlinearity are presented in Section 3. Numerical examples illustrated the obtained results and are given in Section 4. The paper ends with conclusions in Section 5 and cited references.

2. Problem Formulation and Preliminaries

The following notation will be used in this paper: $\mathbb{R}^+$ denotes the set of all real nonnegative numbers; $\mathbb{R}^n$ denotes the n-dimensional space and the vector norm $\|\cdot\|$; $M^{n\times r}$ denotes the space of all matrices of $(n\times r)$-dimensions. A^T denotes the transpose of matrix A; A is symmetric if $A = A^T$; I denotes the identity matrix; $\lambda(A)$ denotes the set of all eigenvalues of A; $\lambda_{\max}(A) = \max\{\operatorname{Re}\lambda;\ \lambda\in\lambda(A)\}$. $x_t := \{x(t+s) : s\in[-h,0]\}$, $\|x_t\| = \sup_{s\in[-h,0]}\|x(t+s)\|$; $C([0,t],\mathbb{R}^n)$ denotes the set of all $\mathbb{R}^n$-valued continuous functions on $[0,t]$; Matrix A is called semipositive definite ($A\ge 0$) if $x^TAx\ge 0$, for all $x\in\mathbb{R}^n$; A is positive definite ($A>0$) if $x^TAx>0$ for all $x\neq 0$; $A>B$ means $A-B>0$; $\operatorname{diag}(c_1,c_2,\dots,c_m)$ denotes block diagonal matrix with diagonal elements c_i, $i=1,2,\dots,m$. The symmetric term in a matrix is denoted by $*$.

Consider the following uncertain Lur'e system of neutral type with interval time-varying delays and sector-bounded nonlinearity:

$$\begin{gathered}\dot{x}(t) - C\dot{x}(t-\eta(t)) = (A+\Delta A(t))x(t) + (A_1+\Delta A_1(t))x(t-h(t)) + (B+\Delta B(t))f(\sigma(t)),\\ \sigma(t) = H^Tx(t) = \begin{bmatrix}\overline{h}_1 & \overline{h}_2 & \cdots & \overline{h}_m\end{bmatrix}^T x(t),\quad \forall t\ge 0,\\ x(t+s)=\phi(t+s),\quad \dot{x}(t+s)=\varphi(t+s),\quad s\in[-m,0],\ m=\max\{h_2,\eta\},\end{gathered}\tag{2.1}$$

where $x(t)\in\mathbb{R}^n$ is the state vector; $\sigma(t)\in\mathbb{R}^n$ is the output vector; $A\in\mathbb{R}^{n\times n}$, $B\in\mathbb{R}^{n\times m}$, $C\in\mathbb{R}^{n\times n}$, $A_1\in\mathbb{R}^{n\times n}$, $H\in\mathbb{R}^{n\times m}$ are constant known matrices; $f(\sigma(t))\in\mathbb{R}^m$ is the nonlinear function in the feedback path, which is denoted as f for simplicity in the sequel. Its form is formulated as

$$\begin{gathered}f(\sigma(t)) = \begin{bmatrix}f_1(\sigma_1(t)) & f_2(\sigma_2(t)) & \cdots & f_m(\sigma_m(t))\end{bmatrix}^T,\\ \sigma(t) = \begin{bmatrix}\sigma_1(t) & \sigma_2(t) & \cdots & \sigma_m(t)\end{bmatrix}^T = \begin{bmatrix}h_1^Tx(t) & h_2^Tx(t) & \cdots & h_m^Tx(t)\end{bmatrix},\end{gathered}\tag{2.2}$$

wherein, each term $f_i(\sigma_i(t))$, $i=1,2,\dots,m$ satisfies any one of the following sector conditions:

$$f_i(\sigma_i(t))\in K_{[0,k_i]} = \left\{f_i(\sigma_i(t)) \mid f_i(0)=0,\ 0<\sigma_i(t)f_i(\sigma_i(t))\le k_i\sigma_i(t)^2,\ \ \sigma_i(t)\neq 0\right\}\tag{2.3}$$

or

$$f_i(\sigma_i(t))\in K_{[0,\infty]} = \left\{f_i(\sigma_i(t)) \mid f_i(0)=0,\ \sigma_i(t)f_i(\sigma_i(t))>0,\ \sigma_i(t)\neq 0\right\}.\tag{2.4}$$

$\Delta A(t)$, $\Delta B(t)$, and $\Delta A_1(t)$ are time-varying uncertainties of appropriate dimensions, which are assumed to be of the following form:

$$[\Delta A(t) \quad \Delta B(t) \quad \Delta A_1(t)] = DF(t)[E_1 \quad E_2 \quad E_3], \tag{2.5}$$

where D, E_1, E_2, and E_3 are known matrices of appropriate dimensions, and the time-varying matrix $F(t)$ satisfies

$$F^T(t)F(t) \leq I, \quad \forall t \geq 0. \tag{2.6}$$

The delays $h(t)$ and $\tau(t)$ are time-varying continuous functions that satisfy

$$0 \leq h_1 \leq h(t) \leq h_2, \quad 0 \leq \eta(t) \leq \eta, \quad \dot{\eta}(t) \leq \eta_d < 1. \tag{2.7}$$

We introduce the following technical well-known propositions and definition, which will be used in the proof of our results.

Definition 2.1. If there exist $\gamma > 0$ and $\psi(\gamma) > 0$ such that

$$\|x(t)\| \leq \psi(\gamma)e^{-\gamma t}, \quad \forall t > 0, \tag{2.8}$$

system (2.1) is said to be exponentially stable at the equilibrium point, where γ is called the degree of exponential stability.

Proposition 2.2 (Cauchy inequality). *For any symmetric positive definite matrix $N \in M^{n\times n}$ and $x, y \in \mathbb{R}^n$ we have*

$$\pm 2x^T y \leq x^T N x + y^T N^{-1} y. \tag{2.9}$$

Proposition 2.3 (see [2]). *For any symmetric positive definite matrix $M > 0$, scalar $\gamma > 0$, and vector function $\omega : [0, \gamma] \to \mathbb{R}^n$ such that the integrations concerned are well defined, the following inequality holds*

$$\left(\int_0^\gamma \omega(s)ds\right)^T M \left(\int_0^\gamma \omega(s)ds\right) \leq \gamma \left(\int_0^\gamma \omega^T(s) M \omega(s)ds\right). \tag{2.10}$$

Proposition 2.4 (Schur complement lemma, [2]). *Given constant symmetric matrices X, Y, Z with appropriate dimensions satisfying $X = X^T$, $Y = Y^T > 0$. Then $X + Z^T Y^{-1} Z < 0$ if and only if*

$$\begin{pmatrix} X & Z^T \\ Z & -Y \end{pmatrix} < 0 \quad \text{or} \quad \begin{pmatrix} -Y & Z \\ Z^T & X \end{pmatrix} < 0. \tag{2.11}$$

3. Main Results

Now we present a new delay-dependent condition for the uncertain system (2.1) satisfying the sector conditions (2.3).

Assumption 3.1. All the eigenvalues of matrix C are inside the unit circle.

Theorem 3.2. *Under Assumption 3.1, given $\alpha > 0$. The system* (2.1) *satisfying the sector condition* (2.3) *is α-exponentially stabilizable if there exist symmetric positive definite matrices P, Q, R, U, F, L; symmetric positive semidefinite matrices $Z = \mathrm{diag}(z_1, z_2, \ldots, z_m)$ and $J = \mathrm{diag}(j_1, j_2, \ldots, j_m)$; scalars $\epsilon_1 > 0$ and $\epsilon_2 > 0$; matrices N_i; $i = 1, 2, 3$ of appropriate dimension such that the following LMI holds:*

$$\begin{aligned}\mathcal{M}_1 &= \mathcal{M} - \begin{bmatrix}0 & I & -I & 0 & 0 & 0 & 0 & 0 & 0\end{bmatrix}^T \\ &\quad \times e^{-2\alpha h_2} U \begin{bmatrix}0 & -I & I & 0 & 0 & 0 & 0 & 0 & 0\end{bmatrix} < 0, \\ \mathcal{M}_2 &= \mathcal{M} - \begin{bmatrix}0 & -I & 0 & I & 0 & 0 & 0 & 0 & 0\end{bmatrix}^T \\ &\quad \times e^{-2\alpha h_2} U \begin{bmatrix}0 & I & 0 & -I & 0 & 0 & 0 & 0 & 0\end{bmatrix} < 0,\end{aligned} \tag{3.1}$$

$$\mathcal{M} = \begin{bmatrix} \phi_{11} & \phi_{12} & \phi_{13} & \phi_{14} & \phi_{15} & \phi_{16} & \phi_{17} & \phi_{18} & \phi_{19} \\ * & \phi_{22} & \phi_{23} & \phi_{24} & \phi_{25} & \phi_{26} & \phi_{27} & \phi_{28} & \phi_{29} \\ * & * & \phi_{33} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \phi_{44} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & \phi_{55} & \phi_{56} & \phi_{57} & \phi_{58} & \phi_{59} \\ * & * & * & * & * & \phi_{66} & \phi_{67} & \phi_{68} & 0 \\ * & * & * & * & * & * & \phi_{77} & 0 & 0 \\ * & * & * & * & * & * & * & \phi_{88} & 0 \\ * & * & * & * & * & * & * & * & \phi_{99} \end{bmatrix}, \tag{3.2}$$

where

$$\begin{aligned}
&\phi_{11} = PA + A^T P + Q + Q^T - e^{-2\alpha h_1} R - e^{-2\alpha h_2} R + 2\alpha x^T(t) P x(t), \qquad \phi_{12} = PA_1 + A^T N_1^T, \\
&\phi_{13} = e^{-2\alpha h_1} R, \qquad \phi_{14} = e^{-2\alpha h_2} R, \qquad \phi_{17} = PC, \qquad \phi_{18} = PD, \qquad \phi_{19} = \epsilon_i E_1^T, \\
&\phi_{16} = A^T L + A^T N_3^T, \qquad \phi_{15} = A^T H Z^T + PB + HKJ + A^T N_2^T + 2\alpha H Z, \\
&\phi_{22} = -2e^{-2\alpha h_2} U + N_1 A_1 + A_1^T N_1^T, \qquad \phi_{23} = e^{-2\alpha h_2} U, \qquad \phi_{24} = e^{-2\alpha h_2} U, \\
&\phi_{25} = A_1^T H Z^T + N_1 B + A_1^T N_2^T, \phi_{26} = A_1^T L - N_1 + A_1^T N_3^T, \qquad \phi_{27} = N_1 C, \qquad \phi_{28} = N_1 D, \\
&\phi_{29} = \epsilon_i E_3^T, \qquad \phi_{33} = -e^{-2\alpha h_1} Q - e^{-2\alpha h_1} R - e^{-2\alpha h_2} U, \qquad \phi_{44} = -e^{-2\alpha h_2} Q - e^{-2\alpha h_2} R - e^{-2\alpha h_2} U, \\
&\phi_{57} = Z H^T C + N_2 C, \qquad \phi_{55} = Z H^T B + B^T H Z^T - J - J^T + N_2 B + B^T N_2^T, \\
&\phi_{56} = B^T L - N_2 + B^T N_3^T, \qquad \phi_{59} = \epsilon_i E_2^T, \qquad \phi_{58} = Z H^T D + N_2 D, \qquad \phi_{88} = -\epsilon_i I, \\
&\phi_{99} = -\epsilon_i I, \qquad \phi_{66} = h_1^2 R + h_2^2 R + F + (h_2 - h_1)^2 U - L - L^T - N_3 - N_3^T, \\
&\phi_{67} = LC + N_3 C, \qquad \phi_{68} = LD + N_3 D, \qquad \phi_{77} = -e^{-2\alpha\eta}(1 - \eta_D) F.
\end{aligned} \tag{3.3}$$

The solution $x(t)$ of the system satisfies,

$$\|x(t)\| \leq \sqrt{\frac{a\|\phi\|^2 + b\|M_1\|^2 + c\|M_2\|^2}{\lambda_m(P)}}, \tag{3.4}$$

where $a = \lambda_M(P) + 2h_2\lambda_M(R) + h_2\lambda_M(U) + 2\lambda_M(HZKH^T)$, $b = 2\lambda_M(Q)(1 - e^{-2\alpha h_2})/2\alpha + 2h_2\lambda_M(R)((1-e^{-2\alpha h_2})/2\alpha) + h_2\lambda_M(U)((1-e^{-2\alpha h_2})/2\alpha)$, and $c = \lambda_M(F)((1-e^{-2\alpha\eta})/2\alpha)$, $\|M_1\| = \sup_{-m\leq s\leq 0}\|x(s)\|$, $\|M_2\| = \sup_{-m\leq s\leq 0}\|\dot{x}(s)\|$.

Proof. Using (2.5), the uncertain system (2.1) can be represented as

$$\begin{gathered}
\dot{x}(t) = Ax(t) + A_1x(t-h(t)) + Bf(\sigma(t)) + C\dot{x}(t-\eta(t)) + Dp(t), \\
p(t) = F(t)\big(E_1x(t) + E_2f(\sigma(t)) + E_3x(t-h(t))\big), \\
\sigma(t) = H^Tx(t) = \begin{bmatrix}\overline{h}_1 & \overline{h}_2 & \cdots & \overline{h}_m\end{bmatrix}^T x(t), \quad \forall t \geq 0, \\
x(s) = \phi(s), \quad s \in \left[-\max(h_2, \eta_2), 0\right].
\end{gathered} \tag{3.5}$$

We consider the following Lyapunov-Krasovskii functional

$$V(x(t)) = \sum_{i=1}^{8} V_i, \tag{3.6}$$

where

$$\begin{aligned}
V_1(x(t)) &= e^{2\alpha t}x^T(t)Px(t), \\
V_2(x(t)) &= \int_{t-h_1}^{t} e^{2\alpha s}x^T(s)Qx(s)ds, \\
V_3(x(t)) &= \int_{t-h_2}^{t} e^{2\alpha s}x^T(s)Qx(s)ds, \\
V_4(x(t)) &= h_1\int_{-h_1}^{0}\int_{t+s}^{t} e^{2\alpha s}\dot{x}^T(\tau)R\dot{x}(\tau)d\tau\, ds, \\
V_5(x(t)) &= h_2\int_{-h_2}^{0}\int_{t+s}^{t} e^{2\alpha s}\dot{x}^T(\tau)R\dot{x}(\tau)d\tau\, ds,
\end{aligned}$$

$$V_6(x(t)) = (h_2 - h_1)\int_{-h_2}^{-h_1}\int_{t+s}^{t} e^{2\alpha s}\dot{x}^T(\tau)U\dot{x}(\tau)d\tau\, ds,$$

$$V_7(x(t)) = \int_{t-\eta(t)}^{t} e^{2\alpha s}\dot{x}^T(s)F\dot{x}(s)ds,$$

$$V_8(x(t)) = 2\sum_{i=1}^{m}\lambda_i e^{2\alpha t}\int_0^{\sigma_i} f_i(\sigma_i)d\sigma_i. \tag{3.7}$$

Taking the derivative of $V(x_t)$ along the solution of system (3.5), we have

$$\begin{aligned}
\dot{V}_1(x(t)) &= 2\alpha e^{2\alpha t}x^T(t)Px(t) + 2e^{2\alpha t}x^T(t)P\dot{x}(t),\\
&= e^{2\alpha t}\Big[2\alpha x^T(t)Px(t) + 2x^T(t)P(Ax(t) + A_1x(t-h(t))\\
&\qquad +Bf(\sigma(t)) + C\dot{x}(t-\eta(t))Dp(t))\Big], \dot{V}_2(x(t))\\
&= e^{2\alpha t}\Big[x^T(t)Qx(t) - e^{-2\alpha h_1}x^T(t-h_1)Qx(t-h_1)\Big], \dot{V}_3(x(t))\\
&= e^{2\alpha t}\Big[x^T(t)Qx(t) - e^{-2\alpha h_2}x^T(t-h_2)Qx(t-h_2)\Big], \dot{V}_4(x(t))\\
&\le e^{2\alpha t}\left[h_1^2\dot{x}^T(t)R\dot{x}(t) - h_1e^{-2\alpha h_1}\int_{t-h_1}^{t}\dot{x}^T(s)R\dot{x}(s)ds\right], \dot{V}_5(x(t))\\
&\le e^{2\alpha t}\left[h_2^2\dot{x}^T(t)R\dot{x}(t) - h_2e^{-2\alpha h_2}\int_{t-h_2}^{t}\dot{x}^T(s)R\dot{x}(s)ds\right], \dot{V}_6(x(t))\\
&\le e^{2\alpha t}\left[(h_2-h_1)^2\dot{x}^T(t)U\dot{x}(t) - (h_2-h_1)e^{-2\alpha h_2}\int_{t-h_2}^{t-h_1}\dot{x}^T(s)U\dot{x}(s)ds\right], \dot{V}_7(x(t))\\
&\le e^{2\alpha t}\Big[\dot{x}^T(t)F\dot{x}(t) - e^{-2\alpha\eta}(1-\eta_d)\dot{x}^T(t-\eta(t))F\dot{x}(t-\eta(t))\Big], \dot{V}_8(x(t))\\
&= 2\sum_{i=1}^{m}\lambda_i e^{2\alpha t}\left(2\alpha\int_0^{\sigma_i(t)} f_i(\sigma_i)d\sigma_i + f_i(\sigma_i(t))\dot{\sigma}_i(t)\right)\\
&\le e^{2\alpha t}\Big[4\alpha f^T(\sigma(t))Z\sigma(t) + 2f^T(\sigma(t))Z\dot{\sigma}(t)\Big]\\
&\le e^{2\alpha t}\Big[4\alpha f^T(\sigma(t))ZHx(t) + 2f^T(\sigma(t))ZH^T\dot{x}(t)\Big]\\
&= e^{2\alpha t}\Big[4\alpha f^T(\sigma(t))ZHx(t) + 2f^T(\sigma(t))ZH^T\\
&\qquad \times(Ax(t) + A_1x(t-h(t)) + Bf(\sigma(t)) + C\dot{x}(t-\eta(t)) + Dp(t))\Big].
\end{aligned} \tag{3.8}$$

Applying Proposition 2.3 and the Leibniz-Newton formula, we have

$$\begin{aligned}
-h_1\int_{t-h_1}^{t}\dot{x}^T(s)R\dot{x}(s)ds &\le -\left[\int_{t-h_1}^{t}\dot{x}(s)\right]^T R\left[\int_{t-h_1}^{t}\dot{x}(s)\right]\\
&\le -[x(t)-x(t-h_1)]^T R[x(t)-x(t-h_1)]\\
&= -x^T(t)Rx(t)+2x^T(t)Rx(t-h_1)-x^T(t-h_1)Rx(t-h_1),\\
-h_2\int_{t-h_2}^{t}\dot{x}^T(s)R\dot{x}(s)ds &\le -\left[\int_{t-h_2}^{t}\dot{x}(s)\right]^T R\left[\int_{t-h_2}^{t}\dot{x}(s)\right]\\
&\le -[x(t)-x(t-h_2)]^T R[x(t)-x(t-h_2)]\\
&= -x^T(t)Rx(t)+2x^T(t)Rx(t-h_2)-x^T(t-h_2)Rx(t-h_2).
\end{aligned} \tag{3.9}$$

Note that

$$\begin{aligned}
-(h_2-h_1)\int_{t-h_2}^{t-h_1}\dot{x}^T(s)U\dot{x}(s)ds =& -(h_2-h_1)\int_{t-h_2}^{t-h(t)}\dot{x}^T(s)U\dot{x}(s)ds\\
&-(h_2-h_1)\int_{t-h(t)}^{t-h_1}\dot{x}^T(s)U\dot{x}(s)ds\\
=& -(h_2-h(t))\int_{t-h_2}^{t-h(t)}\dot{x}^T(s)U\dot{x}(s)ds\\
&-(h(t)-h_1)\int_{t-h_2}^{t-h(t)}\dot{x}^T(s)U\dot{x}(s)ds\\
&-(h(t)-h_1)\int_{t-h(t)}^{t-h_1}\dot{x}^T(s)U\dot{x}(s)ds\\
&-(h_2-h(t))\int_{t-h(t)}^{t-h_1}\dot{x}^T(s)U\dot{x}(s)ds.
\end{aligned} \tag{3.10}$$

Using Proposition 2.3 gives

$$\begin{aligned}
-(h_2-h(t))\int_{t-h_2}^{t-h(t)}\dot{x}^T(s)U\dot{x}(s)ds &\le -\left[\int_{t-h_2}^{t-h(t)}\dot{x}(s)ds\right]^T U\left[\int_{t-h_2}^{t-h(t)}\dot{x}(s)ds\right]\\
&\le -[x(t-h(t))-x(t-h_2)]^T U[x(t-h(t))-x(t-h_2)],\\
-(h(t)-h_1)\int_{t-h(t)}^{t-h_1}\dot{x}^T(s)U\dot{x}(s)ds &\le -\left[\int_{t-h(t)}^{t-h_1}\dot{x}(s)ds\right]^T U\left[\int_{t-h(t)}^{t-h_1}\dot{x}(s)ds\right]\\
&\le -[x(t-h_1)-x(t-h(t))]^T U[x(t-h_1)-x(t-h(t))].
\end{aligned} \tag{3.11}$$

Let $\beta = (h_2 - h(t))/(h_2 - h_1) \le 1$, so we have

$$\begin{aligned}
-(h_2 - h(t)) \int_{t-h(t)}^{t-h_1} \dot{x}^T(s) U \dot{x}(s) ds &= -\beta \int_{t-h(t)}^{t-h_1} (h_2 - h_1) \dot{x}^T(s) U \dot{x}(s) ds \\
&\le -\beta \int_{t-h(t)}^{t-h_1} (h(t) - h_1) \dot{x}^T(s) U \dot{x}(s) ds \\
&\le -\beta [x(t-h_1) - x(t-h(t))]^T U [x(t-h_1) - x(t-h(t))], \\
-(h(t) - h_1) \int_{t-h_2}^{t-h(t)} \dot{x}^T(s) U \dot{x}(s) ds &= -(1-\beta) \int_{t-h_2}^{t-h(t)} (h_2 - h_1) \dot{x}^T(s) U \dot{x}(s) ds \\
&\le -(1-\beta) \int_{t-h_2}^{t-h(t)} (h_2 - h(t)) \dot{x}^T(s) U \dot{x}(s) ds \\
&\le -(1-\beta) [x(t-h(t)) - x(t-h_2)]^T \\
&\quad \times U [x(t-h(t)) - x(t-h_2)].
\end{aligned} \tag{3.12}$$

Therefore from (3.10)–(3.12), we obtain

$$\begin{aligned}
-(h_2 - h_1) \int_{t-h_2}^{t-h_1} \dot{x}^T(s) U \dot{x}(s) ds \le\; & - [x(t-h(t)) - x(t-h_2)]^T U [x(t-h(t)) - x(t-h_2)] \\
& - [x(t-h_1) - x(t-h(t))]^T U [x(t-h_1) - x(t-h(t))] \\
& - \beta [x(t-h_1) - x(t-h(t))]^T U [x(t-h_1) - x(t-h(t))] \\
& - (1-\beta) [x(t-h(t)) - x(t-h_2)]^T U [x(t-h(t)) - x(t-h_2)].
\end{aligned} \tag{3.13}$$

We add the following zero equation:

$$2\xi^T(t) \overline{N} \left[A x(t) + A_1 x(t-h(t)) + B f(\sigma(t)) + C \dot{x}(t-\eta(t)) + D p(t) - \dot{x}(t) \right] = 0, \tag{3.14}$$

where $\overline{N} = [N_1^T \;\; N_2^T \;\; N_3^T]^T$, $\xi(t) = [x^T(t-h(t)) \;\; f^T(\sigma(t)) \;\; \dot{x}^T(t)]^T$ and by using the identity relation

$$-\dot{x}(t) + A x(t) + A_1 x(t-h(t)) + B f(\sigma(t)) + C \dot{x}(t-\eta(t)) + D p(t) = 0, \tag{3.15}$$

we have

$$\begin{aligned}
& -2\dot{x}^T(t) L \dot{x}(t) + 2\dot{x}^T(t) L A x(t) + 2\dot{x}^T(t) L A_1 x(t-h(t)) + 2\dot{x}^T(t) L B f(\sigma(t)) \\
& \quad + 2\dot{x}^T(t) L C \dot{x}(t-\eta(t)) + 2\dot{x}^T(t) L D p(t) = 0.
\end{aligned} \tag{3.16}$$

For system (2.1) with nonlinearity located in the sectors $[0, k_j]$ $(j = 1, 2, \ldots, m)$, if there exists $J = \mathrm{diag}(j_1, j_2, \ldots, j_m)$, then we have

$$j_i f_i(\sigma_i)\left[k_i h_i^T x(t) - f_i(\sigma_i)\right] \geq 0, \quad i = 1, 2, \ldots, m, \tag{3.17}$$

which is equivalent to

$$x^T(t) HKJf(\sigma(t)) - f^T(\sigma(t)) Jf(\sigma(t)) \geq 0. \tag{3.18}$$

Similarly, for any $\epsilon > 0$, from (3.4), we have

$$\begin{aligned} &-\epsilon p^T(t)p(t) + \epsilon\left(E_1 x(t) + E_2 f(\sigma(t)) + E_3 x(t-h(t))\right)^T \\ &\quad \times \left(E_1 x(t) + E_2 f(\sigma(t)) + E_3 x(t-h(t))\right) \geq 0. \end{aligned} \tag{3.19}$$

Hence, according to (3.9), (3.13), and by adding the zero term (3.16) and (3.18)-(3.19), we get

$$\begin{aligned} \dot{V}(x(t)) \leq e^{2\alpha t}\Big\{ & 2\alpha x^T(t)Px(t) + 2x^T(t)P\big(Ax(t) + A_1 x(t-h(t)) + Bf(\sigma(t)) + C\dot{x}(t-\eta(t)) \\ & + Dp(t)\big) + x^T(t)Qx(t) - e^{-2\alpha h_1} x^T(t-h_1)Qx(t-h_1) \\ & - e^{-2\alpha h_2} x^T(t-h_2)Qx(t-h_2) + h_1^2 \dot{x}^T(t)R\dot{x}(t) + h_2^2 \dot{x}^T(t)R\dot{x}(t) - e^{-2\alpha h_1} x^T(t)Rx(t) \\ & + 2e^{-2\alpha h_1} x^T(t)Rx(t-h_1) + x^T(t)Qx(t) - e^{-2\alpha h_1} x^T(t-h_1)Rx(t-h_1) \\ & - e^{-2\alpha h_2} x^T(t-h_2)Rx(t-h_2) - e^{-2\alpha h_2} x^T(t)Rx(t) + 2e^{-2\alpha h_2} x^T(t)Rx(t-h_2) \\ & \times \dot{x}^T(t)F\dot{x}(t) - e^{-2\alpha\eta}(1-\eta_D)\dot{x}^T(t-\eta(t))F\dot{x}(t-\eta(t)) 2\alpha f^T(\sigma(t))ZHx(t) \\ & + 2f^T(\sigma(t))ZH^T\big(Ax(t) + A_1 x(t-h(t)) + Bf(\sigma(t)) + C\dot{x}(t-\eta(t)) + Dp(t)\big) \\ & - e^{-2\alpha h_2} x^T(t)Rx(t) + 2e^{-2\alpha h_2} x^T(t)Rx(t-h_2)(h_2-h_1)^2 \dot{x}^T(t)U\dot{x}(t) \\ & + 2x^T(t)HKJf(\sigma(t)) - 2f^T(\sigma(t))Jf(\sigma(t)) - 2\dot{x}^T(t)L\dot{x}(t) + 2\dot{x}^T(t)LAx(t) \\ & + 2\dot{x}^T(t)LA_1 x(t-h(t)) + 2\dot{x}^T(t)LBf(\sigma(t)) + 2\dot{x}^T(t)LC\dot{x}(t-\eta(t)) \\ & + 2\dot{x}^T(t)LDp(t) - \epsilon p^T(t)p(t) + \epsilon\big(E_1 x(t) + E_2 f(\sigma(t)) + E_3 x(t-h(t))\big)^T \\ & \times \big(E_1 x(t) + E_2 f(\sigma(t)) + E_3 x(t-h(t))\big) - e^{-2\alpha h_2}[x(t-h(t)) - x(t-h_2)]^T \\ & \times U[x(t-h(t)) - x(t-h_2)] \\ & - e^{-2\alpha h_2}[x(t-h_1) - x(t-h(t))]^T U[x(t-h_1) - x(t-h(t))]\Big\} \end{aligned}$$

$$
\begin{aligned}
&- \beta[x(t-h_1) - x(t-h(t))]^T U[x(t-h_1) - x(t-h(t))] \\
&- (1-\beta)[x(t-h(t)) - x(t-h_2)]^T U[x(t-h(t)) - x(t-h_2)] \\
= {}& e^{2\alpha t}\Big\{\zeta^T(t)\mathcal{M}\zeta(t) - \beta[x(t-h_1) - x(t-h(t))]^T e^{-2\alpha h_2} U[x(t-h_1) - x(t-h(t))] \\
&\quad -(1-\beta)[x(t-h(t)) - x(t-h_2)]^T e^{-2\alpha h_2} U[x(t-h(t)) - x(t-h_2)]\Big\} \\
= {}& e^{2\alpha t}\Big\{\zeta^T(t)\left[(1-\beta)\mathcal{M}_1 + \beta\mathcal{M}_2\right]\zeta(t)\Big\},
\end{aligned}
\tag{3.20}
$$

where $\mathcal{M}_1$ and $\mathcal{M}_2$ are defined as in (3.1), respectively, and $\zeta(t) = [x(t)\ \ x(t-h(t))\ \ x(t-h_1)\ \ x(t-h_2)\ \ f(\sigma(t))\ \ \dot{x}(t)\ \ \dot{x}(t-\eta(t))\ \ p(t)]$. By $(1-\beta)\mathcal{M}_1 + \beta\mathcal{M}_2 < 0$ holds if and only if $\mathcal{M}_1 < 0$ and $\mathcal{M}_2 < 0$. For showing the convergence rate, we have $\dot{V}(x(t)) \le 0$, and then $V(x(t)) \le V(x(0))$. However,

$$
\begin{aligned}
V_1(x(0)) &= e^{2\alpha t} x^T(0) P x(0) \le \lambda_{\max}(P)\|\phi\|^2, V_2(x(0)) \\
&= \int_{-h_1}^{0} e^{2\alpha s} x^T(s) Q x(s) ds, \le \lambda_{\max}(Q) \int_{-h_1}^{0} e^{2\alpha s} ds \|M_1\|^2 \\
&= \lambda_{\max}(Q) \frac{1 - e^{-2\alpha h_2}}{2\alpha} \|M_1\|^2, V_3(x(0)) \\
&= \int_{-h_2}^{0} e^{2\alpha s} x^T(s) Q x(s) ds, \le \lambda_{\max}(Q) \int_{-h_2}^{0} e^{2\alpha s} ds \|M_1\|^2 \\
&= \lambda_{\max}(Q) \frac{1 - e^{-2\alpha h_2}}{2\alpha} \|M_1\|^2, V_4(x(0)) \\
&= h_1 \int_{-h_1}^{0} \int_{s}^{0} e^{2\alpha s} \dot{x}^T(\tau) R \dot{x}(\tau) d\tau\, ds \\
&= h_1 \int_{-h_1}^{0} e^{2\alpha s} \left[x^T(0) R x(0) - x^T(s) R x(s)\right] ds \\
&\le h_2 \lambda_M(R) \int_{-h_1}^{0} e^{2\alpha s} ds \|\phi\|^2 - h_2 \lambda_M(R) \int_{-h_1}^{0} e^{2\alpha s} ds \|M_1\|^2 \\
&= h_2 \lambda_M(R) \frac{1 - e^{-2\alpha h_2}}{2\alpha} \|\phi\|^2 - h_2 \lambda_M(R) \frac{1 - e^{-2\alpha h_2}}{2\alpha} \|M_1\|^2, V_5(x(0)) \\
&= h_2 \int_{-h_2}^{0} \int_{s}^{0} e^{2\alpha s} \dot{x}^T(\tau) R \dot{x}(\tau) d\tau\, ds \\
&\le h_2 \lambda_M(R) \frac{1 - e^{-2\alpha h_2}}{2\alpha} \|\phi\|^2 - h_2 \lambda_M(R) \frac{1 - e^{-2\alpha h_2}}{2\alpha} \|M_1\|^2, V_6(x(0)) \\
&= h_2 \int_{-h_2}^{-h_1} \int_{s}^{0} e^{2\alpha s} \dot{x}^T(\tau) U \dot{x}(\tau) d\tau\, ds
\end{aligned}
$$

$$\begin{aligned}
&\leq h_2\lambda_M(U)\frac{1-e^{-2\alpha h_2}}{2\alpha}\|\phi\|^2 - h_2\lambda_M(U)\frac{1-e^{-2\alpha h_2}}{2\alpha}\|M_1\|^2, V_7(x(0))\\
&= \int_{\eta(0)}^{0} e^{2\alpha s}\dot{x}^T(s)F\dot{x}(s)ds, \leq \lambda_M(F)\int_{-\eta}^{0} e^{2\alpha s}ds\|M_2\|^2\\
&\leq \lambda_M(F)\frac{1-e^{-2\alpha\eta}}{2\alpha}\|M_2\|^2, V_8(x(t))\\
&= 2\sum_{i=1}^{m}\lambda_i e^{2\alpha t}\int_0^{\sigma_i} f_i(\sigma_i)d\sigma_i \leq 2\sum_{i=1}^{m}\lambda_i e^{2\alpha t}\int_0^{\sigma_i} k\sigma_i(t)d\sigma_i\\
&\leq 2\sum_{i=1}^{m}e^{2\alpha t}\lambda_i k\sigma_i^2(t) \leq 2\sum_{i=1}^{m}e^{2\alpha t}\lambda_i kx^T(t)HH^Tx(t), V_8(x(0))\\
&\leq 2\lambda_M\left(HZKH^T\right)\|\phi\|^2,
\end{aligned} \tag{3.21}$$

and $V(x(t)) \geq e^{2\alpha t}x^T(t)Px(t) \geq e^{2\alpha t}\lambda_m(P)\|x(t)\|^2$. Then, from Definition 2.1, we conclude that the equilibrium point is globally exponentially stable. This completes the proof. □

Remark 3.3. For system (2.1) with uncertainty located in the sector conditions (2.4), if there exists $J = \mathrm{diag}(j_1, j_2, \ldots, j_m)$, it follows that

$$r_i f_i(\sigma_i(t))h_i^T x(t) \geq 0, \quad i = 1, 2, \ldots, m, \tag{3.22}$$

which is equivalent to

$$x^T(t)HJf(\sigma(t)) \geq 0. \tag{3.23}$$

By replacing (3.18) with (3.23) and letting $\alpha = 0$, we obtain the following robust stability criterion.

Corollary 3.4. *Under Assumption 3.1, the system* (2.1) *satisfying the sector condition* (2.4) *is asymptotically stable if there exist symmetric positive definite matrices P, Q, R, U, F, L; symmetric positive semidefinite matrices $Z = \mathrm{diag}(z_1, z_2, \ldots, z_m)$ and $J = \mathrm{diag}(j_1, j_2, \ldots, j_m)$; scalars $\epsilon_1 > 0$ and $\epsilon_2 > 0$; matrices N_i; $i = 1, 2, 3$ of appropriate dimension such that the following LMI holds:*

$$\begin{aligned}
\mathcal{M}_1 &= \mathcal{M} - \begin{bmatrix}0 & I & -I & 0 & 0 & 0 & 0 & 0 & 0\end{bmatrix}^T\\
&\quad\times e^{-2\alpha h_2}U\begin{bmatrix}0 & -I & I & 0 & 0 & 0 & 0 & 0 & 0\end{bmatrix} < 0,\\
\mathcal{M}_2 &= \mathcal{M} - \begin{bmatrix}0 & -I & 0 & I & 0 & 0 & 0 & 0 & 0\end{bmatrix}^T\\
&\quad\times e^{-2\alpha h_2}U\begin{bmatrix}0 & I & 0 & -I & 0 & 0 & 0 & 0 & 0\end{bmatrix} < 0,
\end{aligned}$$

$$\mathcal{M} = \begin{bmatrix} \phi_{11} & \phi_{12} & \phi_{13} & \phi_{14} & \varpi_{15} & \phi_{16} & \phi_{17} & \phi_{18} & \phi_{19} \\ * & \phi_{22} & \phi_{23} & \phi_{24} & \phi_{25} & \phi_{26} & \phi_{27} & \phi_{28} & \phi_{29} \\ * & * & \phi_{33} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \phi_{44} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & \varpi_{55} & \phi_{56} & \phi_{57} & \phi_{58} & \phi_{59} \\ * & * & * & * & * & \phi_{66} & \phi_{67} & \phi_{68} & 0 \\ * & * & * & * & * & * & \phi_{77} & 0 & 0 \\ * & * & * & * & * & * & * & \phi_{88} & 0 \\ * & * & * & * & * & * & * & * & \phi_{99} \end{bmatrix}, \tag{3.24}$$

where

$$\varpi_{15} = A^T H Z^T + PB + HJ, \qquad \varpi_{55} = ZH^T B + B^T H Z^T. \tag{3.25}$$

Remark 3.5. The following stability criteria are presented for finite and infinite sector conditions, for systems without uncertainties.

Corollary 3.6. *Under Assumption 3.1, given $\alpha > 0$. The system* (2.1) *without uncertainties satisfying the sector condition* (2.3) *is α-exponentially stabilizable if there exist symmetric positive definite matrices P, Q, R, U, F, L; symmetric positive semidefinite matrices $Z = \operatorname{diag}(z_1, z_2, \dots, z_m)$ and $J = \operatorname{diag}(j_1, j_2, \dots, j_m)$; matrices N_i; $i = 1, 2, 3$ of appropriate dimension such that the following LMI holds:*

$$\begin{aligned} \mathcal{M}_1 &= \mathcal{M} - \begin{bmatrix} 0 & I & -I & 0 & 0 & 0 & 0 \end{bmatrix}^T \\ &\quad \times e^{-2\alpha h_2} U \begin{bmatrix} 0 & -I & I & 0 & 0 & 0 & 0 \end{bmatrix} < 0, \\ \mathcal{M}_2 &= \mathcal{M} - \begin{bmatrix} 0 & -I & 0 & I & 0 & 0 & 0 \end{bmatrix}^T \\ &\quad \times e^{-2\alpha h_2} U \begin{bmatrix} 0 & I & 0 & -I & 0 & 0 & 0 \end{bmatrix} < 0, \\ \mathcal{M} &= \begin{bmatrix} \phi_{11} & \phi_{12} & \phi_{13} & \phi_{14} & \phi_{15} & \phi_{16} & \phi_{17} \\ * & \phi_{22} & \phi_{23} & \phi_{24} & \phi_{25} & \phi_{26} & \phi_{27} \\ * & * & \phi_{33} & 0 & 0 & 0 & 0 \\ * & * & * & \phi_{44} & 0 & 0 & 0 \\ * & * & * & * & \phi_{55} & \phi_{56} & \phi_{57} \\ * & * & * & * & * & \phi_{66} & \phi_{67} \\ * & * & * & * & * & * & \phi_{77} \end{bmatrix}. \end{aligned} \tag{3.26}$$

Corollary 3.7. *Under Assumption 3.1, the system* (2.1) *without uncertainties satisfying the sector condition* (2.4) *is asymptotically stable if there exist symmetric positive definite matrices P, Q, R, U, F, L; symmetric positive semidefinite matrices $Z = \operatorname{diag}(z_1, z_2, \dots, z_m)$ and*

$J = \operatorname{diag}(j_1, j_2, \ldots, j_m)$*; matrices* N_i*;* $i = 1, 2, 3$ *of appropriate dimension such that the following LMI holds:*

$$\begin{aligned}
\mathcal{M}_1 &= \mathcal{M} - \begin{bmatrix}0 & I & -I & 0 & 0 & 0 & 0\end{bmatrix}^T \\
&\quad \times e^{-2\alpha h_2} U \begin{bmatrix}0 & -I & I & 0 & 0 & 0 & 0\end{bmatrix} < 0, \\
\mathcal{M}_2 &= \mathcal{M} - \begin{bmatrix}0 & -I & 0 & I & 0 & 0 & 0\end{bmatrix}^T \\
&\quad \times e^{-2\alpha h_2} U \begin{bmatrix}0 & I & 0 & -I & 0 & 0 & 0\end{bmatrix} < 0, \\
\mathcal{M} &= \begin{bmatrix}
\phi_{11} & \phi_{12} & \phi_{13} & \phi_{14} & \varpi_{15} & \phi_{16} & \phi_{17} \\
* & \phi_{22} & \phi_{23} & \phi_{24} & \phi_{25} & \phi_{26} & \phi_{27} \\
* & * & \phi_{33} & 0 & 0 & 0 & 0 \\
* & * & * & \phi_{44} & 0 & 0 & 0 \\
* & * & * & * & \varpi_{55} & \phi_{56} & \phi_{57} \\
* & * & * & * & * & \phi_{66} & \phi_{67} \\
* & * & * & * & * & * & \phi_{77}
\end{bmatrix}.
\end{aligned} \tag{3.27}$$

Remark 3.8. In this paper, the restriction that the state delay is differentiable is not required which allows the state delay to be fast time-varying. Meanwhile, this restriction is required in some existing result, see [8, 9, 16, 17].

Remark 3.9. It is worth pointing out that we can extend this method to more complex dynamical network models, such as neutral-type neural networks [18, 19] or BAM neutral-type neural networks [20, 21].

4. Numerical Examples

In this section, we provide numerical examples to show the effectiveness of our theoretical results.

Example 4.1. Consider the following nominal Lur'e system with time-varying delays which is studied in [8, 9, 17]:

$$\begin{aligned}
\dot{x}(t) - C\dot{x}(t - \eta(t)) &= Ax(t) + A_1 x(t - h(t)) + Bf(\sigma(t)), \\
\sigma(t) &= H^T x(t) = [h_1 \quad h_2]^T x(t), \quad \forall \geq 0,
\end{aligned} \tag{4.1}$$

where

$$\begin{aligned}
A &= \begin{bmatrix}-2 & 0.5 \\ 0 & -1\end{bmatrix}, \quad A_1 = \begin{bmatrix}1 & 0.4 \\ 0.4 & -1\end{bmatrix}, \quad B = \begin{bmatrix}-0.5 \\ -0.75\end{bmatrix}, \\
C &= \begin{bmatrix}0.2 & 0.1 \\ 0.1 & 0.2\end{bmatrix}, \quad H = \begin{bmatrix}0.2 \\ 0.6\end{bmatrix}.
\end{aligned} \tag{4.2}$$

Table 1: Upper delay bound h_2 with $h_1 = 0$.

η_D	Method	h_D	0.6	0.8	No restriction on $\dot{h}_D$
0.1	[8]	h_2	0.9888	0.7228	—
	Corollary 3.4	h_2	—	—	1.0651
0.5	[8]	h_2	0.7793	0.6282	—
	Corollary 3.4	h_2	—	—	0.9210
0.9	[8]	h_2	0.0983	0.0967	—
	Corollary 3.4	h_2	—	—	0.1177

Table 2: Upper delay bound h_2 with $h_1 = 0$.

η_D	Method	h_D	0.6	0.8	No restriction on $\dot{h}_D$
0.5	[8]	h_2	0.7793	0.6282	—
	[17]	h_2	0.7901	0.6321	—
	Corollary 3.6	h_2	—	—	0.9209
0.9	[8]	h_2	0.0983	0.0967	—
	[17]	h_2	0.0994	0.0981	—
	[9]	h_2	0.1086	0.1086	—
	Corollary 3.6	h_2	—	—	0.1181

Tables 1 and 2 give comparison of maximum allowable value of h_2 for (4.1) obtained in Corollary 3.6 with nonlinearity satisfying (2.3), where $k = 100$ and Corollary 3.7 with nonlinearity satisfying (2.4), respectively. We see that, when $h_1 = 0$, the maximum allowable bounds for h_2 obtained from Corollaries 3.6 and 3.7 are much better than those obtained in [8, 9, 17]. The results obtained in [8, 9] may not be used for the case when $h_1 \neq 0$. Moreover, the differentiability of the time delay $h(t)$ is not required in Corollaries 3.6 and 3.7.

We let $h(t) = 0.7|\cos t|$, $\eta(t) = 1$, $\phi(t) = [-0.1\cos t, \cos t]$, for all $t \in [-1, 0]$, and $f(x(t)) = \delta|x(t)|$, $|\delta| \leq 0.5$. Figure 1 shows the trajectories of solutions $x_1(t)$ and $x_2(t)$ of the nominal Lur'e system with time-varying delays (4.1).

Example 4.2. Consider the following uncertain Lur'e system with interval time-varying delay with the following parameters:

$$\begin{aligned} \dot{x}(t) - C\dot{x}(t-\eta(t)) = {} & (A + \Delta A(t))x(t) + (A_1 + \Delta A_1(t))x(t-h(t)) \\ & + (B + \Delta B(t))f(\sigma(t)), \end{aligned} \tag{4.3}$$

where

$$\begin{gathered} A = \begin{bmatrix} -2 & 0 \\ 0 & -0.9 \end{bmatrix}, \quad A_1 = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} -0.2 \\ -0.3 \end{bmatrix}, \quad C = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}, \\ H = \begin{bmatrix} 0.4 \\ 0.8 \end{bmatrix}, \quad D = E_1 = E_3 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, \quad E_2 = \begin{bmatrix} 0.1 \\ 0 \end{bmatrix}. \end{gathered} \tag{4.4}$$

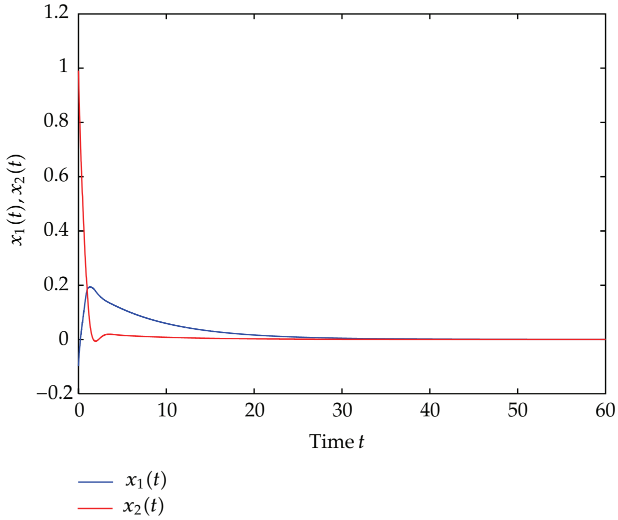

Figure 1: The trajectories of $x_1(t)$ and $x_2(t)$ of the the nominal Lur'e system with time-varying delays (4.1).

Solution. From the conditions (3.1) of Theorem 3.2, we let $\alpha = 0.4$, $h_1 = 0.1$, $h_2 = 0.4$, $t = 0.8$, $\eta = 0.1$, and $K = 0.5$. By using the LMI Tool-box in MATLAB, we obtain

$$\begin{gathered} P = \begin{bmatrix} 28.6713 & -0.2237 \\ -0.2237 & 2.4026 \end{bmatrix}, \qquad Q = \begin{bmatrix} 25.1176 & 0.7616 \\ 0.7616 & 1.0772 \end{bmatrix}, \qquad U = \begin{bmatrix} 23.6798 & 3.0045 \\ 3.0045 & 3.1148 \end{bmatrix}, \\ F = \begin{bmatrix} 7.0939 & -0.3657 \\ -0.3657 & 0.8018 \end{bmatrix}, \qquad N1 = \begin{bmatrix} -3.1736 & 0.4426 \\ -1.4601 & 1.2008 \end{bmatrix}, \qquad L = \begin{bmatrix} 16.5554 & 0 \\ 0 & 16.5554 \end{bmatrix}, \\ N3 = \begin{bmatrix} -5.5006 & -0.4441 \\ -0.9892 & -14.4101 \end{bmatrix}, \qquad N2 = \begin{bmatrix} -1.6410 & 0.2843 \end{bmatrix}, \qquad R = \begin{bmatrix} 12.0906 & 0.2313 \\ 0.2313 & 2.2354 \end{bmatrix}, \\ Z = 0.9717, \qquad J = 4.7851, \qquad e_1 = 10.2112, \qquad e_2 = 18.1870. \end{gathered} \tag{4.5}$$

Thus, the system (2.1), is 0.4-exponentially stabilizable. Given $\alpha > 0$, we will give the values of the maximum allowable upper bounds of the uncertain Lur'e system with interval time-varying delay (4.3) for difference η_d of the delay for different decay rates $0.1 \le \alpha \le 0.4$. From Theorem 3.2, we obtain the maximum allowable upper bound of the time-varying delay h_2, as shown in Table 3.

We let $h(t) = 0.1 + 0.65|\sin t|$, $\eta(t) = 0.5 + 0.4|\sin t|$, $\phi(t) = [-\cos t,\ \cos t]$, for all $t \in [-0.9, 0]$, and $f(x(t)) = \delta|x(t)|$, $|\delta| \le 0.5$. Figure 2 shows the trajectories of solutions $x_1(t)$ and $x_2(t)$ of the uncertain Lur'e system with interval time-varying delay (4.3).

5. Conclusions

In this paper, we have investigated the delay-dependent robust stability criteria for uncertain neutral and Lur'e dynamical systems with sector-bounded nonlinearity. Based on Lyapunov-krasovskii theory, new delay-dependent sufficient conditions for robust stability have been

Table 3: Maximum allowable upper bounds h_2 of the uncertain Lur'e system with interval time-varying delay (4.3) for different values of the η_d and decay rates.

	$\eta_d = 0.2$	$\eta_d = 0.4$	$\eta_d = 0.6$	$\eta_d = 0.8$
$\alpha = 0.1$	0.7423	0.7094	0.6541	0.5294
$\alpha = 0.2$	0.6798	0.6513	0.6031	0.4923
$\alpha = 0.3$	0.6291	0.6037	0.5606	0.4604
$\alpha = 0.4$	0.5870	0.5637	0.5245	0.4326

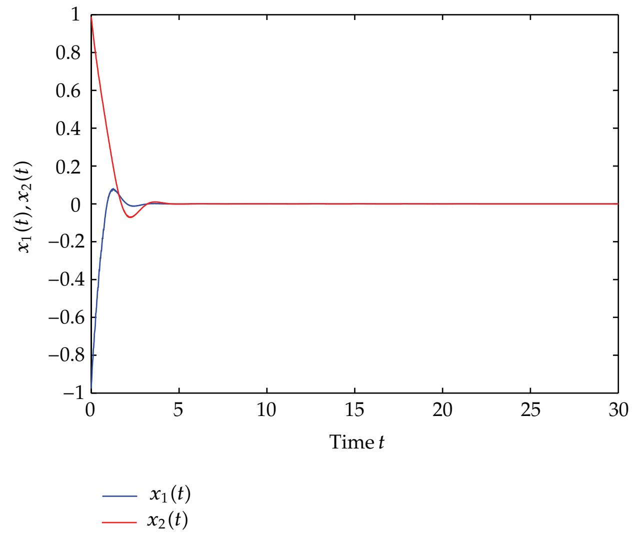

Figure 2: The trajectories of $x_1(t)$ and $x_2(t)$ of the uncertain Lur'e system with interval time-varying delay (4.3).

derived in terms of LMIs. The interval time-varying delay function is not required to be differentiable which allows time-delay function to be a fast time-varying function. The global exponential stability for uncertain neutral and Lur'e dynamical systems with some conditions are investigated. Numerical examples are given to illustrate the effectiveness of the theoretic results which show that our results are much less conservative than some existing results in the literature.

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