Hindawi Publishing Corporation
Abstract and Applied Analysis
Volume 2012, Article ID 948480, 13 pages
doi:10.1155/2012/948480

Research Article

Dynamic Properties of a Forest Fire Model

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Received 11 June 2012; Revised 7 September 2012; Accepted 17 September 2012

Academic Editor: Bashir Ahmad

The reaction-diffusion equations have been widely used in physics, chemistry, and other areas. Forest fire can also be described by such equations. We here propose a fighting forest fire model. By using the normal form approach theory and center manifold theory, we analyze the stability of the trivial solution and Hopf bifurcation of this model. Finally, we give the numerical simulations to illustrate the effectiveness of our results.

1. Introduction

The forest fire is an important issue in the world. It has brought us huge losses. It not only burns our forests but also destroys the local ecological environment. Many factors lead to forest fires. Several authors have studied them in depth [1–6]. Some important organizations, especially the USDA Forest Service, have also researched them in their themes [7].

Reaction-diffusion equations have been applied in forest fire model for several years. Some authors analyzed the dynamical behavior of the fire front propagations using hyperbolic reaction-diffusion equations [8]. Lots of articles related to percolation theory [9] and self-organized criticality [10] are trying to provide a different dynamical model for the spread of the fire. In this paper, the model describes the condition that people are putting out the fire when the fire is spreading. We analyze dynamic properties of the reaction-diffusion equations. Kolmogorov et al. proposed the famous KPP model [11] in the 1930s. From then on, it had been applied in various fields including forest fire:

$$u_t = d_1 u_{xx} + u + f(u), \quad x \in R,\ t \geq 0, \tag{1.1}$$

where $u = u(x,t)$ can be seen as the area of the burned forest. u_{xx} is a diffusion term of u in space, and d_1 is the diffusion coefficient. $f(u)$ is a nonlinear function. The equation can describe the speed of fire spreading. Zeldovich et al. gave the famous theory of combustion and explosions [12]. We can get inspiration from it:

The people will go to put out the fire as soon as they realize the forest fire. We can use a reaction-diffusion equation to describe it.

$$v_t = d_2 v_{xx} - cv + g(v). \tag{1.2}$$

In this equation, $v = v(x,t)$ is the area where the fire has been put out. v_{xx} is a diffusion term of v in space, and d_2 is the diffusion coefficient. c is the resurgence probability of v. $g(v)$ is a nonlinear function which represents the ability of people to put out the fire.

Now, let us consider the two reaction-diffusion equations together. As we know, u and v influence each other. Thus, f and g must be functions of u, v. We define $g(u,v)$ by referring to the combustion model [13]:

$$g(u,v) = \frac{uv}{b(u+1)}, \quad b > 0. \tag{1.3}$$

Since $g(u,v)$ has opposite effect on the fire area (or u), we can also define $f(u,v)$ by taking into account KPP model [8]:

$$f(u,v) = -au^2 - \frac{uv}{b(u+1)}. \tag{1.4}$$

Then we get a new model:

$$\begin{gathered} u_t - d_1 u_{xx} = u - au^2 - \frac{uv}{b(u+1)}, \\ v_t - d_2 v_{xx} = -cv + \frac{uv}{b(u+1)}, \\ u_x(0,t) = v_x(0,t) = 0, \qquad u_x(l\pi,t) = v_x(l\pi,t) = 0, \\ u(x,0) \geq 0, \quad v(x,0) \geq 0, \quad x \in (0, l\pi). \end{gathered} \tag{1.5}$$

Define

$$H = L^2[0,\pi] \times L^2[0,\pi] = \left\{ \begin{pmatrix} f \\ g \end{pmatrix} : f, g \in L^2[0,\pi] \right\}, \tag{1.6}$$

and an inner product is given by

$$\left\langle \begin{pmatrix} f_1 \\ g_1 \end{pmatrix}, \begin{pmatrix} f_2 \\ g_2 \end{pmatrix} \right\rangle = \langle f_1, f_2 \rangle_{L^2} + \langle g_1, g_2 \rangle_{L^2} = \frac{2}{\pi} \int_0^{\pi} \overline{f_1} f_2 \, dx + \frac{2}{\pi} \int_0^{\pi} \overline{g_1} g_2 \, dx, \tag{1.7}$$

where $(f_1, g_1)^T \in H, (f_2, g_2)^T \in H$. From the standpoint of biology, we are only interested in the dynamics of model (1.5) in the region:

$$R_+^2 = \{(u, v) \mid u > 0, v > 0\}. \tag{1.8}$$

2. Stability Analysis

Firstly, we consider the location stability [14, 15] and the number of the equilibria of model (1.5) in R_+^2. We can also study autowave solutions [16] of the model. The interior equilibrium point is a root of the following equation:

$$\begin{aligned} u - au^2 - \frac{uv}{b(u+1)} &= 0, \\ -cv + \frac{uv}{b(u+1)} &= 0. \end{aligned} \tag{2.1}$$

It is obvious that (2.1) has an only real solution $Y_0 = (u_0, v_0)$, where

$$u_0 = \frac{bc}{1-bc}, \qquad v_0 = ab\left(\frac{1}{a} - u_0\right)(1 + u_0), \tag{2.2}$$

and $b < 1/c(a+1)$.

Now, we analyze the asymptotic stability of (u_0, v_0) by Lyapunov function.

Lemma 2.1. *For the model* (1.5),

(1) *if* $a \geq 1$, (u_0, v_0) *is global asymptotic stability.*

(2) *if* $a < 1$ *and* $(1-a)/c \leq b \leq 1/(ac+c)$, (u_0, v_0) *also has global asymptotic stability.*

Proof. Defining

$$\omega(u, v) = \int_0^{l\pi} \int_{u_0}^{u} \frac{r/b(r+1) - c}{r/(r+1)} dr\, dx + \frac{1}{b} \int_0^{l\pi} \int_{v_0}^{v} \frac{s - v_0}{s} ds\, dx, \tag{2.3}$$

we can get

$$\frac{\partial \omega}{\partial t} = \frac{1}{b} \int_0^{l\pi} (h(u) - h(u_0))(p(u) - p(u_0)) dx + Y(t), \tag{2.4}$$

where

$$\begin{aligned} h(u) &= \frac{u}{u+1}, \qquad p(u) = (1 - au)(u+1), \\ Y(t) &= -d_1 c \int_0^{l\pi} \frac{h'(u)}{h^2(u)} u_x^2 dx + \frac{d_2 v_0}{b} \int_0^{l\pi} \frac{v_x^2}{v^2} dx. \end{aligned} \tag{2.5}$$

In what follows, we split it into two cases to prove. If $a \geq 1$, for all $u > 0$, $p'(u) < 0$, since $h'(u) = 1/(u+1)^2 > 0$, we can get

$$(h(u) - h(u_0))(p(u) - p(u_0)) \leq 0. \tag{2.6}$$

If $a < 1$ and $(1-a)/c \leq b \leq 1/(ac+c)$ (equal to $v_0 \leq b$), we can still get (2.6). That is to say

$$w_t(u, v) < 0. \tag{2.7}$$

We prove the conclusion. □

Because of the conclusion of Lemma 2.1, we always assume $a < 1$ and $0 < b < (1-a)/(2ac-c)$. Introducing perturbations $u^* = u - u_0, v^* = v - v_0$, and replace (u^*, v^*) with (u, v), for which model (1.5) yields

$$\begin{gathered}
u_t - d_1 u_{xx} = u + u_0 - a(u+u_0)^2 - \frac{(u+u_0)(v+v_0)}{b(u+u_0+1)}, \\
v_t - d_2 v_{xx} = -c(v+v_0) + \frac{(u+u_0)(v+v_0)}{b(u+u_0+1)}, \\
u_x(0,t) = v_x(0,t) = 0, \qquad u_x(l\pi, t) = v_x(l\pi, t) = 0, \\
u(x,0) \geq 0, \quad v(x,0) \geq 0, \quad x \in (0, l\pi).
\end{gathered} \tag{2.8}$$

Now we can get the linearized system of parametric model (2.8) at $(0,0)$,

$$\begin{pmatrix} u_t \\ v_t \end{pmatrix} = \left(\widetilde{\Delta} + L(b)\right)\begin{pmatrix} u \\ v \end{pmatrix}, \tag{2.9}$$

where

$$\widetilde{\Delta} = \begin{pmatrix} 0 & d_1 \dfrac{\partial^2}{\partial x^2} \\ d_2 \dfrac{\partial^2}{\partial x^2} & 0 \end{pmatrix}, \qquad L(b) = \begin{pmatrix} \dfrac{u_0(1-a-au_0)}{1+u_0} & -c \\ \dfrac{1-au_0}{1+u_0} & 0 \end{pmatrix}. \tag{2.10}$$

The eigenvalues of $\widetilde{\Delta}$ are as follows:

$$\left\{ -d_2 \frac{n^2}{l^2}, -d_1 \frac{n^2}{l^2} \right\}_{n=0}^{+\infty}, \tag{2.11}$$

and the corresponding eigenvectors as follows:

$$\left\{ \beta_n^1, \beta_{nk}^2 \right\}_{n=0}^{+\infty}, \tag{2.12}$$

where

$$\beta_n^1 = \begin{pmatrix} 0 \\ \cos\left(\frac{n}{l}x\right) \end{pmatrix}, \qquad \beta_n^2 = \begin{pmatrix} \cos\left(\frac{n}{l}x\right) \\ 0 \end{pmatrix}. \tag{2.13}$$

Define for all $y \in H$

$$y = \sum_{k=1}^{n} Y_k^T \begin{pmatrix} \beta_k^1 \\ \beta_k^2 \end{pmatrix}, \qquad Y_k = \begin{pmatrix} \langle y, \beta_k^1 \rangle \\ \langle y, \beta_k^2 \rangle \end{pmatrix}. \tag{2.14}$$

It is easy to get, $\lambda \in (\widetilde{\Delta} + L(b))$, if and only if the equation

$$\sum_{k=1}^{n} Y_k^T \left(E\lambda - L(b) - \begin{pmatrix} -d_1 \dfrac{k^2}{l^2} & 0 \\ 0 & -d_2 \dfrac{k^2}{l^2} \end{pmatrix} \right) \begin{pmatrix} \beta_k^1 \\ \beta_k^2 \end{pmatrix} = 0 \tag{2.15}$$

is held.

We obtain

$$\left| E\lambda - L(b) - \begin{pmatrix} -d_1 \dfrac{n^2}{l^2} & 0 \\ 0 & -d_2 \dfrac{n^2}{l^2} \end{pmatrix} \right| = 0. \tag{2.16}$$

Rewrite it as

$$\lambda^2 - T_n(b)\lambda + D_n(b) = 0, \tag{2.17}$$

where

$$\begin{aligned} T_n(b) &= bc\left(1 - a - 2a\frac{bc}{1-bc}\right) - \frac{(d_1+d_2)n^2}{l^2}, \\ D_n(b) &= c(1 - bc - abc) - d_2 bc\left(1 - a - \frac{2abc}{1-bc}\right)\frac{n^2}{l^2} + \frac{d_1 d_2 n^4}{l^4}. \end{aligned} \tag{2.18}$$

From (2.18), when $(1-a)/(ac+c) < b < (1-a)/c$ is held, we can get $T_n(b) < 0$, $D_n(b) > 0$. So the system's eigenvalues have negative real part, and (u_0, v_0) has local asymptotic stability. Then, we can conclude that the system has Hopf bifurcation [14] in $b \in (0, (1-a)/(ac+c))$.

Define

$$B = \left\{ b_0 \mid T_n(b_0) = 0, D_n(b_0) > 0, T_j(b_0) \neq 0, D_j(b_0) \neq 0, \forall j \neq n \right\}. \tag{2.19}$$

$b_0 \in B \cup (0, (1-a)/(ac+c))$, $\alpha(b) \pm i\omega(b)$ are characteristic roots of $\tilde{\Delta} + L(b)$, where

$$\alpha(b) = \frac{1}{2}\left(A(b) - \frac{d_1 + d_2}{l^2}n^2\right), \qquad \omega(b) = \sqrt{D_n(b) - \alpha^2(b)},$$
$$A(b) = bc\left(1 - a - 2a\frac{bc}{1-bc}\right). \tag{2.20}$$

Now we compute transversality condition:

$$\alpha'(b_0) = \frac{1}{2}a(1-b_0c)^2\left(\frac{1}{a} - 1 - \frac{4b_0c}{1-b_0c} - \left(\frac{b_0c}{1-b_0c}\right)^2\right)$$
$$\times \begin{cases} > 0, & 0 < b_0 < \dfrac{1}{c}\left(1 - \sqrt{\dfrac{2a}{1+a}}\right), \\ < 0, & \dfrac{1}{c}\left(1 - \sqrt{\dfrac{2a}{1+a}}\right) < b_0 < \dfrac{1-a}{ac+c}. \end{cases} \tag{2.21}$$

Now we consider $A(0) = A(b_0^B) = 0$ and $A(b)$ is positive in $(0, b_0^B)$. So we can get the maximum value of $A(b)$ (*defined as* $A(b^*)$):

Define

$$l_n = n\sqrt{\frac{d_1 + d_2}{A(b^*)}}, \quad n \in \mathbb{N}, \ A(b^*) = a\left(\sqrt{\frac{1}{a} + 1} - \sqrt{2}\right)^2, \tag{2.22}$$

for all $l \in (l_n, l_{n+1}]$, $0 \le j \le n$, $b_{j,-}^B$ and $b_{j,+}^B$ are two roots of the equation

$$A(b) = \frac{d_1 + d_2}{l^2}j^2. \tag{2.23}$$

It is easy to get

$$0 < b_{1,-}^B < \cdots < b_{n,-}^B < \frac{1}{c}\left(1 - \sqrt{\frac{2a}{1+a}}\right) < b_{n,+}^B < \cdots < b_{1,+}^B < b_0^B. \tag{2.24}$$

Then we give the condition of $D_n(b_{j,\pm}^B) \ne 0$ especially $D_n(b_{j,\pm}^B) > 0$.

As we know

$$D_n(b) \ge ac - d_2A(b^*)\frac{n^2}{l^2} + d_1d_2\frac{n^4}{l^4}. \tag{2.25}$$

Then $D_n(b) > 0$ is held if and only if

$$d_1 d_2 > 0,$$
$$(d_2 A(b^*))^2 - 4d_1 d_2 ac > 0. \tag{2.26}$$

Theorem 2.2. *Assume that $d_1, d_2, c > 0,\ 0 < a < 1$, and the equation is held:*

$$d_2 A^2(b^*) - 4d_1 ac > 0, \tag{2.27}$$

where

$$l_n = n\sqrt{\frac{d_1 + d_2}{A(b^*)}}, \quad n \in \mathbb{N},\ A(b^*) = a\left(\sqrt{\frac{1}{a} + 1} - \sqrt{2}\right)^2, \tag{2.28}$$

then for all $l \in (l_n, l_{n+1}]$, existing $b = b^B_{j,\pm}$ or $b = b^B_0$; there are Hopf bifurcations at the real solution of model (1.5).

Furthermore

$$0 < b^B_{1,-} < \cdots < b^B_{n,-} < \frac{1}{c}\left(1 - \sqrt{\frac{2a}{1+a}}\right) < b^B_{n,+} < \cdots < b^B_{1,+} < b^B_0. \tag{2.29}$$

3. Hopf Bifurcation

In the above section, we have already obtained the conditions which ensure that model (2.8) undergoes the Hopf bifurcation at the critical values b_0 or $b_{j,\pm}$ $(j = 1, \cdots)$. In the following part, we will study the direction and stability of the Hopf bifurcation based on the normal form approach theory and center manifold theory introduced by Hassard at al. [14].

Firstly, by the transformation $u^* = u - u_0,\ v^* = v - v_0$, and replacing (u^*, v^*) with (u, v), the parametric system (1.5) is equivalent to the following functional differential equation (FDE) system:

$$\frac{\partial U}{\partial t} = \left(\tilde{\Delta} + L(b_0)\right) + F(b_0, U), \tag{3.1}$$

where

$$U = (u, v)^T, \qquad F(b_0, U) = \begin{pmatrix} f - f_u u - f_v v \\ g - g_u u - g_v v \end{pmatrix}. \tag{3.2}$$

The adjoint operator of $L_n(b)$ is defined as

$$L^*(b_0) = \begin{pmatrix} d_1\dfrac{\partial^2}{\partial x^2} + f_u(b_0) & f_v(b_0) \\ g_u(b_0) & d_2\dfrac{\partial^2}{\partial x^2} + g_v(b_0) \end{pmatrix}. \tag{3.3}$$

It is easy to get

$$\langle u, L(b_0)v\rangle = \langle L^*(b_0)u, v\rangle. \tag{3.4}$$

From the discussions in Section 2, define $q^* = (a_n^*, b_n^*)^T \cos(n/l)x$. We have

$$L^*(b_0)q^* = -i\omega q^*, \qquad \langle q^*, q\rangle = 1, \qquad \langle q^*, \overline{q}\rangle = 0. \tag{3.5}$$

Decompose X as $X = X^c \oplus X^s$, where $X^c = \{zq + \overline{zq} \mid z \in C\}$ and $X^s = \{u \in x \mid \langle q^*, u\rangle = 0\}$. For all $(u,v) \in X$, existing $z \in \mathbb{C}$ and $\omega = (\omega_1, \omega_2) \in X^s$, we can obtain

$$\begin{pmatrix} u \\ v \end{pmatrix} = zq + \overline{zq} + \begin{pmatrix} \omega_1 \\ \omega_2 \end{pmatrix}. \tag{3.6}$$

Rewrite (3.1) as

$$\begin{aligned} \dot{z} &= i\omega_0 z + \langle q^*, F_0^*\rangle, \\ \dot{\omega} &= L(b_0)\omega + H(z, \overline{z}, \omega), \end{aligned} \tag{3.7}$$

where

$$F_0^* = zq + \overline{zq} + \omega, \qquad H(z, \overline{z}, \omega) = F_0^* - \langle q^*, F_0^*\rangle q - \left\langle \overline{q^*}, F_0^* \right\rangle \overline{q}. \tag{3.8}$$

Using the same notations as in [11],

$$F_0^*(U) = \frac{1}{2}Q(U,U) + \frac{1}{6}C(U,U,U) + O\left|U^4\right|, \tag{3.9}$$

where $U = (u, v)$ and Q, C are symmetrical multilinear functions. We can compute

$$Q(q,q) = \begin{pmatrix} A_n^1 \\ A_n^2 \end{pmatrix}\cos^2\frac{n}{l}x, \qquad Q(q,\overline{q}) = \begin{pmatrix} B_n^1 \\ B_n^2 \end{pmatrix}\cos^2\frac{n}{l}x, \qquad C(q,q,\overline{q}) = \begin{pmatrix} C_n^1 \\ C_n^2 \end{pmatrix}\cos^3\frac{n}{l}x, \tag{3.10}$$

where

$$\begin{aligned}
A_n^1 &= f_{uu} a_n^2 + 2 f_{uv} a_n b_n + f_{vv} b_n^2, \\
A_n^2 &= g_{uu} a_n^2 + 2 g_{uv} a_n b_n + g_{vv} b_n^2, \\
B_n^1 &= f_{uu} |a_n|^2 + f_{uv} \left(a_n \overline{b_n} + \overline{a_n} b_n \right) + f_{vv} |b_n|^2, \\
B_n^2 &= g_{uu} |a_n|^2 + g_{uv} \left(a_n \overline{b_n} + \overline{a_n} b_n \right) + g_{vv} |b_n|^2, \\
C_n^1 &= f_{uuu} |a_n|^2 a_n + f_{uuv} \left(2|a_n|^2 b_n + a_n^2 \overline{b_n} \right) + f_{uvv} \left(2 b_n^2 a_n + b_n^2 \overline{a_n} \right) + f_{vvv} |b_n|^2 b_n, \\
C_n^2 &= g_{uuu} |a_n|^2 a_n + g_{uuv} \left(2|a_n|^2 b_n + a_n^2 \overline{b_n} \right) + g_{uvv} \left(2 b_n^2 a_n + b_n^2 \overline{a_n} \right) + g_{vvv} |b_n|^2 b_n.
\end{aligned} \tag{3.11}$$

Define

$$H(z, \overline{z}, \omega) = \frac{1}{2} H_{20} z^2 + H_{11} z \overline{z} + \frac{1}{2} H_{02} \overline{z}^2 + o(|z||\omega|), \tag{3.12}$$

where

$$\begin{aligned}
H_{20} &= Q(q, q) - \langle q^*, Q(q, q) \rangle q - \left\langle \overline{q^*}, Q(q, q) \right\rangle \overline{q}, \\
H_{11} &= Q(q, \overline{q}) - \langle q^*, Q(q, \overline{q}) \rangle q - \left\langle \overline{q^*}, Q(q, \overline{q}) \right\rangle \overline{q}.
\end{aligned} \tag{3.13}$$

On the center manifold, we have

$$\omega = \frac{1}{2} \omega_{20} z^2 + \omega_{11} z \overline{z} + \frac{1}{2} \omega_{02} \overline{z}^2 + o\left(|z|^3 \right). \tag{3.14}$$

We can obtain

$$w_{20} = [2 i \omega_0 I - L_n(b_0)]^{-1} H_{20}, \qquad \omega_{11} = [L_n(b_0)]^{-1} H_{11}. \tag{3.15}$$

Comparing (3.9) and (3.13), we can get

$$H_{20} = \begin{cases} Q(q, q) - \begin{pmatrix} A_n^1 \\ A_n^2 \end{pmatrix} \cos^2 \dfrac{n}{l} x - \begin{pmatrix} A_n^1 \\ A_n^2 \end{pmatrix} \left(\dfrac{1}{2} \cos^2 \dfrac{n}{2} x + \dfrac{1}{2} \right), & n \in \mathbb{N}^*, \\ \begin{pmatrix} A_0^1 \\ A_0^2 \end{pmatrix} - \langle q^*, Q(q, q) \rangle \begin{pmatrix} a_0 \\ b_0 \end{pmatrix} - \langle q^*, Q(q, q) \rangle \begin{pmatrix} \overline{a_0} \\ \overline{b_0} \end{pmatrix}, & n = 0. \end{cases} \tag{3.16}$$

Similarly

$$\omega_{11}=\begin{cases}-\dfrac{1}{2}[L(b_o)]^{-1}\begin{pmatrix}C_n^1\\ C_n^2\end{pmatrix}\left(\cos^2\dfrac{n}{2}+1\right), & n\in\mathbb{N}^*,\\ -[L(b_0)]^{-1}\left[\begin{pmatrix}c_0^1\\ c_0^2\end{pmatrix}-\langle q^*,Q(q,\overline{q})\rangle\begin{pmatrix}a_0^1\\ b_0^2\end{pmatrix}-\left\langle\overline{q^*},Q(q,\overline{q})\right\rangle\begin{pmatrix}\overline{a_0}\\ \overline{b_0}\end{pmatrix}\right], & n=0.\end{cases}\tag{3.17}$$

Then on the center manifold rewrite (dU) as

$$\frac{dz}{dt}=i\omega_0 z+\langle q^*,F_0^*\rangle=i\omega_0 z+\sum_{2\le i+j\le 3}\frac{g_{ij}}{i!j!}z^i\overline{z}^j+o\left(|z|^4\right),\tag{3.18}$$

where

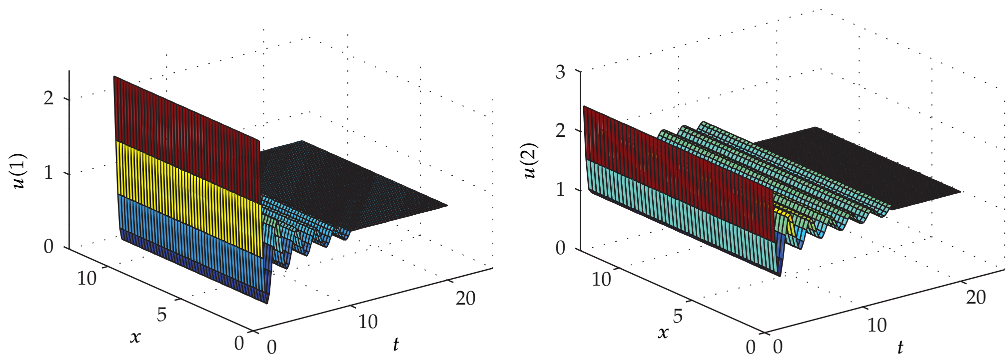
$$\begin{aligned}g_{20}&=\langle q^*,Q(q,q)\rangle=\frac{4c\omega_0a^2-a(1-a)^2\omega_0-2ca^2(3a-1)i}{(1-a^2)\omega_0},\\ g_{11}&=\langle q^*,Q(q,\overline{q})\rangle=\frac{a(a-q)\omega_0-2ca^2i}{(1+a)\omega_0},\\ g_{02}&=\langle q^*,Q(\overline{q},\overline{q})\rangle=\frac{a(1-a)^2+2c\omega_0a^2-4ca^2i}{(1-a^2)\omega_0},\\ g_{21}&=2\langle q^*,Q(w_{11},q)\rangle+\langle q^*,Q(w_{20},\overline{q})\rangle+\langle q^*,C(q,q,\overline{q})\rangle\\ &=\frac{-12a^3(1-a)\omega_0-8ca^3\omega_0+4ca^{3(3-5a)i}}{(1-a)(1+a)^2\omega_0}.\end{aligned}\tag{3.19}$$

Using conclusions in [14] we can get

$$C_1(b)=\frac{g_{20}g_{11}(3\alpha(b)+i\omega(b))}{2(\alpha^2(b)+\omega^2(b))}+\frac{|g_{11}|^2}{\alpha(b)+i\omega(b)}+\frac{|g_{02}|^2}{2(\alpha(b)+3i\omega(b))}+\frac{g_{21}}{2},\tag{3.20}$$

then

$$\begin{gathered}C_1(b_0)=\frac{i}{2\omega_0}\left(g_{20}g_{11}-2|g_{11}|^2-\frac{1}{3}|g_{02}|^2\right)+\frac{g_{21}}{2},\\ \mu_2=-\frac{\operatorname{Re}\{c_1(0)\}}{\operatorname{Re}\{b'(\tau_n)\}},\qquad \beta_2=2\operatorname{Re}\{c_1(0)\},\\ T_2=\frac{2\pi}{\omega_0}\left(1+\tau_2s^2\right)+o\left(s^4\right),\end{gathered}\tag{3.21}$$



(a) (b)

Figure 1: When $b = 0.1$, the positive equilibrium point Y_0 is asymptotically stable.

where

$$\tau_2 = -\frac{1}{\omega_0}\left[\operatorname{Im}(c_1(b_0)) - \frac{\operatorname{Re}(c_1(b_0))}{\alpha'(b_0)}\omega'(b_0)\right]. \tag{3.22}$$

Now we give a conclusion.

Conclusion. (1) The sign of μ_2 determines the direction of Hopf bifurcation. When $\mu_2 > 0$, the Hopf bifurcation is supercritical; when $\mu_2 < 0$, the Hopf bifurcation is subcritical.

(2) β_2 determines the stability of bifurcated periodic solutions. When $\beta_2 < 0$, the periodic solutions are stable; when $\beta_2 > 0$, the periodic solutions are unstable.

(3) T_2 determines the period of bifurcated periodic solutions. When $T_2 > 0$, the period increases; when $T_2 < 0$, the period decreases.

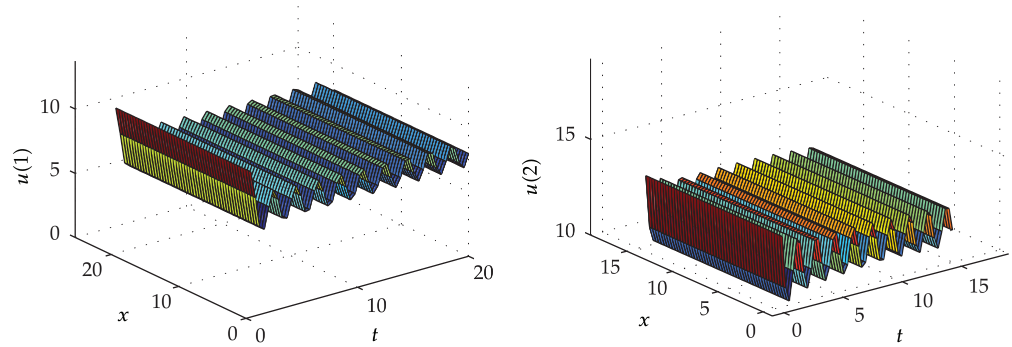
4. Example

In this section, we use a numerical simulation to illustrate the analytical results we obtained in previous sections.

Let $x \in (0, l\pi),\ d_1 = 1,\ d_2 = 3,\ c = 4,\ a = 0.0588$. The system (1.5) is

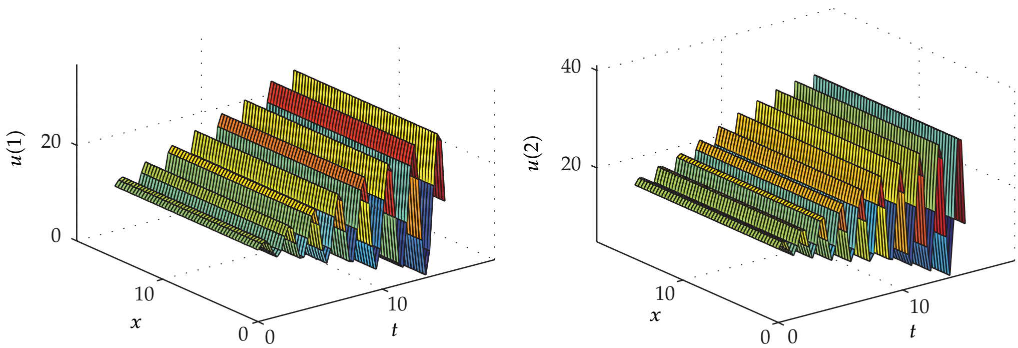
$$\begin{gathered}
u_t - u_{xx} = u - 0.0588u^2 - \frac{uv}{b(u+1)}, \\
v_t - 3v_{xx} = -4v + \frac{uv}{b(u+1)}, \\
u_x(0,t) = v_x(0,t) = 0, \qquad u_x(l\pi,t) = v_x(l\pi,t) = 0, \\
u(x,0) \geq 0, \quad v(x,0) \geq 0, \quad x \in (0, l\pi).
\end{gathered} \tag{4.1}$$

Now we determine the direction of a Hopf bifurcation with $b \in B$ and the other properties of bifurcating periodic solutions based on the theory of Hassard et al. [14], as discussed before. By means of software MATLAB 7.0, we can get some figures to illustrate the effectiveness of



(a) (b)

Figure 2: When $b = 0.1253$, periodic solutions occur from Y_0.



(a) (b)

Figure 3: When $b = 1.3$, the positive equilibrium point Y_0 is unstable.

our results. $b_0^B = 0.2222$, $b^* = 0.1667$, $A(b^*) = 0.4706$, $l_n = 2.9155n$, and (2.27) is held. When $n = 2$, $l = 3.1162$, $(l_n, l_{n+1}] = (2.9155, 5.8302]$, $l \in (l_n, l_{n+1}]$. We can get $B = (0.1253, 0.1944, 0.2222)$. The only positive equilibrium point of (4.1) is $Y_0 = (4b/(1-4b), b(0.0588-u_0)(1+u_0)/0.0588)$. When $b = 0.1253$, we can compute

$$\operatorname{Re} c_1(0.1253) = 0.2856 > 0, \qquad \mu_2 = -0.3423 < 0, \qquad T_2 = 1.2346 > 0. \tag{4.2}$$

The positive equilibrium point of (4.1) is unstable and the Hopf bifurcation is supercritical. The positive equilibrium point Y_0 of system (4.1) is locally asymptotically stable when $b = 0.1$ as is illustrated by computer simulations in Figure 1. And periodic solutions occur from Y_0 when $b = 0.1253$ as is illustrated by computer simulations in Figure 2. When $b = 1.3$, we can easily show that the positive equilibrium point Y_0 is unstable as is illustrated in Figure 3. From the above results, we can conclude that the stability properties of the system could switch with parameter b.

Acknowledgments

This research was supported by the National Natural Science Foundations of Heilongjiang province (C200941). The authors are grateful to the anonymous reviewers for their valuable comments and suggestions which essentially improve the quality of the paper.

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