Hindawi Publishing Corporation
Journal of Applied Mathematics
Volume 2012, Article ID 783657, 9 pages
doi:10.1155/2012/783657

Research Article

Further Results on Derivations of Ranked Bigroupoids

Young Bae Jun,[1] Hee Sik Kim,[2] and Eun Hwan Roh[3]

[1] *Department of Mathematics Education, Gyeongsang National University, Jinju 660-701, Republic of Korea*
[2] *Department of Mathematics, Research Institute for Natural Sciences, Hanyang University, Seoul 133-791, Republic of Korea*
[3] *Department of Mathematics Education, Chinju National University of Education, Jinju 660-756, Republic of Korea*

Correspondence should be addressed to Hee Sik Kim, heekim@hanyang.ac.kr

Received 16 May 2012; Revised 2 August 2012; Accepted 4 August 2012

Academic Editor: Hak-Keung Lam

Further properties on $(X, *, \&)$-self-(co)derivations of ranked bigroupoids are investigated, and conditions for an $(X, *, \&)$-self-(co)derivation to be regular are provided. The notion of ranked $*$-subsystems is introduced, and related properties are investigated.

1. Introduction

Several authors [1–4] have studied derivations in rings and near rings. Jun and Xin [5] applied the notion of derivation in ring and near-ring theory to BCI-algebras, and as a result they introduced a new concept, called a (regular) derivation, in BCI-algebras. Zhan and Liu [6] studied f-derivations in BCI-algebras. Alshehri [7] applied the notion of derivations to incline algebras. Alshehri et al. [8] introduced the notion of ranked bigroupoids and discussed $(X, *, \&)$-self-(co)derivations. In this paper, we investigate further properties on $(X, *, \&)$-self-(co)derivations and provide conditions for an $(X, *, \&)$-self-(co)derivation to be regular. We introduce the notion of ranked $*$-subsystems and investigate related properties.

2. Preliminaries

In a nonempty set X with a constant 0 and a binary operation $*$, we consider the following axioms:

(a1) $((x * y) * (x * z)) * (z * y) = 0$,

(a2) $(x * (x * y)) * y = 0$,

(a3) $x * x = 0$,

(a4) $x * y = 0$ and $y * x = 0$ imply $x = y$,

(b1) $x * 0 = x$,

(b2) $(x * y) * z = (x * z) * y$,

(b3) $((x * z) * (y * z)) * (x * y) = 0$,

(b4) $x * (x * (x * y)) = x * y$.

If X satisfies axioms (a1), (a2), (a3), and (a4), then we say that $(X, *, 0)$ is a BCI-algebra. Note that a BCI-algebra $(X, *, 0)$ satisfies conditions (b1), (b2), (b3), and (b4) (see [9]).

In a p-semisimple BCI-algebra X, the following hold:

(b5) $(x * z) * (y * z) = x * y$,

(b6) $0 * (0 * x) = x$.

3. Derivations on Ranked Bigroupoids

A *ranked bigroupoid* (see [8]) is an algebraic system $(X, *, \bullet)$ where X is a non-empty set and "$*$" and "$\bullet$" are binary operations defined on X. We may consider the first binary operation $*$ as the major operation and the second binary operation $\bullet$ as the minor operation.

Given a ranked bigroupoid $(X, *, \&)$, a map $d : X \to X$ is called an $(X, *, \&)$-*self-derivation* (see [8]) if for all $x, y \in X$,

$$d(x * y) = (d(x) * y)\&(x * d(y)). \tag{3.1}$$

In the same setting, a map $d : X \to X$ is called an $(X, *, \&)$-*self-coderivation* (see [8]) if for all $x, y \in X$,

$$d(x * y) = (x * d(y))\&(d(x) * y). \tag{3.2}$$

Note that if $(X, *)$ is a commutative groupoid, then $(X, *, \&)$-self-derivations are $(X, *, \&)$-self-coderivations. A map $d : X \to X$ is called an $abelian$-$(X, *, \&)$-*self-derivation* (see [8]) if it is both an $(X, *, \&)$-self-derivation and an $(X, *, \&)$-self-coderivation.

Proposition 3.1. *Let $(X, *, \&)$ be a ranked bigroupoid with distinguished element 0 in which the minor operation & is defined by $x\&y = y * (y * x)$ for all $x, y \in X$.*

(1) *Assume that X satisfies axioms (b1), (b2), (b3), (a3), and (a4). If a map $d : X \to X$ is an $(X, *, \&)$-self-derivation, then $d(x) = d(x)\&x$ for all $x \in X$.*

(2) *If X satisfies two axioms (b1) and (a3) and a map $d : X \to X$ is an $(X, *, \&)$-self-coderivation, then the following are equivalent:*

(2.1) $d(0) = 0$;

(2.2) $(\forall x \in X)(d(x) = x\&d(x))$.

Proof. (1) Let $x \in X$. Using (b1) and (b2), we have

$$\begin{aligned} d(x) &= d(x * 0) = (d(x) * 0)\&(x * d(0)) \\ &= d(x)\&(x * d(0)) \\ &= (x * d(0)) * ((x * d(0)) * d(x)) \\ &= (x * d(0)) * ((x * d(x)) * d(0)). \end{aligned} \tag{3.3}$$

It follows from (b3) that

$$d(x) * (d(x)\&x) = ((x * d(0)) * ((x * d(x)) * d(0))) * (d(x)\&x) = 0. \tag{3.4}$$

Using (b2) and (a3), we have $(d(x)\&x) * d(x) = 0$, and so $d(x) = d(x)\&x$ for all $x \in X$ by (a4).

(2) Let d be an $(X, *, \&)$-self-coderivation. If $d(0) = 0$, then

$$d(x) = d(x * 0) = (x * d(0))\&(d(x) * 0) = x\&d(x) \tag{3.5}$$

for all $x \in X$. Assume that $d(x) = x\&d(x)$ for all $x \in X$. Taking $x = 0$ implies that $d(0) = 0\&d(0) = 0$. □

Corollary 3.2. *Let $(X, *, \&)$ be a ranked bigroupoid in which $(X, *, 0)$ is a BCI-algebra and the minor operation & is defined by $x\&y = y * (y * x)$ for all $x, y \in X$.*

(1) *If a map $d : X \to X$ is an $(X, *, \&)$-self-derivation, then $d(x) = d(x)\&x$ for all $x \in X$.*

(2) *If a map $d : X \to X$ is an $(X, *, \&)$-self-coderivation, then the following are equivalent:*

(2.1) $d(0) = 0$*;*

(2.2) $(\forall x \in X)\ (d(x) = x\&d(x))$.

Lemma 3.3. *Let $(X, *, \&)$ be a ranked bigroupoid with distinguished element 0 in which three axioms (b2), (a3), and (a4) are valid and the minor operation & is defined by $x\&y = y*(y*x)$ for all $x, y \in X$.*

(1) *For every $x \in X$ with $x\&0 = x$, one has*

$$(\forall y \in X) \quad (y * x = 0 \Longrightarrow y = x). \tag{3.6}$$

(2) *If an element a of X satisfies $a\&0 = a$, then $a\&x = a$ for all $x \in X$.*

Proof. (1) Let $y \in X$ be such that $y * x = 0$. Then

$$\begin{aligned} x * y = (x\&0) * y &= (0 * y) * (0 * x) \\ &= ((y * x) * y) * (0 * x) = (0 * x) * (0 * x) = 0, \end{aligned} \tag{3.7}$$

and so $y = x$ by (a4).

(2) Since $(a\&x) * a = 0$, it follows from (3.6) that $a\&x = a$ for all $x \in X$. □

Corollary 3.4. *Let $(X, *, \&)$ be a ranked bigroupoid in which $(X, *, 0)$ is a BCI-algebra and the minor operation & is defined by $x\&y = y * (y * x)$ for all $x, y \in X$.*

(1) *For every $x \in X$ with $x\&0 = x$, one has*

$$(\forall y \in X) \quad (y * x = 0 \Longrightarrow y = x). \tag{3.8}$$

(2) *If an element a of X satisfies $a\&0 = a$, then $a\&x = a$ for all $x \in X$.*

Proposition 3.5. *Let $(X, *, \&)$ be a ranked bigroupoid with distinguished element 0 in which four axioms (b2), (b4), (a3), and (a4) are valid and the minor operation & is defined by $x\&y = y * (y * x)$ for all $x, y \in X$. If a map $d : X \to X$ is an $(X, *, \&)$-self-coderivation, then $0 * d(x) = d(x)$ for all $x \in X$ with $0 * x = x$.*

Proof. Let $x \in X$ be such that $0 * x = x$. Since $(0 * d(x))\&0 = 0 * d(x)$, it follows from Lemma 3.3(2) that $d(x) = d(0 * x) = (0 * d(x))\&(d(0) * x) = 0 * d(x)$. □

Corollary 3.6. *Let $(X, *, \&)$ be a ranked bigroupoid in which $(X, *, 0)$ is a BCI-algebra and the minor operation & is defined by $x\&y = y * (y * x)$ for all $x, y \in X$. If a map $d : X \to X$ is an $(X, *, \&)$-self-coderivation, then $0 * d(x) = d(x)$ for all $x \in X$ with $0 * x = x$.*

Using Proposition 3.5, we can find an $(X, *, \&)$-self-derivation which is not an $(X, *, \&)$-self-coderivation.

Example 3.7. Let $(\mathbb{Z}, -, \&)$ be a ranked bigroupoid where $\mathbb{Z}$ is the set of all integers with the minus operation "−" and the minor operation "&" defined by $x\&y = y-(y-x)$ for all $x, y \in \mathbb{Z}$. Let d be a self map of $\mathbb{Z}$ given by $d(x) = x-1$ for all $x \in \mathbb{Z}$. Then d is a $(\mathbb{Z}, -, \&)$-self-derivation since

$$\begin{aligned} d(x-y) &= (x-y)-1 = (x-y+1)-2 \\ &= (x-y-1)\&(x-y+1) = ((x-1)-y)\&(x-(y-1)) \\ &= (d(x)-y)\&(x-d(y)). \end{aligned} \tag{3.9}$$

Note that $0 - d(0) = 0 - (0-1) = 1 \neq -1 = 0 - 1 = d(0)$. Hence d is not a $(\mathbb{Z}, -, \&)$-self-coderivation by Proposition 3.5.

Proposition 3.8. *Let $(X, *, \&)$ be a ranked bigroupoid with distinguished element 0 and the minor operation & is defined by $x\&y = y * (y * x)$ for all $x, y \in X$. For an $(X, *, \&)$-self-derivation $d : X \to X$, if $(X, *, 0)$ satisfies axioms (b2),(b5), and (b6), then $d(x) = d(0) * (0 * x)$ for all $x \in X$. Moreover, if $d(0) = 0$, then d is an identity map.*

Proof. Assume that $(X, *, 0)$ satisfies axioms (b2), (b5), and (b6). Then

$$\begin{aligned} d(x) = d(x\&0) &= (d(0) * (0 * x))\&(0 * d(0 * x)) \\ &= (0 * d(0 * x)) * ((0 * d(0 * x)) * (d(0) * (0 * x))) \\ &= (0 * d(0 * x)) * ((0 * (d(0) * (0 * x))) * d(0 * x)) \\ &= 0 * (0 * (d(0) * (0 * x))) \\ &= d(0) * (0 * x), \end{aligned} \tag{3.10}$$

for all $x \in X$. Moreover, if $d(0) = 0$ then $d(x) = d(0) * (0 * x) = x\&0 = x$ for all $x \in X$, and so d is an identity map. □

Corollary 3.9. *Let $(X, *, \&)$ be a ranked bigroupoid in which $(X, *, 0)$ is a BCI-algebra and the minor operation & is defined by $x\&y = y * (y * x)$ for all $x, y \in X$. If a map $d : X \to X$ is an $(X, *, \&)$-self-derivation, then*

(1) $d(0) = d(0)\&0$;

(2) *if $(X, *, 0)$ is p-semisimple, then $d(x) = d(0) * (0 * x)$ for all $x \in X$;*

(3) *if $(X, *, 0)$ is p-semisimple and $d(0) = 0$, then d is an identity map.*

Definition 3.10. Let $(X, *, \&)$ be a ranked bigroupoid with distinguished element 0. A self map d of $(X, *, \&)$ is said to be *regular* if $d(0) = 0$.

Example 3.11. Consider a ranked bigroupoid $(X, *, \&)$ in which $X = \{0, a, b, c, d, e\}$ and binary operations "$*$" and "&" are defined by

$$x * y = \begin{cases} 0 & \text{if } (x, y) \in \{(0, a), (b, d), (c, e)\} \cup \{(z, z) \mid z \in X\}, \\ a & \text{if } (x, y) \in \{(a, 0), (d, b), (e, c)\}, \\ b & \text{if } (x, y) \in \{(b, 0), (0, c), (0, e), (a, e), (b, a), (c, b), (c, d), (d, a), (e, d)\}, \\ c & \text{if } (x, y) \in \{(c, 0), (c, a), (e, a), (0, b), (b, c), (0, d), (a, d), (b, e), (d, e)\}, \\ d & \text{if } (x, y) \in \{(d, 0), (e, b), (a, c)\}, \\ e & \text{if } (x, y) \in \{(a, b), (d, c), (e, 0)\} \end{cases} \quad (3.11)$$

&	0	a	b	c	d	e
0	0	0	0	0	0	0
a	0	a	0	0	a	a
b	b	b	0	b	b	b
c	c	c	c	c	c	c
d	b	d	b	b	d	d
e	c	e	c	c	e	e.

Define a map $d : X \to X$ by

$$d(x) = \begin{cases} 0 & \text{if } x \in \{0, a\}, \\ b & \text{if } x \in \{b, d\}, \\ c & \text{if } x \in \{c, e\}. \end{cases} \quad (3.12)$$

Then d is an abelian $(X, *, \&)$-self-derivation which is regular.

Proposition 3.12. *Let $(X, *, \&)$ be a ranked bigroupoid with distinguished element 0 in which the minor operation & is defined by $x\&y = y * (y * x)$ for all $x, y \in X$ and $0 * x = 0$ for all $x \in X$. Then every $(X, *, \&)$-self-derivation is regular. Moreover, if X satisfies the axioms (b1) and (a3) then every $(X, *, \&)$-self-coderivation is regular.*

Proof. Let d be an $(X, *, \&)$-self-derivation. Then

$$d(0) = d(0 * x) = (d(0) * x)\&(0 * d(x)) = (d(0) * x)\&0 = 0. \quad (3.13)$$

If d is an $(X, *, \&)$-self-coderivation, then

$$d(0) = d(0 * x) = (0 * d(x))\&(d(0) * x) = 0\&(d(0) * x) = 0. \quad (3.14)$$

Hence every $(X, *, \&)$-self-(co)derivation is regular. □

Proposition 3.13. *Let $(X, *, \&)$ be a ranked bigroupoid with distinguished element 0 in which the minor operation $\&$ is defined by $x\&y = y * (y * x)$ for all $x, y \in X$ and two axioms $(a3)$ and $(b1)$ are satisfied. Let d be a self map of X and $a \in X$ such that $d(x) * a = 0$ (resp., $a * d(x) = 0$) for all $x \in X$. If d is an $(X, *, \&)$-self-derivation (resp., $(X, *, \&)$-self-coderivation), then it is regular.*

Proof. Assume that d is an $(X, *, \&)$-self-derivation. For any $x \in X$, we have

$$0 = d(x * a) * a = ((d(x) * a)\&(x * d(a))) * a = (0\&(x * d(a))) * a = 0 * a, \quad (3.15)$$

which implies that

$$d(0) = d(0 * a) = (d(0) * a)\&(0 * d(a)) = 0\&(0 * d(a)) = 0. \quad (3.16)$$

Hence d is regular. Now, let d be an $(X, *, \&)$-self-coderivation such that $a * d(x) = 0$ for all $x \in X$. Then

$$0 = a * d(a * x) = a * ((a * d(x))\&(d(a) * x)) = a * (0\&(d(a) * x)) = a * 0, \quad (3.17)$$

and so

$$d(0) = d(a * 0) = (a * d(0))\&(d(a) * 0) = 0\&(d(a) * 0) = 0\&d(a) = 0. \quad (3.18)$$

Therefore d is regular. □

Definition 3.14. Let $(X, *, \&)$ be a ranked bigroupoid with distinguished element 0. Let d be a self map of $(X, *, \&)$. A subset A of X is called a ranked $*$-subsystem of X if it satisfies the following:

(r1) $0 \in A$,

(r2) $(\forall x, y \in X)(x \in A, y * x \in A \Rightarrow y \in A)$.

Moreover, if a ranked $*$-subsystem A of X satisfies $d(A) \subseteq A$, then we say that A is *ranked d-invariant*.

Example 3.15. Consider a ranked bigroupoid $(X, *, \&)$ in which $X = \{0, a, b, c, d, e\}$ and binary operations "$*$"and "&" are defined by

$$x * y = \begin{cases} 0 & \text{if } (x, y) \in \{(0, a), (b, c), (b, d), (b, e), (c, d), (c, e)\} \cup \{(z, z) \mid z \in X\}, \\ a & \text{if } (x, y) \in \{(a, 0), (c, b), (d, b), (e, b), (d, c), (e, c), (e, d), (d, e)\}, \\ c & \text{if } (x, y) = (c, 0), \\ d & \text{if } (x, y) = (d, 0), \\ e & \text{if } (x, y) = (e, 0), \\ b & \text{otherwise}, \end{cases} \tag{3.19}$$

and $x\&y = y * (y * x)$ for all $x, y \in X$. Define a map d: $X \to X$ by

$$d(x) = \begin{cases} b & \text{if } x \in \{0, a\} \\ 0 & \text{otherwise.} \end{cases} \tag{3.20}$$

Then d is an abelian $(X, *, \&)$-self-derivation which is not regular. It is easily check that $A = \{0, a\}$ is a ranked $*$-subsystem of X. Since $d(A) = \{b\} \nsubseteq A$, d is not ranked d-invariant.

Example 3.16. In Example 3.11, $A = \{0, a\}$ is a ranked d-invariant $*$-subsystem of X.

Theorem 3.17. *Let $(X, *, \&)$ be a ranked bigroupoid with distinguished element 0 in which three axioms (b1),(b2), and (a3) are valid and the minor operation & is defined by $x\&y = y * (y * x)$ for all $x, y \in X$. For an $(X, *, \&)$-self-coderivation d, if d is regular then every ranked $*$-subsystem of X is ranked d-invariant.*

Proof. Assume that d is regular and let A be a ranked $*$-subsystem of X. Then $d(x) = x\&d(x)$ for all $x \in X$ by Proposition 3.1(2). Let $y \in d(A)$. Then $y = d(a)$ for some $a \in A$. Thus $y * a = d(a) * a = (a\&d(a)) * a = 0 \in A$, and so $y \in A$ by (r2). Hence $d(A) \subseteq A$ and A is ranked d-invariant. □

Corollary 3.18. *Let d be an $(X, *, \&)$-self-coderivation where $(X, *, 0)$ is a BCI-algebra and the minor operation & is defined by $x\&y = y * (y * x)$ for all $x, y \in X$. If d is regular, then every ideal of X is ranked d-invariant.*

Example 3.15 shows that Theorem 3.17 is not true if we drop the regularity of d.

We consider the converse of Theorem 3.17.

Theorem 3.19. *Let d be an $(X, *, \&)$-self-coderivation where $(X, *, \&)$ is a ranked bigroupoid with distinguished element 0 in which the minor operation & is defined by $x\&y = y*(y*x)$ for all $x, y \in X$ and there does not exist a nonzero element x of X such that $x * 0 = 0$. If every ranked $*$-subsystem of X is ranked d-invariant, then d is regular.*

Proof. Assume that every ranked $*$-subsystem of X is ranked d-invariant. Note that $A = \{0\}$ is a ranked $*$-subsystem of X. Thus $d(A) = d(\{0\}) \subseteq \{0\}$, and therefore $d(0) = 0$, that is, d is regular. □

Corollary 3.20. *Let d be an $(X,*,\&)$-self-coderivation where $(X,*,0)$ is a BCI-algebra and the minor operation & is defined by $x\&y = y*(y*x)$ for all $x,y\in X$. Then d is regular if and only if every ranked $*$-subsystem of X is ranked d-invariant.*

Proposition 3.21. *Let $(X,*,\&)$ be a ranked bigroupoid where $(X,*,0)$ is a BCI-algebra and the minor operation & is defined by $x\&y = y*(y*x)$ for all $x,y\in X$. For any $\alpha\in X$, let d_α be a self map of X defined by $d_\alpha(x) = x*\alpha$ for all $x\in X$. If X satisfies the following conditions:*

(1) $((x*y)*z)*(x*(y*z)) = 0$ *for all* $x,y,z\in X$,

(2) $(\forall x,y\in X)\,(x*y=0\Rightarrow x=y)$,

then d_α is an abelian $(X,*,\&)$-self-derivation.

Proof. If X satisfies two given conditions, then the following identity is valid (see [9]):

$$(\forall x,y,z\in X)\big((x*y)*z = x*(y*z)\big). \tag{3.21}$$

It follows from (b1), (a3), and (b2) that

$$\begin{aligned} d_\alpha(x*y) &= (x*y)*\alpha = (x*(y*\alpha))*0 \\ &= (x*(y*\alpha))*((x*(y*\alpha))*(x*(y*\alpha))) \\ &= (x*(y*\alpha))*((x*(y*\alpha))*((x*\alpha)*y)) \\ &= (d_\alpha(x)*y)\&(x*d_\alpha(y)). \end{aligned} \tag{3.22}$$

Hence d_α is an $(X,*,\&)$-self-derivation. Similarly, we can verify that d_α is an $(X,*,\&)$-self-coderivation. □

Corollary 3.22. *Let $(X,*,\&)$ be a ranked bigroupoid where $(X,*,0)$ is a BCI-algebra and the minor operation & is defined by $x\&y = y*(y*x)$ for all $x,y\in X$. For any $\alpha\in X$, let d_α be a self map of X defined by $d_\alpha(x) = x*\alpha$ for all $x\in X$. If X satisfies (b1) and the following conditions:*

(1) $((x*y)*z)*(x*(y*z)) = 0$ *for all* $x,y,z\in X$,

(2) $(x*y)*(x*z) = z*y$ *for all* $x,y,z\in X$,

then d_α is an abelian $(X,*,\&)$-self-derivation.

Proof. If X satisfies both (b1) and the second condition, then X is a p-semisimple BCI-algebra (see [9]). Hence the second condition of Proposition 3.21 is valid. Therefore d_α is an abelian $(X,*,\&)$-self-derivation. □

4. Conclusion

Alshehri et al. [8] introduced the notion of ranked bigroupoids and discussed $(X,*,\&)$-self-(co)derivations.

A nonempty set X together with maps $* : X \times X \to X$ and $\& : X \times X \to X$ is called a *ranked bigroupoid*. For a ranked bigroupoid $(X, *, \&)$, a map $d : X \to X$ is called:

(1) an $(X, *, \&)$-*self-derivation* if

$$d(x * y) = (d(x) * y)\&(x * d(y)) \tag{4.1}$$

for all $x, y \in X$;

(2) an $(X, *, \&)$-*self-coderivation* if

$$d(x * y) = (x * d(y))\&(d(x) * y) \tag{4.2}$$

for all $x, y \in X$.

In this paper, we have investigated further properties on $(X, *, \&)$-self-(co)derivations and have provided conditions for an $(X, *, \&)$-self-(co)derivation to be regular. We have introduced the notion of ranked $*$-subsystems and have investigated related properties.

In general, there are many kind of derivations (generalized derivations, biderivations, triderivations, etc.) in algebraic structures, for example, (near) rings, prime rings, semiprime rings, Γ-near-rings, incline algebras, Banach algebras, lattices, MV-algebras, and BCK/BCI-algebras.

Based on this paper together with related papers on derivations, we will consider several kind of derivations in ranked bigroupoids.

Acknowledgment

The authors wish to thank the anonymous reviewers for their valuable suggestions.

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