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Research Article

Option-Game Approach to Analyze Technology Innovation Investment under Fuzzy Environment

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Based on the model of symmetric and asymmetric duopoly option game, this paper discusses the present value of profit flows and the sunk investment costs for the trapezoidal fuzzy number. It constructs the fuzzy expressions of the investment value and investment threshold of followers and leaders under fuzzy environment and conducts numerical analysis. This offers a kind of explanation to the investment strategies under fuzzy environment.

1. Introduction

Under the condition of uncertain symmetric duopoly game model, the model was firstly proposed by Smets [1]. For the research of asymmetric enterprises, Huisman [2] considered the initial investment cost and two asymmetry enterprise option-game models. Technology investment extends the already existing real option models by the introduction of the game theory. Under the environment of imperfect competition, the original idea of decision-making model of investment cost asymmetry investment came from the Grenadier's [3] duopoly model. Kong and Kwok [4] studied the real options problem of strategic investment game between two asymmetry enterprises. Pawlina and Kort [5] studied the influence to the investment decision caused by the difference of the enterprise and discussed the relationship of the value of enterprises and the different investment cost. Zmeškal [6] used the European call option of fuzzy random variables to assess the value of the enterprise. Yoshida [7] constructed symmetric triangular fuzzy numbers with the assumption that the stock price was fuzzy and stochastic; the fuzzy objective definition was introduced with fuzzy expectation with the assumption that the fuzzy degree and stock price were in proportion. It gave rise to the pricing formula of the European option and the fuzzy hedge strategy. Hui and Yong [8] studied the influence of the enterprise investment strategy given by the investment cost's variance and the time required by the success of technical innovation strategy. Carlsson et al. [9] considered the rates' fuzzy option formula of fuzzy relation and used the optimization theory to build R & D project's investment decision model. Liu [10] applied the fuzzy theory to build the model of currency option pricing, which converts the risk-free interest rate. The volatility and the original asset price to the fuzzy number under fuzzy environment, based on the model of equivalent martingale measuring and Black-Scholes.

The Problem of Trapezoidal Fuzzy Number

The concept of fuzzy set was initiated by Zadeh [11]. From the definition of Carlsson and Fullér [12], a fuzzy set A is a fuzzy set of the real line, (fuzzy) convex, and continuous membership function of bounded support; the family of fuzzy numbers will be denoted by F, for any for all $A \in F$; we will use the notation $[A]^{\gamma} = [a_1(\gamma), a_2(\gamma)]$ as γ-sets of A; If $A \in F$ is a fuzzy number and $x \in \mathbb{R}$ is a real number, then $A(x)$ may be interpreted as the degree of possibility of the statement "x is A".

Definition 1.1 (See [7, 12]). A fuzzy set $A \in F$ is called a trapezoidal fuzzy number with core $[a, b]$, left width α, and right width β, if its membership function has the following form:

$$A(t) = \begin{cases} 1 - \dfrac{a-t}{\alpha} & \text{if } a - \alpha \leq t < a \\ 1 & \text{if } a \leq t \geq b \\ 1 - \dfrac{b-t}{\beta} & \text{if } b \leq t \leq b + \beta \\ 0 & \text{Otherwise.} \end{cases} \tag{1.1}$$

And we use the notation $A = (a, b, \alpha, \beta)$, for all $\gamma \in [0, 1]$, then it can easily be shown that

$$[A]^{\gamma} = \left[a - (1-\gamma)\alpha, b - (1-\gamma)\beta\right]. \tag{1.2}$$

The support of A is $(a - \alpha, b + \beta)$.

Let $[A]^{\gamma} = (a_1(\gamma), b_1(\gamma), \alpha_1(\gamma), \beta_1(\gamma))$, $[B]^{\gamma} = (a_2(\gamma), b_2(\gamma), \alpha_2(\gamma), \beta_2(\gamma))$ be fuzzy numbers and let $\lambda \in \mathbb{R}$ be a real number; using the extension principle, we can verify the following rules for addition and scalar multiplication of fuzzy numbers

$$[A + B]^{\gamma} = \left[a_1(\gamma) + a_2(\gamma), b_1(\gamma) + b_2(\gamma), \alpha_1(\gamma) + \alpha_2(\gamma), \beta_1(\gamma) + \beta_2(\gamma)\right], \tag{1.3}$$

$$[A - B]^{\gamma} = \left[a_1(\gamma) - b_2(\gamma), b_1(\gamma) - a_2(\gamma), \alpha_1(\gamma) + \alpha_2(\gamma), \beta_1(\gamma) + \beta_2(\gamma)\right], \tag{1.4}$$

$$[\lambda A]^{\gamma} = \lambda [A]^{\gamma}. \tag{1.5}$$

From Carlsson and Fullér [12] and Yoshida [7], it is easy to see that the (crisp) possibility mean (or expected) value of A and the (possibility) variance of A:

$$E(A) = \int_0^1 \gamma(a - (1-\gamma)\alpha + b + (1-\gamma)\beta) = \frac{a+b}{2} + \frac{\beta - \alpha}{6},$$
$$\delta^2(A) = \frac{(b-a)^2}{4} + \frac{(b-a)(\alpha+\beta)}{6} + \frac{(\alpha+\beta)^2}{24}. \tag{1.6}$$

This paper reviews the symmetrical and asymmetrical enterprises technology innovation investment decision problem under fuzzy environment, discusses the model under fuzzy environment, and concludes followers' and leaders' fuzzy expression of the investment and the critical value under fuzzy environment. By the technology of fuzzy simulation and data, we can find that the symmetrical and asymmetrical enterprises have the optimal investment strategy under fuzzy environment.

2. Symmetric Model in Fuzzy Environment

2.1. Basic Assumptions

Assuming that there exist two technology innovation investment enterprises with neutral and rational risk in market and assuming that both of them have opportunities of technology innovation and irreversible investment, where such investment opportunity exists forever, their competition and strategies are both symmetrical. The sunk investment cost I is a fuzzy number, because the enterprise technology innovation investment sunk cost is often difficult to be expressed by a number; using fuzzy numbers to estimate is more objective and actual. Market is in the symmetrical duopoly, supposing that the two enterprises are of nonconstraint conspiracy. The present value of expected profit value of technology innovation investment project has uncertainty and fuzziness. In order to incorporate uncertainty, assuming the uncertainty factor $X(t)$ of market demanding follows a geometric Brown motion process

$$dX(t) = \mu X(t)dt + \sigma X(t)dz, \tag{2.1}$$

where $\mu \in (0, r)$ is drift, and σ is the rate of fluctuation. For the technical innovation of enterprises, assuming their anticipated profits of the present value's stream depends on $X(t)$, and the firms' investment behavior at the same time, the expected profit of the present value's stream is a trapezoidal fuzzy number and can be given by

$$\pi(t) = X(t)D(N_i, N_j), \tag{2.2}$$

where $D(N_i, N_j)$ is determined by the market demand parameters. Therefore, according to the trapezoidal fuzzy number (1.5), the nature of $X(t)$ is also for the trapezoidal fuzzy number. For any $k \in \{i, j\}$, we can conclude

$$N_k = \begin{cases} 0 & \text{The enterprise } k \text{ has not invest} \\ 1 & \text{The enterprise } k \text{ has to invest.} \end{cases} \tag{2.3}$$

If the investment can increase the expected income stream and have advantage of the first mover, the following restrictions on $D(N_i, N_j)$ are implied:

$$D(1,0) > D(1,1) > D(0,0) > D(0,1). \tag{2.4}$$

Further, we assume that there is a first mover advantage to investment

$$D(1,0) - D(0,0) > D(1,1) - D(0,1). \tag{2.5}$$

2.2. The Investment Value and Investment Threshold Value of Followers

According to the calculation rules of the trapezoidal fuzzy number, the solution $(X/X^f)^{\theta_1}$ is difficult, according to the literature [6, 7, 9, 10, 12] in solving similar problems by expectations to approximate estimation methods; we also can use $(E(X)/E(X^f))^{\theta_1}$ to estimate. We have taken the similar approach, just using $E(X) < E(X^f)$ instead of $X < X^f$. According to the equation of Itô's lemma and the Behrman equation, the option value $F(X)$ of followers can be shown by the following partial differential equation. According to the method of Dixit and Pindyck [13] and Huisman [2], we have the following investment value:

$$F^f(X) = \begin{cases} \left(\dfrac{X}{X^f}\right)^{\theta_1} \dfrac{X^f(D(1,1) - D(0,1))}{(r-\mu)\theta_1} \\ \qquad + \dfrac{(X_1, X_2, \alpha_1, \beta_1) D(0,1)}{r-\mu} & \text{if } E(X) < E(X^f) \\ \dfrac{(X_1, X_2, \alpha_1, \beta_1) D(1,1)}{r-\mu} - (I_1, I_2, \alpha_2, \beta_2) & \text{if } E(X) \geq E(X^f). \end{cases} \tag{2.6}$$

Here, it is optimal for the investment to invest when $E(X) \geq E(X^f)$. Equation (2.6) is derived by solving the optimal stopping problem with use of Itô's lemma. X^f expresses the followers' investment value and the investment threshold value under fuzzy environment, then

$$X^f = \frac{\theta_1}{\theta_1 - 1} \frac{(r-\mu)}{D(1,1) - D(0,1)} (I_1, I_2, \alpha_2, \beta_2), \tag{2.7}$$

where

$$\theta_1 = \frac{-(\mu - (1/2)\sigma^2) + \sqrt{(\mu - (1/2)\sigma^2)^2 + 2r\sigma^2}}{\sigma^2}. \tag{2.8}$$

2.3. *Investment Value and Investment Threshold Value of Leaders*

Similar with the followers' discussion, we have the leading value function $F^l(X)$ and the optimal investment's threshold X^l, using X^m as the monopolist's critical value of investment

$$F^l(X) = \begin{cases} \left(\dfrac{X}{X^f}\right)^{\theta_1} \dfrac{X^f(D(1,1) - D(1,0))}{r-\mu} \\ \quad + \dfrac{(X_1, X_2, \alpha_1, \beta_1)D(1,0)}{r-\mu} - (I_1, I_2, \alpha_2, \beta_2) & \text{if } E(X) < E(X^f) \\ \dfrac{(X_1, X_2, \alpha_1, \beta_1)D(1,1)}{r-\mu} - (I_1, I_2, \alpha_2, \beta_2) & \text{if } E(X) \geq E(X^f). \end{cases} \tag{2.9}$$

When there does not exist an initial investment, the critical value of the optimal investment strategy in a critical point is

$$X^m = \frac{\theta_1}{\theta_1 - 1} \frac{(r-\mu)}{D(1,0) - D(0,0)} I. \tag{2.10}$$

When there exists an initial investment, X^l is the leader's' critical value of investment, then the leader's optimal investment strategy is

$$X^l = \text{Inf}\left\{0 < E\left(X^l\right) < E\left(X^f\right), F^l\left(X^l\right) = F^f\left(X^l\right)\right\}. \tag{2.11}$$

3. Establishments of the Asymmetric Model under Fuzzy Environment

3.1. *Review of Model and Basic Assumptions*

Under imperfect competition environment, the thought of the investment cost asymmetry investment decision-making's model is from the Grenadier's [3] duopoly model, according to Hui and Yong's [8] model, using standard backward induction of solving dynamic game. Promote the symmetry enterprises considered by Grenadiar to the enterprise of asymmetry. Suppose there are two technology innovation enterprises in the market, neutral risk and pursuing the largest expected value, expressed as i and j, they have opportunity to do a technical innovation investment to increase their profit flow and investment cost is asymmetry; that is, $(I_{1i}, I_{2i}, \alpha_{2i}, \beta_{2i})_1 \neq (I_{1i}, I_{2i}, \alpha_{2i}, \beta_{2i})_2$, followers' profit flow begins to be immediately affected, and the net profit of leaders changes to zero in the process of implementation of technology innovation. From the beginning to the successful implementation of investment, it needs a fixed period of time, δ years; the inverse demanding curve faced by enterprises can be shown by market prices $P(t)$ of unit product of enterprise i:

$$P(t) = X(t) D_{N_i N_j}. \tag{3.1}$$

Here, $X(t)$ shows product market demanding uncertainty, if we assume that the two enterprises' fuzzy uncertainty is equal, and is $(X_1, X_2, \alpha_1, \beta_1)$; in order not to cause confusion of circumstances, we use $X(t)$ to express, assuming that it follows the geometric Brown motion

$$dX(t) = \alpha X(t)dt + \sigma X(t)dW_t, \tag{3.2}$$

where α is the drift term, σ is a variable rate, dW_t is the increment of standard Wiener process, and $D_{N_iN_j}$ is the deterministic demand parameters of enterprises i, showing the effect of the strategic decision to profit flow between enterprises, which depends on the identity of enterprise $k \in \{i, j\}$, and the following inequality is established:

$$D_{10} > D_{11} > D_{00} > D_{01} > 0, \tag{3.3}$$

where $D_{10} > D_{00}$ shows that the profit of unsuccessful innovation enterprise decreases because of competitor's success; $D_{00} > D_{01}$ shows that the profits of enterprise's innovation success exceeds the failed; $D_{11} > D_{00}$ shows that when the competitor succeeds, the success of enterprise innovation can increase the profits; $D_{11} < D_{10}$ shows that if the competition has innovation success cases, successful innovation will improve their profit level; $D_{N_iN_j} > 0$ said corporate profits for non negative. In addition, also assumes the existence of investment first mover advantage over a competitor is enterprise innovation successful case of the comparative income greater than after competitor's innovation successful case of the comparative income, and have the following relations (refer to Grenadier [3]):

$$D_{10} - D_{00} > D_{11} - D_{01}. \tag{3.4}$$

3.2. The Followers' Investment Value and Investment Threshold Value under Fuzzy Environment

When the leader has invested, investment value of followers is the combination of the profit stream $XD(0,1)$ and the investment option value. Inspired by the [2–5] and other relative literatures, according to Itô's lemma and the Behrman equation [13], we can obtain the option-game model of enterprise technology innovation under fuzzy environment (avoiding causing confusion circumstances, we still use the original symbol, $F(X)$, X_{iF} express the followers' investment value and investment threshold value under fuzzy environment)

$$F_i(X) = \begin{cases} \left(\dfrac{X}{X_{iF}}\right)^{\beta} \dfrac{(I_{1i}, I_{2i}, \alpha_{2i}, \beta_{2i})_i}{\beta - 1} + \dfrac{(X_1, X_2, \alpha_1, \beta_1)D_{01}}{(r-\alpha)}, & E(X) < E(X_{iF}) \\ \dfrac{(X_1, X_2, \alpha_1, \beta_1)D_{01}}{(r-\alpha)} - (I_{1i}, I_{2i}, \alpha_{2i}, \beta_{2i})_i & \\ \quad + \dfrac{(X_1, X_2, \alpha_1, \beta_1)(D_{11} - D_{01})}{(r-\alpha)} e^{-(r-\alpha)\delta}, & E(X) \geq E(X_{iF}), \end{cases} \tag{3.5}$$

where $(I_{1i}, I_{2i}, \alpha_{2i}, \beta_{2i})_i$ $(i = 1, 2)$ is a follower of cost of the enterprise technological innovation investment. Assuming that $(I_{1i}, I_{2i}, \alpha_{2i}, \beta_{2i})_1 \neq (I_{1i}, I_{2i}, \alpha_{2i}, \beta_{2i})_2$, the β in (3.5) is the positive root of the following quadratic equation

$$\frac{1}{2}\sigma^2\beta(\beta - 1) + \alpha\beta - r = 0. \tag{3.6}$$

From Dixit and Pindyck [13], we can conclude the investment threshold of followers

$$X_{iF} = \left(\frac{\beta}{\beta - 1}\right)\frac{(r-\alpha)(I_{1i}, I_{2i}, \alpha_{2i}, \beta_{2i})_i}{D_{11} - D_{01}}e^{-(r-\alpha)\delta}. \tag{3.7}$$

According to the method from the literature [6, 7, 9, 10, 12], using the expectations of X to approximate estimation in solving similar problems, we can also use $(E(X)/E(X_{iF}))^{\beta}$ to estimate; in the comparison, we adopt a similar approach too, namely, using $E(X) < E(X_{iF})$ to be instead of $X < X_{iF}$.

Conclusion 1. The optimal investment strategy of followers to invest is at the time of $T_F = \{t \geq 0 : X(t) \geq X_{iF}\}$. The optimal investment threshold of followers is $X_F = (\beta/\beta - 1)((r - \alpha)(I_{1i}, I_{2i}, \alpha_{2i}, \beta_{2i})_i/D_{11} - D_{01})e^{-(r-\alpha)\delta}$. The coefficient $\beta/(\beta - 1) > 1$ is more expanded with the rule of the optimal investment than that with the rule of net present value; in the symmetric case, X_{iF} is influenced under the previous symmetric case $e^{-(r-\alpha)\delta}$; it is different from the discussion. Using the same method, we can establish the value function and the investment threshold of the enterprise as the leader under fuzzy environment

$$L_i(X) = \begin{cases} \left(\dfrac{X}{X_{jF}}\right)^{\beta}\dfrac{\beta}{\beta - 1}\dfrac{D_{11} - D_{10}}{D_{11} - D_{01}}(I_{1i}, I_{2i}, \alpha_{2i}, \beta_{2i})_i \\ \quad + \dfrac{(X_1, X_2, \alpha_1, \beta_1)D_{10}}{(r-\alpha)}e^{-(r-\alpha)\delta} - (I_{1i}, I_{2i}, \alpha_{2i}, \beta_{2i})_i, & E(X) < E(X_{jF}) \\ \dfrac{(X_1, X_2, \alpha_1, \beta_1)D_{11}}{(r-\alpha)}e^{-(r-\alpha)\delta} - (I_{1i}, I_{2i}, \alpha_{2i}, \beta_{2i})_i, & E(X) \geq E(X_{jF}). \end{cases} \tag{3.8}$$

Under fuzzy environment, the value function and monopoly investment threshold of the enterprise i and its competitors are given by $S_i(X)$ when they invest at the same time

$$S_i(X) = \begin{cases} \dfrac{(X_1, X_2, \alpha_1, \beta_1)D_{00}}{(r-\alpha)} + \dfrac{(I_{1i}, I_{2i}, \alpha_{2i}, \beta_{2i})_i}{\beta - 1}\left(\dfrac{X}{X_{iS}}\right)^{\beta}, & E(X) < E(X_{iS}) \\ \dfrac{(X_1, X_2, \alpha_1, \beta_1)D_{00}}{(r-\alpha)} - (I_{1i}, I_{2i}, \alpha_{2i}, \beta_{2i})_i \\ \quad + \dfrac{(X_1, X_2, \alpha_1, \beta_1)(D_{11} - D_{00})}{(r-\alpha)}e^{-(r-\alpha)\delta} & E(X) \geq E(X_{iS}). \end{cases} \tag{3.9}$$

Conclusion 2. Under the situation of two companies investing at the same time, the value of each enterprise investment

$$S_i(X) = \frac{\beta}{\beta - 1} \frac{(r-\alpha)}{(D_{11} - D_{00})} \left(I_{1i}, I_{2i}, \alpha_{2i}, \beta_{2i}\right)_i e^{-(r-\alpha)\delta}. \tag{3.10}$$

Conclusion 3. The optimal investment strategy of two businesses is to invest at the time of $T_S = \inf\{t \geq 0 : X \geq X_{iS}\}$.

4. Analysis with Comparison

(1) In most cases, under the condition of symmetry and asymmetry, the present value and investment sunk of expected profit flow uncertainty factor are not a definite number, but an estimate. This provides the basis for dealing with the above problem.

(2) From symmetry model, we can see the following.

When $E(X) < E(X^f)$, $F^f(X)$ and $(D(1,1) - D(0,1))$, X is in correlation positively, with $(r-\mu)$, I negative correlation.

At the same time, influenced by σ, θ_1, $(r-\mu)$, $D(1,1) - D(0,1)$ and the fuzzy of investment sunk cost according to the formula (2.6). From the model of asymmetry.

When $E(X) < E(X_{iF})$, $F(X)$ and $D_{11} - D_{01}$, $(I_{1i}, I_{2i}, \alpha_{2i}, \beta_{2i})_i$, and $(X_1, X_2, \alpha_1, \beta_1)$ are related positively. This is different according to the formula (3.5), from the symmetric case.

(3) To solve the symmetry model, firstly we calculate the θ_1, X^f, X^l, then we get the X range by fuzzy mathematics, we not only know the optimal strategy of two enterprise which can also be calculated through the simulation, but also can calculate the investment value of followers and leaders.

When $E(X) \geq E(X^l)$ and $E(X) \geq E(X^f)$ the investment value of followers is equal to the leader, that is, $F^f(X) = F^l(X)$. Due to the assumption that the two enterprises have symmetry, the leader and followers have no difference for any enterprise. But for the asymmetric model, this is different.

5. Conclusion

Due to space limitations, we do not use the fuzzy simulation method to compute. This method can be found in [10]. As long as we are given the estimation value of parameter, we can find the investment strategy and estimate the strategy of investment value according to the relative model. We also can make in-depth analysis according to the features of different districts caused by the different parameters.

Of course, when the uncertain factor of the enterprise technology innovation investment project cash flow, and the investment sunk cost of the enterprise technology innovation are trapezoidal fuzzy numbers, we consider the cost difference and balanced relationship types of operating costs of the symmetrical and asymmetrical enterprise, and obtain the value function and the corresponding threshold value of investment of the leaders and followers under fuzzy environment. Through numerical analysis, we find the symmetrical and asymmetrical enterprises under fuzzy environment still have the optimal investment strategies and make comparison. Of course, since in reality there exist the situation of symmetry and asymmetry of market demanding under fuzzy environment and technology, two or more factors, and the problems in process of dynamic stage, we will try our best to solve these problems in the future.

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