Hindawi Publishing Corporation
Abstract and Applied Analysis
Volume 2012, Article ID 825643, 17 pages
doi:10.1155/2012/825643

Research Article

Dynamical Analysis for High-Order Delayed Hopfield Neural Networks with Impulses

Dengwang Li

College of Physics and Electronics, Shandong Normal University, Jinan 250014, China

Correspondence should be addressed to Dengwang Li, dengwangli6@126.com

Received 25 June 2012; Accepted 3 September 2012

Academic Editor: José J. Oliveira

The global exponential stability and uniform stability of the equilibrium point for high-order delayed Hopfield neural networks with impulses are studied. By utilizing Lyapunov functional method, the quality of negative definite matrix, and the linear matrix inequality approach, some new stability criteria for such system are derived. The results are related to the size of delays and impulses. Two examples are also given to illustrate the effectiveness of our results.

1. Introduction

In the last several years, Hopfield neural networks (HNNs) have received especially considerable attention due to their extensive applications in solving optimization problem, traveling salesman problem, and many other subjects, see [1–17]. However such neural networks are shown to have limitations such as limited capacity when used in pattern recognition problems, see [2, 3]. This led many researchers to use neural networks with high order connections. The high-order neural networks have stronger approximation property, faster convergence rate, greater storage capacity, and higher fault tolerance than lower-order neural networks. Recently, various results on stability of high-order delayed HNN are obtained, see [11–15]. For example, Lou and Cui [13] studied the global asymptotic stability of high-order HNN with time-varying delays by using Lyapunov method, linear matrix inequality (LMI), and analytic technique as follows:

$$\begin{aligned} x_i'(t) = & -c_i x_i(t) + \sum_{j=1}^{n} a_{ij} f_j(x_j(t)) + \sum_{j=1}^{n} b_{ij} g_j(x_j(t-\tau_j(t))) \\ & + \sum_{j=1}^{n}\sum_{l=1}^{n} T_{ijl} g_l(x_l(t-\tau_l(t))) g_j(x_j(t-\tau_j(t))) + I_i, \quad t \geq t_0,\ i = 1, 2, \ldots, n. \end{aligned} \tag{1.1}$$

But the authors only obtained some global asymptotic stability criteria for the above high-order HNN. Those results cannot ensure the global exponential stability of the equilibrium point. It is well known that global exponential stability plays an important role in many areas such as designs and applications of neural networks and synchronization in secure communication [5, 17–23]. One purpose of this paper is to improve the results in [13]. We obtain several new criteria on global exponential stability and uniform stability for the above high-order HNN.

On the other hand, it is well known that the artificial electronic networks are subject to instantaneous perturbations and experience change of the state abruptly, that is, do exhibit impulsive effects. Such systems are described by impulsive differential systems which have been used successfully in modeling many practical problems arisen in the fields of natural sciences and technology, see [12, 24–30]. Hence, it is very important and, in fact, necessary to investigate the issue of the stability of high-order delayed HNN with impulses. However, to the best of the authors' knowledge, there are few results on the stability of high-order delayed HNN with impulses. In [12], Liu et al. obtained some sufficient conditions for ensuring global exponential stability of impulsive high order HNN with time-varying delays by using the method of Lyapunov functions.

The purpose of this paper is to present some new criteria concerning the global exponential stability and uniform stability for a class of high-order delayed HNN with impulses by utilizing Lyapunov functional method, the quality of negative definite matrix, and the linear matrix inequality approach. The conditions on impulses are different from that presented in [12]. The effects of impulses and delays on the solutions are stressed here. As a special case, several new criteria on global exponential stability and uniform stability for the corresponding high-order HNN without impulses (see [13]) are obtained. To illustrate the validity of those results, two examples are given to illustrate the effectiveness of the results obtained.

2. Preliminaries

Let $\mathbb{R}$ denote the set of real numbers, $\mathbb{R}_+$ the set of nonnegative real numbers, $\mathbb{Z}_+$ the set of positive integers, and $\mathbb{R}^n$ the n-dimensional real space equipped with the Euclidean norm $\|\cdot\|$.

Consider the following high-order delayed HNN model with impulses

$$\begin{aligned} x_i'(t) = &-c_i x_i(t) + \sum_{j=1}^{n} a_{ij} f_j\big(x_j(t)\big) + \sum_{j=1}^{n} b_{ij} g_j\big(x_j(t-\tau(t))\big) \\ &+ \sum_{j=1}^{n}\sum_{l=1}^{n} T_{ijl} g_l(x_l(t-\tau(t))) g_j\big(x_j(t-\tau(t))\big) + I_i, \quad t \neq t_k,\ t \geq t_0, \\ &\Delta x_i|_{t=t_k} = x_i(t_k) - x_i\big(t_k^-\big), \quad i \in \Lambda,\ k \in \mathbb{Z}_+, \end{aligned} \tag{2.1}$$

where $\Lambda = \{1, 2, \ldots, n\}$, $n \geq 2$ corresponds to the number of units in a neural network; the impulse times t_k satisfy $0 \leq t_0 < t_1 < \cdots < t_k < \cdots$, $\lim_{k\to+\infty} t_k = +\infty$; x_i corresponds to the membrane potential of the unit i at time t; c_i is positive constant; f_j, g_j denote, respectively, the measures of response or activation to its incoming potentials of the unit j at time t and $t-\tau(t)$; T_{ijl} is the second-order synaptic weights of the neural networks; constant a_{ij} denotes the synaptic connection weight of the unit j on the unit i at time t; constant b_{ij} denotes the synaptic connection weight of the unit j on the unit i at time $t-\tau(t)$; I_i is the input of the unit i; $\tau(t)$ is the transmission delay such that $0<\tau(t)\le\tau$ and $\dot{\tau}(t)\le\rho<1$, $t\ge t_0$; τ, ρ are constants.

The initial conditions associated with system (2.1) are of the form

$$x(s)=\phi(s),\qquad s\in[t_0-\tau,t_0],\tag{2.2}$$

where $x(s)=(x_1(s),x_2(s),\dots,x_n(s))^T$, $\phi(s)=(\phi_1(s),\phi_2(s),\dots,\phi_n(s))^T\in PC([-\tau,0],\mathbb{R}^n)$, $PC([-\tau,0],\mathbb{R}^n)=\{\varphi:[-\tau,0]\to\mathbb{R}^n$ is continuous everywhere except at finite number of points t_k, at which $\varphi(t_k^+)$ and $\varphi(t_k^-)$ exist and $\varphi(t_k^+)=\varphi(t_k)\}$. For $\varphi\in PC([-\tau,0],\mathbb{R}^n)$, the norm of φ is defined by $\|\varphi\|_\tau=\sup_{-\tau\le\theta\le0}|\varphi(\theta)|$. For any $t_0\ge0$, let $PC_\delta(t_0)=\{\varphi\in PC([-\tau,0],\mathbb{R}^n):\|\varphi\|<\delta\}$.

Assume that $x^*=(x_1^*,x_2^*,\dots,x_n^*)^T$ is an equilibrium point of system (2.1). Impulsive operator is viewed as perturbation of the equilibrium point x^* of such system without impulsive effects. We assume that

$$\Delta x_i|_{t=t_k}=x_i(t_k)-x_i(t_k^-)=d_k^{(i)}\left(x_i(t_k^-)-x_i^*\right),\quad d_k^{(i)}\in\mathbb{R},\ i\in\Lambda,\ k\in\mathbb{Z}_+.\tag{2.3}$$

Since x^* is an equilibrium point of system (2.1), one can derive from system (2.1)-(2.2) that the transformation $y_i=x_i-x_i^*$, $i\in\Lambda$ transforms such system into the following system (for more details, please see papers [12, 13]):

$$\begin{aligned}y'(t)=&-Cy(t)+AF(y(t))+BG(y(t-\tau(t)))\\&+\Gamma^T T^\star G(y(t-\tau(t))),\quad t\ne t_k,\ t\ge t_0,\\y(t_k)=&\,D_k y(t_k^-),\quad k\in\mathbb{Z}_+,\\y(t_0+\theta)=&\,\varphi(\theta),\quad\theta\in[-\tau,0],\end{aligned}\tag{2.4}$$

where

$$\begin{gathered}\varphi(\theta)=x(t_0+\theta)-x^*,\qquad y(t)=(y_1(t),y_2(t),\dots,y_n(t))^T,\\y(t-\tau(t))=(y_1(t-\tau(t)),y_2(t-\tau(t)),\dots,y_n(t-\tau(t)))^T,\\F(y(t))=[F_1(y_1(t)),F_2(y_2(t)),\dots,F_n(y_n(t))]^T,\\G(y(t-\tau(t)))=[G_1(y_1(t-\tau(t))),G_2(y_2(t-\tau(t))),\dots,G_n(y_n(t-\tau(t)))]^T,\\F_j(y_j(t))=f_j\left(x_j^*+y_j(t)\right)-f_j\left(x_j^*\right),\qquad G_j(y_j(t-\tau(t)))=g_j\left(x_j^*+y_j(t-\tau(t))\right)-g_j\left(x_j^*\right),\\C=\operatorname{diag}[c_1,c_2,\dots,c_n],\qquad A=(a_{ij})_{n\times n},\qquad B=(b_{ij})_{n\times n},\qquad T_i=(T_{ijl})_{n\times n},\\T^\star=\left(T_1+T_1^T,T_2+T_2^T,\dots,T_n+T_n^T\right)^T,\end{gathered}$$

$$\Gamma = \operatorname{diag}[\varsigma, \varsigma, \ldots, \varsigma], \qquad \varsigma = (\varsigma_1, \varsigma_2, \ldots, \varsigma_n)^T,$$
$$D_k = \operatorname{diag}\left[1 + d_k^{(1)}, 1 + d_k^{(2)}, \ldots, 1 + d_k^{(n)}\right], \tag{2.5}$$

in which ς_l is a real value between $g_l(x_l(t - \tau(t)))$ and $g_l(x_l^*)$, $l \in \Lambda$.

Remark 2.1. Obviously, $(0, 0, \ldots, 0)^T$ is an equilibrium point of (2.4). Therefore, there exists at least one equilibrium point of system (2.1). So, the stability analysis of the equilibrium point x^* of (2.1) can now be transformed to the stability analysis of the trivial solution $y = 0$ of (2.4).

In the following, the notations X^T and X^{-1} mean the transpose of and the inverse of a square matrix X. We will use the notation $X > 0$ (or $X < 0$, $X \geq 0$, $X \leq 0$) to denote that the matrix X is a symmetric and positive definite (negative definite, positive semidefinite, negative semidefinite) matrix. Let $\lambda_{\max}(X)$, $\lambda_{\min}(X)$, respectively, denote the largest and smallest eigenvalue of matrix X.

Throughout this paper, we assume that there exist constants $\chi_i > 0, M, N \geq 0$ such that $|g_i(x_i)| \leq \chi_i$, $i \in \Lambda$, $F^T(y)F(y) \leq My^Ty$, $G^T(y)G(y) \leq Ny^Ty$.

We introduce some definitions as follows.

Definition 2.2 (see [5]). Leting $V : \mathbb{R}_+ \times \mathbb{R}^n \to \mathbb{R}_+$, for any $(t, x) \in [t_{k-1}, t_k) \times \mathbb{R}^n$, the upper right-hand Dini derivative of $V(t, x)$ along the solution of (2.4) is defined by

$$D^+V(t, x) = \limsup_{h \to 0^+} \frac{1}{h} \Big\{ V\Big[t + h, x + h\Big(-Cy(t) + AF(y(t)) + BG(y(t - \tau(t))) + \Gamma^T T^\star G(y(t - \tau(t)))\Big)\Big] - V(t, x) \Big\}. \tag{2.6}$$

Definition 2.3 (see [25]). Assume $y(t) = y(t_0, \varphi)(t)$ is the solution of (2.4) through (t_0, φ). Then the zero solution of (2.4) is said to be uniformly stable, if, for any $\varepsilon > 0$ and $t_0 \geq 0$, there exists some $\delta = \delta(\varepsilon) > 0$ such that $\varphi \in PC_\delta(t_0)$ implies $\|y(t)\| < \varepsilon$, $t \geq t_0$.

Definition 2.4 (see [5]). The equilibrium point x^* of the system (2.1) is globally exponentially stable, if there exists constant $\mu > 0, \mathbb{M} \geq 1$ such that, for any initial value ϕ,

$$\|x(t_0, \phi)(t) - x^*\| < \mathbb{M}\|\phi - x^*\|_\tau e^{-\mu(t - t_0)}, \quad t \geq t_0. \tag{2.7}$$

Next, in order to obtain our results, we need to establish the following lemma.

Lemma 2.5 (see [13]). *For any vectors $a, b \in \mathbb{R}^n$, the inequality*

$$\pm 2a^Tb \leq a^TXa + b^TX^{-1}b \tag{2.8}$$

holds, in which X is any $n \times n$ matrix with $X > 0$.

Lemma 2.6 (see [31]). *Let $X \in R^{n\times n}$, then*

$$\lambda_{\min}(X)a^T a \leq a^T X a \leq \lambda_{\max}(X)a^T a \tag{2.9}$$

for any $a \in \mathbb{R}^n$ if X is a symmetric matrix.

3. Main Results

In this section, some sufficient delay-dependent conditions of global exponential stability and uniform stability for system (2.1) are obtained.

Theorem 3.1. *Assume that there exist constants $\varepsilon^\star > 0$, $\delta^\star \in [0, \varepsilon^\star)$ and $n\times n$ symmetric and positive definite matrices P, Q_1, Q_2 such that*

(i)

$$\begin{aligned}
&\varepsilon^\star P - PC - CP + PAQ_1^{-1}A^T P + \lambda_{\max}(Q_1)ME + \frac{N\lambda_{\max}\left(Q_2 + T^{\star T}T^\star\right)}{1-\rho}E \\
&\quad + e^{\tau\varepsilon^\star} PBQ_2^{-1}B^T P + e^{\tau\varepsilon^\star}\|\chi\|^2 P^2 \leq 0,
\end{aligned} \tag{3.1}$$

where $\chi = (\chi_1, \chi_2, \ldots, \chi_n)^T$,

(ii) *there exists constant $\mathbb{W} \geq 0$ such that*

$$\sum_{k=1}^{m} \ln \max\{\eta_k, 1\} - \delta^\star(t_m - t_0) \leq \mathbb{W} \quad \forall m \in \mathbb{Z}_+ \text{ holds}, \tag{3.2}$$

where η_k is the largest eigenvalue of $P^{-1}D_k P D_k$, $k \in \mathbb{Z}_+$.

Then the equilibrium point of the system (2.1) is globally exponentially stable and the approximate exponential convergent rate is $(\varepsilon^\star - \delta^\star)/2$.

Proof. We only need to prove that the zero solution of system (2.4) is globally exponentially stable. For any $t_0 \geq 0$, let $y(t) = y(t_0, \varphi)(t)$ be a solution of (2.4) through (t_0, φ).

Consider the Lyapunov functional as follows:

$$V(t) = e^{\varepsilon^\star t} y^T(t) P y(t) + \frac{1}{1-\rho}\int_{t-\tau(t)}^{t} e^{\varepsilon^\star s} G^T(y(s))\left(Q_2 + T^{\star T}T^\star\right)G(y(s))\,ds, \tag{3.3}$$

then we have

$$\begin{aligned}\lambda_{\min}(P)e^{\varepsilon^{\star}t}\|y(t)\|^2 &< V(t)\\ &\leq \lambda_{\max}(P)e^{\varepsilon^{\star}t}\|y(t)\|^2 + \frac{\lambda_{\max}\left(Q_2 + T^{\star T}T^{\star}\right)Ne^{\varepsilon^{\star}t}\left(1 - e^{-\varepsilon^{\star}\tau(t)}\right)}{\varepsilon^{\star}(1-\rho)}\|y(t)\|_{\tau}^2\\ &\leq \left(\lambda_{\max}(P) + \frac{\lambda_{\max}\left(Q_2 + T^{\star T}T^{\star}\right)N\left(1 - e^{-\varepsilon^{\star}\tau}\right)}{\varepsilon^{\star}(1-\rho)}\right)e^{\varepsilon^{\star}t}\|y(t)\|_{\tau}^2.\end{aligned} \tag{3.4}$$

By Lemma 2.5, we get

$$\begin{aligned}2y^T(t)PAF(y(t)) &= 2F^T(y(t))A^TPy(t)\\ &\leq F^T(y(t))Q_1F(y(t)) + y^T(t)PAQ_1^{-1}A^TPy(t)\\ &\leq \lambda_{\max}(Q_1)F^T(y(t))F(y(t)) + y^T(t)PAQ_1^{-1}A^TPy(t)\\ &\leq y^T(t)\left[PAQ_1^{-1}A^TP + \lambda_{\max}(Q_1)ME\right]y(t),\end{aligned} \tag{3.5}$$

$$\begin{aligned}2y^T(t)PBG(y(t-\tau(t))) &= 2G^T(y(t-\tau(t)))B^TPy(t)\\ &= 2\left[G(y(t-\tau(t)))\sqrt{e^{-\tau\varepsilon^{\star}}}\right]^T\left(B^TPy(t)\sqrt{e^{\tau\varepsilon^{\star}}}\right)\\ &\leq e^{-\tau\varepsilon^{\star}}G^T(y(t-\tau(t)))Q_2G(y(t-\tau(t)))\\ &\quad + e^{\tau\varepsilon^{\star}}y^T(t)PBQ_2^{-1}B^TPy(t).\end{aligned} \tag{3.6}$$

On the other hand, since $\Gamma^T\Gamma = \|\varsigma\|^2E$ and $\|\varsigma\| \leq \|\chi\|$, then we have

$$y^T(t)P\Gamma^T\Gamma Py(t) \leq \|\chi\|^2 y^T(t)P^2y(t), \tag{3.7}$$

where $\chi = (\chi_1, \chi_2, \ldots, \chi_n)^T$.

Thus, we obtain

$$\begin{aligned}2y^T(t)P\Gamma^TT^{\star}G(y(t-\tau(t))) &= 2G^T(y(t-\tau(t)))T^{\star T}\Gamma Py(t)\\ &= 2\left[T^{\star}G(y(t-\tau(t)))\sqrt{e^{-\tau\varepsilon^{\star}}}\right]^T\left(\Gamma Py(t)\sqrt{e^{\tau\varepsilon^{\star}}}\right)\\ &\leq e^{-\tau\varepsilon^{\star}}G^T(y(t-\tau(t)))T^{\star T}T^{\star}G(y(t-\tau(t))) + e^{\tau\varepsilon^{\star}}y^T(t)P\Gamma^T\Gamma Py(t)\\ &\leq e^{-\tau\varepsilon^{\star}}G^T(y(t-\tau(t)))T^{\star T}T^{\star}G(y(t-\tau(t))) + e^{\tau\varepsilon^{\star}}\|\chi\|^2y^T(t)P^2y(t).\end{aligned} \tag{3.8}$$

Now we consider the derivation of V along the trajectories of system (2.4), for $t \in [t_k, t_{k+1})$, $k \in \mathbb{Z}_+$,

$$
\begin{aligned}
D^+V(t)|_{(2.3)} = {} & e^{\varepsilon^\star t}\varepsilon^\star y^T(t)Py(t) + e^{\varepsilon^\star t}\left\{ y'^T(t)Py(t) + y^T(t)Py'(t)\right\} \\
& + \frac{1}{1-\rho}e^{\varepsilon^\star t}G^T(y(t))\left(Q_2 + T^{\star T}T^\star\right)G(y(t)) \\
& - \frac{1-\dot{\tau}(t)}{1-\rho}e^{\varepsilon^\star(t-\tau(t))}G^T(y(t-\tau(t)))\left(Q_2 + T^{\star T}T^\star\right)G(y(t-\tau(t))) \\
\le {} & e^{\varepsilon^\star t}\varepsilon^\star y^T(t)Py(t) + e^{\varepsilon^\star t}\Big\{ y^T(t)(-CP-PC)y(t) + 2y^T(t)PAF(y(t)) \\
& \qquad + 2y^T(t)PBG(y(t-\tau(t))) \\
& \qquad + 2y^T(t)P\Gamma^T T^\star G(y(t-\tau(t)))\Big\} \\
& + \frac{1}{1-\rho}e^{\varepsilon^\star t}G^T(y(t))\left(Q_2 + T^{\star T}T^\star\right)G(y(t)) \\
& - e^{\varepsilon^\star(t-\tau)}G^T(y(t-\tau(t)))\left(Q_2 + T^{\star T}T^\star\right)G(y(t-\tau(t))) \\
\le {} & e^{\varepsilon^\star t}y^T(t)\Bigg\{ \varepsilon^\star P - PC - CP + PAQ_1^{-1}A^TP + \lambda_{\max}(Q_1)ME \\
& + \frac{N\lambda_{\max}\left(Q_2 + T^{\star T}T^\star\right)}{1-\rho}E + e^{\tau\varepsilon^\star}PBQ_2^{-1}B^TP + e^{\tau\varepsilon^\star}\|\chi\|^2P^2\Bigg\} y(t) \\
\le {} & 0.
\end{aligned}
\tag{3.9}
$$

Moreover, we note

$$
\begin{aligned}
V(t_k) &= e^{\varepsilon^\star t_k}y^T(t_k)Py(t_k) + \frac{1}{1-\rho}\int_{t_k-\tau(t_k)}^{t_k} e^{\varepsilon^\star s}G^T(y(s))\left(Q_2 + T^{\star T}T^\star\right)G(y(s))ds \\
&= e^{\varepsilon^\star t_k}y^T(t_k^-)D_kPD_ky(t_k^-) + \frac{1}{1-\rho}\int_{t_k^- -\tau(t_k^-)}^{t_k^-} e^{\varepsilon^\star s}G^T(y(s))\left(Q_2 + T^{\star T}T^\star\right)G(y(s))ds \\
&\le e^{\varepsilon^\star t_k}\eta_k y^T(t_k^-)Py(t_k^-) + \frac{1}{1-\rho}\int_{t_k^- -\tau(t_k^-)}^{t_k^-} e^{\varepsilon^\star s}G^T(y(s))\left(Q_2 + T^{\star T}T^\star\right)G(y(s))ds \\
&\le \max\{\eta_k, 1\}V(t_k^-).
\end{aligned}
\tag{3.10}
$$

By simple induction, considering (3.4)–(3.10), we get, for $k \geq 1$,

$$\lambda_{\min}(P)e^{\varepsilon^{\star}t}\|y(t)\|^2 \leq V(t) \leq V(t_0)\prod_{t_0<t_k\leq t}\max\{\eta_k, 1\}. \tag{3.11}$$

On the other hand, from (3.4), we get

$$V(t_0) \leq \left(\lambda_{\max}(P) + \frac{\lambda_{\max}\left(Q_2 + T^{\star T}T^{\star}\right)N\left(1 - e^{-\varepsilon^{\star}\tau}\right)}{\varepsilon^{\star}(1-\rho)}\right)e^{\varepsilon^{\star}t_0}\|\varphi\|_{\tau}^2. \tag{3.12}$$

Substituting the above inequality into (3.11), we obtain

$$\|y(t)\|^2 \leq \left(\frac{\lambda_{\max}(P)}{\lambda_{\min}(P)} + \frac{\lambda_{\max}\left(Q_2 + T^{\star T}T^{\star}\right)N\left(1 - e^{-\varepsilon^{\star}\tau}\right)}{\varepsilon^{\star}(1-\rho)\lambda_{\min}(P)}\right)e^{-\varepsilon^{\star}(t-t_0)}\|\varphi\|_{\tau}^2\prod_{t_0<t_k\leq t}\max\{\eta_k, 1\}. \tag{3.13}$$

In view of condition (ii), we furthermore have

$$\|y(t)\| \leq \mathbb{M}e^{-((\varepsilon^{\star}-\delta^{\star})/2)(t-t_0)}\|\varphi\|_{\tau}, \quad t \geq t_0, \tag{3.14}$$

where

$$\mathbb{M} = \sqrt{\left(\frac{\lambda_{\max}(P)}{\lambda_{\min}(P)} + \frac{\lambda_{\max}\left(Q_2 + T^{\star T}T^{\star}\right)N\left(1 - e^{-\varepsilon^{\star}\tau}\right)}{\varepsilon^{\star}(1-\rho)\lambda_{\min}(P)}\right)e^{\mathbb{W}}} \geq 1. \tag{3.15}$$

Hence, the zero solution of system (2.4) is globally exponentially stable; that is, the equilibrium point of system (2.1) is globally exponentially stable and the approximate exponential convergent rate is $(\varepsilon^{\star}-\delta^{\star})/2$. The proof of Theorem 3.1 is therefore complete. □

Remark 3.2. In Theorem 3.1, we find that condition (i) can be replaced by

$$\begin{aligned}&\varepsilon^{\star}P - PC - CP + PAQ_1^{-1}A^TP + \lambda_{\max}(Q_1)ME + \frac{N\lambda_{\max}(Q_2)}{1-\rho}E + \frac{N\lambda_{\max}\left(T^{\star T}T^{\star}\right)}{1-\rho}E \\ &\quad + e^{\tau\varepsilon^{\star}}PBQ_2^{-1}B^TP + e^{\tau\varepsilon^{\star}}\|\chi\|^2P^2 \leq 0.\end{aligned} \tag{3.16}$$

Leting $P = Q_1 = Q_2 = E$ in Theorem 3.1, then we have the following.

Corollary 3.3. *Assume that there exist constants $\varepsilon^\star > 0$, $\delta^\star \in [0, \varepsilon^\star)$ such that*

(i)

$$\lambda^\star_{\max} \le -\varepsilon^\star - M - \frac{N}{1-\rho} - \frac{N\lambda_{\max}\left(T^{\star T}T^\star\right)}{1-\rho} - e^{\tau\varepsilon^\star}\|\chi\|^2, \tag{3.17}$$

where $\lambda^\star_{\max}$ is the largest eigenvalue of $-2C + AA^T + e^{\tau\varepsilon^\star}BB^T$;

(ii) *there exists constant $\mathbb{W} \ge 0$ such that*

$$\sum_{k=1}^{m} \ln \max\left\{\max_{i\in\Lambda}\left(1+d_k^{(i)}\right)^2, 1\right\} - \delta^\star(t_m - t_0) < \mathbb{W} \quad \forall m \in \mathbb{Z}_+ \text{ holds.} \tag{3.18}$$

The equilibrium point of the system (2.1) *is globally exponentially stable and the approximate exponential convergent rate is* $(\varepsilon^\star - \delta^\star)/2$.

Furthermore, if $d_k^{(i)} \in [-2, 0]$ in Corollary 3.3, then we have the following result.

Corollary 3.4. *The equilibrium point of the system* (2.1) *is globally exponentially stable, if $d_k^{(i)} \in [-2, 0]$, and there exists constant $\varepsilon^\star > 0$ such that*

$$-2C + AA^T + e^{\tau\varepsilon^\star}BB^T + \left[\varepsilon^\star + M + \frac{N\left(1+\lambda_{\max}\left(T^{\star T}T^\star\right)\right)}{1-\rho} + e^{\tau\varepsilon^\star}\|\chi\|^2\right]E \le 0. \tag{3.19}$$

Remark 3.5. In fact, Theorem 3.1 implies that if $\sup_{k\in\mathbb{Z}_+}\prod_{s=1}^{k}(1+\beta_s^{(i)})^2 < \infty$, then one may choose $\delta^\star = 0$. On the other hand, Luo an Cui [13] obtained some results on global asymptotic stability. However, those results cannot ensure the global exponential stability. Let $d_k^{(i)} = 0$ (i.e., $D_k = E$) in Corollary 3.4, then we can obtain the desirable result as follows.

Corollary 3.6. *The equilibrium point of the system* (2.1) *without impulses is globally exponentially stable, if there exist $n \times n$ symmetric and positive definite matrices P, Q_1, Q_2 such that*

$$\begin{aligned} &- PC - CP + PAQ_1^{-1}A^TP + \lambda_{\max}(Q_1)ME + \frac{N\lambda_{\max}\left(Q_2 + T^{\star T}T^\star\right)}{1-\rho}E \\ &+ PBQ_2^{-1}B^TP + \|\chi\|^2P^2 < 0. \end{aligned} \tag{3.20}$$

Furthermore, if $P = Q_1 = Q_2 = E$ in Corollary 3.6, then it becomes as follows.

Corollary 3.7. *The equilibrium point of the system* (2.1) *without impulses is globally exponentially stable, if the following condition holds:*

$$-2C + AA^T + BB^T + \frac{N\lambda_{\max}\left(T^{\star T}T^{\star}\right)}{1-\rho} + \left(\frac{N}{1-\rho} + M + \|\chi\|^2\right)E < 0. \tag{3.21}$$

Remark 3.8. Corollaries 3.6 and 3.7 imply that if the above inequality holds, then there exists enough small $\varepsilon^{\star} > 0$ such that all conditions in Corollary 3.4 are satisfied. Hence, Corollaries 3.6 and 3.7 supplied a new criteria for global exponential stability of equilibrium point of the system (2.1) without impulses.

Next we can establish a theorem which provide sufficient conditions for uniform stability of system (2.1) by constructing another Lyapunov functional. Here we shall emphasize the effects of impulses.

Theorem 3.9. *Assume that there exist $n \times n$ symmetric and positive definite matrices P, Q_1, Q_2 such that the following condition*

$$\begin{aligned} &-PC - CP + PAQ_1^{-1}A^TP + \lambda_{\max}(Q_1)ME + \frac{N\lambda_{\max}\left(Q_2 + T^{\star T}T^{\star}\right)}{1-\rho}\left(\prod_{s=1}^{k}\eta_s\right)E \\ &\quad + \left(\prod_{s=1}^{k}\eta_s\right)^{-1} PBQ_2^{-1}B^TP + \left(\prod_{s=1}^{k}\eta_s\right)^{-1} \|\chi\|^2 P^2 \le 0 \quad \forall k \in \mathbb{Z}_+ \text{ holds,} \end{aligned} \tag{3.22}$$

where $\sup_{k\in\mathbb{Z}_+}\prod_{s=1}^{k}\eta_s < \infty$, η_k is the largest eigenvalue of $P^{-1}D_kPD_k$.

Then the equilibrium point of the system (2.1) *is uniformly stable.*

Proof. We only prove the zero solution of system (2.4) is uniformly stable. For any $\varepsilon > 0$, $t_0 \ge 0$, $\varphi \in PC_{\delta}(t_0)$, let $y(t) = y(t_0, \varphi)(t)$ be a solution of (2.4) through (t_0, φ), $t_0 \ge 0$, then we can prove that $\|y(t)\| < \varepsilon$, $t \ge t_0$,

where

$$\delta = \frac{\varepsilon\sqrt{\lambda_{\min}(P)}}{\sqrt{\eta}\sqrt{\lambda_{\max}(P) + \left(\lambda_{\max}\left(Q_2 + T^{\star T}T^{\star}\right)N\eta\tau/(1-\rho)\right)}}, \qquad \eta \doteq \sup_{k\in\mathbb{Z}_+}\prod_{s=1}^{k}\eta_s. \tag{3.23}$$

Consider the following Lyapunov functional

$$V(t) = y^T(t)Py(t) + \frac{1}{1-\rho}\int_{t-\tau(t)}^{t}\left(\prod_{t_s\le t}\eta_s\right)G^T(y(s))\left(Q_2 + T^{\star T}T^{\star}\right)G(y(s))ds, \tag{3.24}$$

then we have

$$\begin{aligned}
\lambda_{\min}(P)\|y(t)\|^2 &< V(t) \\
&\leq \lambda_{\max}(P)\|y(t)\|^2 + \frac{\lambda_{\max}\left(Q_2 + T^{\star T}T^{\star}\right)N\eta\tau}{1-\rho}\|y(t)\|_{\tau}^2 \\
&\leq \left(\lambda_{\max}(P) + \frac{\lambda_{\max}\left(Q_2 + T^{\star T}T^{\star}\right)N\eta\tau}{1-\rho}\right)\|y(t)\|_{\tau}^2.
\end{aligned} \tag{3.25}$$

Applying the same argument as Theorem 3.1, we get

$$\begin{aligned}
2y^T(t)PAF(y(t)) &\leq y^T(t)\left[PAQ_1^{-1}A^TP + \lambda_{\max}(Q_1)ME\right]y(t), \\
2y^T(t)PBG(y(t-\tau(t))) &\leq \left(\prod_{s=1}^{k}\eta_s\right)G^T(y(t-\tau(t)))Q_2G(y(t-\tau(t))) \\
&\quad + \left(\prod_{s=1}^{k}\eta_s\right)^{-1} y^T(t)PBQ_2^{-1}B^TPy(t), \\
2y^T(t)P\Gamma^TT^{\star}G(y(t-\tau(t))) &\leq \left(\prod_{s=1}^{k}\eta_s\right)G^T(y(t-\tau(t)))T^{\star T}T^{\star}G(y(t-\tau(t))) \\
&\quad + \left(\prod_{s=1}^{k}\eta_s\right)^{-1} \|\chi\|^2 y^T(t)P^2y(t).
\end{aligned} \tag{3.26}$$

By simple calculation, we can obtain, for $t \in [t_k, t_{k+1})$, $k \in \mathbb{Z}_+$,

$$\begin{aligned}
D^+V(t)|_{(2.3)} &= y^T(t)(-CP - PC)y(t) + 2y^T(t)PAF(y(t)) + 2y^T(t)PBG(y(t-\tau(t))) \\
&\quad + 2y^T(t)P\Gamma^TT^{\star}G(y(t-\tau(t))) + \frac{1}{1-\rho}\left(\prod_{s=1}^{k}\eta_s\right)G^T(y(t))\left(Q_2 + T^{\star T}T^{\star}\right)G(y(t)) \\
&\quad - \frac{1-\dot{\tau}(t)}{1-\rho}\left(\prod_{s=1}^{k}\eta_s\right)G^T(y(t-\tau(t)))\left(Q_2 + T^{\star T}T^{\star}\right)G(y(t-\tau(t)))
\end{aligned}$$

$$
\begin{aligned}
&\leq y^T(t)\Bigg\{-PC - CP + PAQ_1^{-1}A^TP + \lambda_{\max}(Q_1)ME \\
&\qquad + \frac{N\lambda_{\max}\left(Q_2 + T^{\star T}T^{\star}\right)}{1-\rho}\left(\prod_{s=1}^{k}\eta_s\right)E + \left(\prod_{s=1}^{k}\eta_s\right)^{-1} PBQ_2^{-1}B^TP \\
&\qquad + \left(\prod_{s=1}^{k}\eta_s\right)^{-1}\|\chi\|^2P^2\Bigg\}y(t). \\
&\leq 0.
\end{aligned}
\tag{3.27}
$$

Moreover, we know

$$
\begin{aligned}
V(t_k) &= y^T(t_k)Py(t_k) + \frac{1}{1-\rho}\int_{t_k-\tau(t_k)}^{t_k}\left(\prod_{t_s\leq t_k}\eta_s\right)\Gamma^T(y(s))\left(Q_2 + T^{\star T}T^{\star}\right)\Gamma(y(s))ds \\
&= y^T(t_k^-)D_kPD_ky(t_k^-) + \frac{1}{1-\rho}\int_{t_k^- -\tau(t_k)}^{t_k^-}\left(\prod_{t_s\leq t_k}\eta_s\right)\Gamma^T(y(s))\left(Q_2 + T^{\star T}T^{\star}\right)\Gamma(y(s))ds \\
&\leq \eta_k y^T(t_k^-)Py(t_k^-) + \frac{1}{1-\rho}\eta_k\int_{t_k^- -\tau(t_k)}^{t_k^-}\left(\prod_{t_s\leq t_{k-1}}\eta_s\right)\Gamma^T(y(s))\left(Q_2 + T^{\star T}T^{\star}\right)\Gamma(y(s))ds \\
&= \eta_k V(t_k^-).
\end{aligned}
\tag{3.28}
$$

By simple induction, from (3.27) and (3.28) we may prove that, for $k \geq 1$,

$$
\lambda_{\min}(P)\|y(t)\|^2 \leq V(t) \leq V(t_0)\prod_{t_0<t_k\leq t}\eta_k. \tag{3.29}
$$

Employing the fact (3.25), we obtain

$$
\lambda_{\min}(P)\|y(t)\|^2 \leq \left(\lambda_{\max}(P) + \frac{\lambda_{\max}\left(Q_2 + T^{\star T}T^{\star}\right)N\eta\tau}{1-\rho}\right)\|\varphi\|_\tau^2\eta, \quad t \geq t_0, \tag{3.30}
$$

which implies that

$$
\|y(t)\| < \varepsilon, \quad t \geq t_0. \tag{3.31}
$$

Therefore, the zero solution of system (2.4) is uniformly stable, that is, the equilibrium point of system (2.1) is uniformly stable. The proof of Theorem 3.9 is complete. □

Corollary 3.10. *The equilibrium point of the system* (2.1) *is uniformly stable, if there exist* $n \times n$ *symmetric and positive definite matrices* P, Q_1, Q_2 *such that the following condition holds:*

$$\Xi_k \leq -\lambda_{\max}(Q_1)M - \frac{N\lambda_{\max}\left(Q_2 + T^{\star T}T^{\star}\right)}{1-\rho}\left(\prod_{s=1}^{k}\eta_s\right), \tag{3.32}$$

where Ξ_k *is the largest eigenvalue of* $-PC - CP + PAQ_1^{-1}A^TP + (\prod_{s=1}^{k}\eta_s)^{-1}[PBQ_2^{-1}B^TP + \|\chi\|^2P^2]$.

If $P = Q_1 = Q_2 = E$ in Theorem 3.9, then we have the following.

Corollary 3.11. *The equilibrium point of the system* (2.1) *is uniformly stable, if the following condition*

$$\begin{aligned} &-2C + AA^T + \left[M + \frac{N\lambda_{\max}\left(E + T^{\star T}T^{\star}\right)}{1-\rho}\left(\prod_{s=1}^{k}\max_{i\in\Lambda}\left(1 + d_s^{(i)}\right)^2\right)\right]E \\ &+ \left[\prod_{s=1}^{k}\max_{i\in\Lambda}\left(1 + d_s^{(i)}\right)^2\right]^{-1}\left[BB^T + \|\chi\|^2E\right] \leq 0 \quad \forall k \in \mathbb{Z}_+ \text{ holds,} \end{aligned} \tag{3.33}$$

where $\sup_{k\in\mathbb{Z}_+}\prod_{s=1}^{k}\max_{i\in\Lambda}(1 + d_s^{(i)})^2 < \infty$.

4. Examples

In this section we give two examples to demonstrate our results.

Example 4.1. Consider the following high-order delayed Hopfield-type neural network with impulses

$$\begin{aligned} x_i'(t) = &-c_ix_i(t) + \sum_{j=1}^{3}a_{ij}f_j\left(x_j(t)\right) + \sum_{j=1}^{3}b_{ij}g_j\left(x_j(t-\tau(t))\right) \\ &+ \sum_{j=1}^{3}\sum_{l=1}^{3}T_{ijl}g_l(x_l(t-\tau(t)))g_j\left(x_j(t-\tau(t))\right), \quad t \neq t_k,\ t \geq t_0, \end{aligned} \tag{4.1}$$

$$\Delta x_i|_{t=t_k} = x_i(t_k) - x_i\left(t_k^-\right) = \beta_k^{(i)}x_i\left(t_k^-\right), \quad i = 1,2,3, k \in \mathbb{Z}_+,$$

where $\beta_k^{(i)} = \sqrt{1+(i/k^2)} - 1$, $\tau(t) = \sin t/2$, $f_1(x_1) = \tanh(0.5x_1)$, $f_2(x_2) = \tanh(0.48x_2)$, $f_3(x_3) = \tanh(0.6x_3)$, $g_1(x_1) = \tanh(0.3x_1)$, $g_2(x_2) = \tanh(0.8x_2)$, $g_3(x_3) = \tanh(0.73x_3)$,

$$C = \operatorname{diag}[c_1, c_2, c_3]^T = \begin{bmatrix} 3.2 & 0 & 0 \\ 0 & 2.5 & 0 \\ 0 & 0 & 2.0 \end{bmatrix}, \qquad A = (a_{ij})_{3\times 3} = \begin{bmatrix} 0.58 & 0.12 & 0.23 \\ -0.08 & 0.36 & -0.05 \\ -0.04 & 0.04 & -0.37 \end{bmatrix},$$
$$B = (b_{ij})_{3\times 3} = \begin{bmatrix} 0.06 & 0 & 0.04 \\ 0.19 & -0.17 & -0.02 \\ -0.03 & 0.13 & 0.44 \end{bmatrix}, \qquad T_1 = (T_{1jl})_{3\times 3} = \begin{bmatrix} 0.03 & -0.20 & -0.05 \\ -0.06 & -0.14 & 0.23 \\ 0.27 & 0.03 & -0.20 \end{bmatrix}, \tag{4.2}$$
$$T_2 = (T_{2jl})_{3\times 3} = \begin{bmatrix} 0.01 & -0.05 & 0.08 \\ -0.06 & -0.03 & -0.09 \\ 0.15 & -0.04 & 0.11 \end{bmatrix}, \qquad T_3 = (T_{3jl})_{3\times 3} = \begin{bmatrix} -0.02 & -0.12 & -0.05 \\ 0.24 & 0.04 & 0.07 \\ -0.02 & 0.08 & 0.01 \end{bmatrix}.$$

In this case, we easily observe that $\tau = \rho = 0.5$, $M = 0.36$, $N = 0.64$, $\|\chi\|^2 = 0.64$.

For Theorem 3.1, choosing $P = Q_1 = Q_2 = E$, then from

$$\prod_{k=1}^{\infty}\left(1+\beta_k^{(i)}\right)^2 = \prod_{k=1}^{\infty}\left(1+\frac{i}{k^2}\right) < \infty, \quad i = 1, 2, 3, \tag{4.3}$$

we may choose $\varepsilon^\star = 0.0976$, $\delta^\star = 0$, $\mathbb{W} = \prod_{k=1}^{\infty}(1+3/k^2) < \infty$.

On the other hand, we can compute

$$-2C + AA^T + e^{\tau\varepsilon^\star}BB^T = \begin{bmatrix} -5.9908 & -0.0036 & -0.0869 \\ -0.0036 & -4.7928 & -0.0023 \\ -0.0869 & -0.0023 & -3.6379 \end{bmatrix} = \Theta \tag{4.4}$$

which implies that $\lambda_{\max}(\Theta) = -3.6347$.

Also, we note that

$$T^{\star T}T^\star = \left[T_1 + T_1^T, T_2 + T_2^T, T_3 + T_3^T\right]^T = \begin{bmatrix} 0.2059 & 0.0832 & -0.0535 \\ 0.0832 & 0.2895 & -0.2735 \\ -0.0535 & -0.2735 & 0.4220 \end{bmatrix} \tag{4.5}$$

implies that

$$-\varepsilon^\star - M - \frac{N}{1-\rho} - \frac{N\lambda_{\max}\left(T^{\star T}T^\star\right)}{1-\rho} - e^{\tau\varepsilon^\star}\|\chi\|^2 \approx -3.2501 > -3.6347. \tag{4.6}$$

By Corollary 3.3, the equilibrium point of (4.1) $(0,0,0)^T$ is global exponential stable with the approximate convergence rate 0.0488.

However, the criteria in [12] are invalid here.

Example 4.2. Consider the high-order delayed Hopfield-type neural network with impulses [13]

$$x_i'(t) = -c_i x_i(t) + \sum_{j=1}^{2} a_{ij} f_j\left(x_j(t)\right) + \sum_{j=1}^{2} b_{ij} g_j\left(x_j(t-\tau(t))\right) + \sum_{j=1}^{2}\sum_{l=1}^{2} T_{ijl} g_l(x_l(t-\tau(t))) g_j\left(x_j(t-\tau(t))\right) + I_i, \quad t \neq t_k,\ t \geq t_0, \tag{4.7}$$

and with impulses

$$\Delta x_i|_{t=t_k} = x_i(t_k) - x_i\left(t_k^-\right) = \beta_k^{(i)}\left(x_i\left(t_k^-\right) - x^*\right), \quad i = 1,2,\ k \in \mathbb{Z}_+, \tag{4.8}$$

where $t_k = k$, $t_0 = 0$, $k \in \mathbb{Z}_+$, $f_1(x_1) = g_1(x_1) = \tanh(0.53x_1)$, $f_2(x_2) = g_2(x_2) = \tanh(0.67x_2)$, $\rho = 0.6$, $J_1 = 1.5$, $J_2 = 2$,

$$\begin{gathered} C = \operatorname{diag}[c_1, c_2]^T = \begin{bmatrix} 1.9 & 0 \\ 0 & 1.89 \end{bmatrix}, \qquad A = \left(a_{ij}\right)_{2\times 2} = \begin{bmatrix} 0.05 & 0.14 \\ 0.20 & 0.31 \end{bmatrix}, \\ B = \left(b_{ij}\right)_{2\times 2} = \begin{bmatrix} 0.09 & 0.25 \\ 0.21 & 0.45 \end{bmatrix}, \qquad T_1 = \left(T_{1jl}\right)_{2\times 2} = \begin{bmatrix} 0.05 & 0.14 \\ -0.06 & 0.05 \end{bmatrix}, \\ T_2 = \left(T_{2jl}\right)_{2\times 2} = \begin{bmatrix} 0.29 & 0.10 \\ 0.23 & -0.14 \end{bmatrix}, \\ \beta_k^{(i)} = e^{0.0625} - 1, \quad k \in \mathbb{Z}_+. \end{gathered} \tag{4.9}$$

It is obvious that $M = N = L^2 = 0.4489$, $\|\chi\|^2 = 0.4489$. Here we consider $\tau = 1$. Choose $P = Q_1 = Q_2 = E$, $\varepsilon^\star = 0.25$, $\delta^\star = 0.128$.

Note that

$$\begin{aligned} \sum_{k=1}^{m} \ln \max\left\{\max_{i\in\Lambda}\left(1 + d_k^{(i)}\right)^2, 1\right\} - \delta^\star(t_m - t_0) &= 0.125m - 0.128m \\ &= -0.003m < 0 \quad \forall m \in \mathbb{Z}_+ \text{ holds.} \end{aligned} \tag{4.10}$$

On the other hand, we can compute

$$-2C + AA^T + e^{\tau\varepsilon^\star} BB^T = \begin{bmatrix} -3.7073 & 0.1848 \\ 0.1848 & -3.3973 \end{bmatrix} = \Delta \tag{4.11}$$

which implies that $\lambda_{\max}(\Delta) = -3.3111$.

One can check that

$$-\varepsilon^\star - M - \frac{N}{1-\rho} - \frac{N\lambda_{\max}\left(T^{\star T} T^\star\right)}{1-\rho} - e^{\tau\varepsilon^\star}\|\chi\|^2 \approx -2.9649 > -3.3111. \tag{4.12}$$

By Corollary 3.3, the equilibrium point of (4.1)–(4.7) x^* is global exponential stable with the approximate convergence rate 0.061.

In fact, for above-given impulsive condition, we only need time-delay τ which satisfies the following condition:

$$\lambda_{\max}\left(-2C + AA^T + e^{0.125\tau}BB^T\right) < -0.4489e^{0.125\tau} - 2.2635. \tag{4.13}$$

Remark 4.3. In [13], the author obtained that the equilibrium point of (4.7) without impulses is globally asymptotically stable. From the example, we obtain that the equilibrium point of (4.7) without impulses is global exponential stability. In fact, if $\beta_k^{(i)} = 0$ in (4.7), then we can choose $\delta^\star = 0$, which implies that, for any given $\tau > 0$, there exists corresponding $\varepsilon^\star > 0$ such that all conditions in Corollary 3.6 are satisfied.

5. Conclusions

In this paper, a class of high-order delayed HNN with impulses is considered. We obtain some new criteria ensuring global exponential stability and uniform stability of the equilibrium point for such system by using Lyapunov functional method, the quality of negative definite matrix, and the linear matrix inequality. Our results show delays and impulsive effects on the stability of HNN. Two examples are given to illustrate the feasibility of the results.

Acknowledgments

This work was jointly supported by the Project of Shandong Province Higher Educational Science and Technology Program (no. J12LN23), Research Fund for Excellent Young and Middle-Aged Scientists of Shandong Province (no. BS2012DX038), National Science Foundation for Postdoctoral Scientists of China (no. 2012M511538), and National Natural Science Foundation of China (no. 61201441).

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