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Research Article

Finite-Time H_∞ Filtering for Linear Continuous Time-Varying Systems with Uncertain Observations

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This paper is concerned with the finite-time H_∞ filtering problem for linear continuous time-varying systems with uncertain observations and $\mathcal{L}_2$-norm bounded noise. The design of finite-time H_∞ filter is equivalent to the problem that a certain indefinite quadratic form has a minimum and the filter is such that the minimum is positive. The quadratic form is related to a Krein state-space model according to the Krein space linear estimation theory. By using the projection theory in Krein space, the finite-time H_∞ filtering problem is solved. A numerical example is given to illustrate the performance of the H_∞ filter.

1. Introduction

Most of the literatures on estimation problem always assume the observations contain the signal to be estimated [1–8]. In [5], the linear matrix inequality technique was applied to solve the finite-time H_∞ filtering problem of singular Markovian jump systems. In [6], new stability and robust stability results for 2D discrete stochastic systems were proposed based on weaker conservative assumptions. In [7], an observer was incorporated to the vaccination control rule for an SEIR propagation disease model. In [8], two linear observer prototypes for a class of linear hybrid systems were proposed based on the prediction error. However, in practice, the observation may contain the signal in a random manner, that is, the observation consists of noise alone in a nonzero probability, and it is commonly called uncertain observations or missing measurements [9, 10]. In this paper, the finite-time H_∞ filtering problem is investigated for linear continuous time-varying systems with uncertain observations and $\mathcal{L}_2$-norm bounded noises.

The H_2-based optimal filtering has been well studied for linear systems with uncertain observations [9–13]. In [9], the recursive least-squares estimator was proposed for linear discrete-time systems with uncertain observations. The robust optimal filter for discrete time-varying systems with missing measurements and norm-bounded parameter uncertainties was designed by optimizing the upper bound of the state estimation error variance in [10]. Using the covariance information, the recursive least-squares filtering and fixed-point smoothing algorithms for linear continuous-time systems with uncertain observations were proposed in [11]. Linear and nonlinear one-step prediction algorithms for discrete-time systems with uncertain observations were presented from a covariance assignment viewpoint in [12]. The statistical convergence properties of the estimation error covariance were studied, and the existence of a critical value for the arrival rate of the observations was shown in [13]. In recent years, due to the fact that the H_∞-based estimation approach does not require the information on statistics of input noise, it has received more and more attention for linear systems with uncertain observations [14–16]. Using Lyapunov function approach, the H_∞ filtering algorithms in terms of linear matrix inequalities were proposed for systems with missing measurements in [14–16]. To authors' best knowledge, research on finite-time H_∞ filtering for linear continuous time-varying systems with uncertain observations has not been fully investigated and remains to be challenging, which motivates the present study.

Although the Krein space linear estimation theory [1, 3] has been applied to fault detection and nonlinear estimation [17, 18], no results have been developed for systems with uncertain observations, which will be an interesting research topic in the future. In this paper, the problem of finite-time H_∞ filtering will be investigated for linear continuous time-varying systems with uncertain observations and $\mathcal{L}_2$-norm bounded input noise. Based on the knowledge of Krein space linear estimation theory [1, 3], a new approach in Krein space will be developed to handle the H_∞ filtering problem for linear continuous time-varying systems with uncertain observations. It will be shown that the H_∞ filtering problem for linear continuous time-varying systems with uncertain observations is partially equivalent to an H_2 filtering problem for a certain Krein space state-space model. Through employing projection theory, both the existence condition and a solution of the H_∞ filtering can be obtained in terms of a differential Riccati equation. The major contribution of this paper can be summarized as follows: (i) it shows that the H_∞ filtering problem for systems with uncertain observations can be converted into an H_2 optimal estimation problem subject to a fictitious Krein space stochastic systems; (ii) it develops a Kalman-like robust estimator for linear continuous time-varying systems with uncertain observations.

Notation. Elements in a Krein space will be denoted by **boldface** letters, and elements in the Euclidean space of complex numbers will be denoted by normal letters. The superscripts "-1" and "$*$" stand for the inverse and complex conjugation of a matrix, respectively. $\delta(t-\tau)=0$ for $t \neq \tau$ and $\delta(t-\tau) = 1$ for $t = \tau$. $\mathbb{R}^n$ denotes the n-dimensional Euclidean space. I is the identity matrix with appropriate dimensions. For a real matrix, $P > 0$ (resp., $P < 0$) means that P is symmetric and positive (resp., negative) definite. $\langle \cdot, \cdot \rangle$ denotes the inner product in Krein space. diag$\{\cdots\}$ denotes a block-diagonal matrix. $\theta(t) \in \mathcal{L}_2[0,T]$ means $\int_{t=0}^{T} \theta^*(t)\theta(t)dt < \infty$. $\mathcal{L}\{\cdots\}$ denotes the linear space spanned by sequence $\{\cdots\}$. An abstract vector space $\{\mathcal{K}, \langle \cdot, \cdot \rangle\}$ that satisfies the following requirements is called a **Krein space** [1].

(i) $\mathcal{K}$ is a linear space over $\mathcal{C}$, the field of complex numbers.

(ii) There exists a bilinear form $\langle \cdot, \cdot \rangle \in \mathcal{C}$ on $\mathcal{K}$ such that

(a) $\langle \mathbf{y}, \mathbf{x} \rangle = \langle \mathbf{x}, \mathbf{y} \rangle^*$,

(b) $\langle a\mathbf{x} + b\mathbf{y}, \mathbf{z} \rangle = a\langle \mathbf{x}, \mathbf{z} \rangle + b\langle \mathbf{y}, \mathbf{z} \rangle$,

for any $\mathbf{x}, \mathbf{y}, \mathbf{z} \in \mathcal{K}$, $a, b \in \mathcal{C}$, and where $*$ denotes complex conjugation.

(iii) The vector space $\mathcal{K}$ admits a direct orthogonal sum decomposition

$$\mathcal{K} = \mathcal{K}_+ \oplus \mathcal{K}_- \tag{1.1}$$

such that $\{\mathcal{K}_+, \langle \cdot, \cdot \rangle\}$ and $\{\mathcal{K}_-, -\langle \cdot, \cdot \rangle\}$ are Hilbert spaces, and

$$\langle \mathbf{x}, \mathbf{y} \rangle = 0 \tag{1.2}$$

for any $\mathbf{x} \in \mathcal{K}_+$ and $\mathbf{y} \in K_-$.

2. System Model and Problem Formulation

In this paper, we consider the following linear continuous time-varying system with uncertain observations

$$\begin{gathered} \dot{x}(t) = A(t)x(t) + B(t)w(t), \\ y(t) = r(t)C(t)x(t) + v(t), \\ z(t) = L(t)x(t), \\ x(0) = x_0, \end{gathered} \tag{2.1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $w(t) \in \mathbb{R}^p$ is an exogenous disturbance belonging to $\mathcal{L}_2[0,T]$, $y(t) \in \mathbb{R}^m$ is the observation, $v(t) \in \mathbb{R}^m$ is the observation noise belonging to $\mathcal{L}_2[0,T]$, $z(t) \in \mathbb{R}^q$ is the signal to be estimated, and $A(t)$, $B(t)$, $C(t)$, and $L(t)$ are known real time-varying matrices with appropriate dimensions.

The stochastic variable $r(t) \in \mathbb{R}$ takes the values of 0 and 1 with

$$\begin{gathered} \text{Prob}\{r(t) = 1\} = E_r\{r(t)\} = p(t), \\ \text{Prob}\{r(t) = 0\} = 1 - E_r\{r(t)\} = 1 - p(t), \\ E_r\{r(t)r(s)\} = p(t)p(s), \quad t \neq s, \\ E_r\left\{r^2(t)\right\} = p(t) \end{gathered} \tag{2.2}$$

[11]. Note that many literatures associated with observer design are based on the assumption that $p(t) = 1$ [1–4], it can be unreasonable in many practical applications [9, 10, 13]. In this paper, we assume that $p(t)$ is a known positive scalar.

The finite-time H_∞ filtering problem under investigation is stated as follows: given a scalar $\gamma > 0$, a matrix $P_0 > 0$, and the observation $\{y(s)|_{0\le s\le t}\}$, find an estimate of the signal $z(t)$, denoted by $\check{z}(t) = \mathcal{F}\{y(s)|_{0\le s\le t}\}$, such that

$$J_{\mathcal{F}} = E_r\left\{\|x_0\|^2_{P_0^{-1}} + \int_0^T \|w(t)\|^2 dt + \int_0^T \|v(t)\|^2 dt - \gamma^{-2}\int_0^T \left\|e_f(t)\right\| dt\right\} > 0, \tag{2.3}$$

where $e_f(t) = \check{z}(t) - z(t)$.

Thus, the finite-time H_∞ filtering problem can be equivalent to the following:

(I) $J_{\mathcal{F}}$ has a minimum with respect to $\{x_0, w(t)|_{0\le t\le T}\}$;

(II) $\check{z}(t)$ can be chosen such that the value of $J_{\mathcal{F}}$ at its minimum is positive.

3. Main Results

In this section, through introducing a fictitious Krein space-state space model, we construct a partially equivalent Krein space projection problem. By using innovation analysis approach, we derive the finite-time H_∞ filter and its existence condition.

3.1. Construct a Partially Equivalent Krein Space Problem

To begin with, we introduce the following state transition matrix:

$$\frac{d}{dt}\Phi(t,\tau) = A(t)\Phi(t,\tau), \quad \Phi(\tau,\tau) = I, \tag{3.1}$$

it follows from the state-space model (2.1) that

$$y(t) = r(t)C(t)\Phi(t,0)x_0 + r(t)C(t)\int_0^t \Phi(t,\tau)B(\tau)w(\tau)d\tau + v(t), \tag{3.2}$$

$$\check{z}(t) = L(t)\Phi(t,0)x_0 + L(t)\int_0^t \Phi(t,\tau)B(\tau)w(\tau)d\tau + e_f(t). \tag{3.3}$$

Thus, we can rewrite $J_{\mathcal{F}}$ as

$$
\begin{aligned}
J_{\mathcal{F}} = {} & E_r\left\{\|x_0\|^2_{P_0^{-1}} + \int_0^T \|w(t)\|^2 dt + \int_0^T \|v(t)\|^2 dt - \gamma^{-2}\int_0^T \left\|e_f(t)\right\|^2 dt\right\} \\
= {} & x_0^* P_0^{-1} x_0 + \int_0^T w^*(t)w(t)dt \\
& + E_r\left\{\int_0^T \left(y(t) - r(t)C(t)\Phi(t,0)x_0 - r(t)C(t)\int_0^t \Phi(t,\tau)B(\tau)w(\tau)d\tau\right)^* \right. \\
& \qquad \left. \times\left(y(t) - r(t)C(t)\Phi(t,0)x_0 - r(t)C(t)\int_0^t \Phi(t,\tau)B(\tau)w(\tau)d\tau\right)dt\right\} \\
& - \gamma^{-2}\int_0^T \left(\check{z}(t) - L(t)\Phi(t,0)x_0 - L(t)\int_0^t \Phi(t,\tau)B(\tau)w(\tau)d\tau\right)^* \\
& \times\left(\check{z}(t) - L(t)\Phi(t,0)x_0 - L(t)\int_0^t \Phi(t,\tau)B(\tau)w(\tau)d\tau\right)dt \\
= {} & x_0^* P_0^{-1} x_0 + \int_0^T w^*(t)w(t)dt \\
& + \int_0^T \left(y_0(t) - C_1(t)\Phi(t,0)x_0 - C_1(t)\int_0^t \Phi(t,\tau)B(\tau)w(\tau)d\tau\right)^* \\
& \times\left(y_0(t) - C_1(t)\Phi(t,0)x_0 - C_1(t)\int_0^t \Phi(t,\tau)B(\tau)w(\tau)d\tau\right)dt \\
& + \int_0^T \left(y_s(t) - C_2(t)\Phi(t,0)x_0 - C_2(t)\int_0^t \Phi(t,\tau)B(\tau)w(\tau)d\tau\right)^* \\
& \times\left(y_s(t) - C_2(t)\Phi(t,0)x_0 - C_2(t)\int_0^t \Phi(t,\tau)B(\tau)w(\tau)d\tau\right)dt \\
& - \gamma^{-2}\int_0^T \left(\check{z}(t) - L(t)\Phi(t,0)x_0 - L(t)\int_0^t \Phi(t,\tau)B(\tau)w(\tau)d\tau\right)^* \\
& \times\left(\check{z}(t) - L(t)\Phi(t,0)x_0 - L(t)\int_0^t \Phi(t,\tau)B(\tau)w(\tau)d\tau\right)dt,
\end{aligned}
\tag{3.4}
$$

where

$$
C_1(t) = p(t)C(t), \qquad C_2(t) = \sqrt{p(t)(1-p(t))}C(t), \qquad y_0(t) = y(t), \quad y_s(t) \equiv 0. \tag{3.5}
$$

Moreover, we introduce the following Krein space stochastic system

$$\begin{aligned}\dot{\mathbf{x}}(t) &= A(t)\mathbf{x}(t) + B(t)\mathbf{w}(t),\\ \mathbf{y}_0(t) &= C_1(t)\mathbf{x}(t) + \mathbf{v}(t),\\ \mathbf{y}_s(t) &= C_2(t)\mathbf{x}(t) + \mathbf{v}_s(t),\\ \check{\mathbf{z}}(t) &= L(t)\mathbf{x}(t) + \mathbf{e}_f(t),\\ \mathbf{x}(0) &= \mathbf{x}_0,\end{aligned} \tag{3.6}$$

where $\mathbf{x}_0$, $\mathbf{w}(t)$, $\mathbf{v}(t)$, $\mathbf{v}_s(t)$, and $\mathbf{e}_f(t)$ are mutually uncorrelated white noises with zero means and known covariance matrices as

$$\left\langle \begin{bmatrix}\mathbf{x}_0\\ \mathbf{w}(t)\\ \mathbf{v}(t)\\ \mathbf{v}_s(t)\\ \mathbf{e}_f(t)\end{bmatrix}, \begin{bmatrix}\mathbf{x}_0\\ \mathbf{w}(\tau)\\ \mathbf{v}(\tau)\\ \mathbf{v}_s(\tau)\\ \mathbf{e}_f(\tau)\end{bmatrix} \right\rangle = \begin{bmatrix}P_0 & 0 & 0 & 0 & 0\\ 0 & I\delta(t-\tau) & 0 & 0 & 0\\ 0 & 0 & I\delta(t-\tau) & 0 & 0\\ 0 & 0 & 0 & I\delta(t-\tau) & 0\\ 0 & 0 & 0 & 0 & -\gamma^2 I\delta(t-\tau)\end{bmatrix}. \tag{3.7}$$

Let

$$\begin{aligned}y_0(t) &= C_1(t)x(t) + v(t),\\ y_s(t) &= C_2(t)x(t) + v_s(t),\end{aligned} \tag{3.8}$$

then it follows from (3.1), (3.3), (3.4), and (3.7) that

$$\begin{aligned}J_{\mp} = {}& x_0^*\langle \mathbf{x}_0, \mathbf{x}_0\rangle^{-1}x_0 + \int_0^T w^*(t)\langle \mathbf{w}(t), \mathbf{w}(t)\rangle^{-1}w(t)dt + \int_0^T v^*(t)\langle \mathbf{v}(t), \mathbf{v}(t)\rangle^{-1}v(t)dt\\ &+ \int_0^T v_s^*(t)\langle \mathbf{v}_s(t), \mathbf{v}_s(t)\rangle^{-1}v_s(t)dt + \int_0^T e_f^*(t)\langle \mathbf{e}_f(t), \mathbf{e}_f(t)\rangle^{-1}e_f(t)dt.\end{aligned} \tag{3.9}$$

According to [1] and [3], we have the following results.

Lemma 3.1. *Consider system* (2.1)*, given a scalar $\gamma > 0$ and a matrix $P_0 > 0$, then $J_{\mp}$ in* (2.3) *has the minimum over $\{x_0, w(t)|_{0\le t\le T}\}$ if and only if the innovation $\tilde{\mathbf{y}}_z(t)$ exists for $0 \le t \le T$, where*

$$\tilde{\mathbf{y}}_z(t) = \mathbf{y}_z(t) - \hat{\mathbf{y}}_z(t), \tag{3.10}$$

$\mathbf{y}_z(t) = [\mathbf{y}_0^*(t) \ \ \mathbf{y}_s^*(t) \ \ \check{\mathbf{z}}^*(t)]^*$, *and* $\hat{\mathbf{y}}_z(t)$ *denote the projection of* $\mathbf{y}_z(t)$ *onto* $\mathcal{L}\{\{\mathbf{y}_z(\tau)\}|_{0\le\tau<t}\}$. *In this case the minimum value of* $J_{\mathcal{F}}$ *is*

$$\begin{aligned}\min J_{\mathcal{F}} = &\int_0^T \left(y_0(t) - C_1(t)\hat{x}(t)\right)^* \left(y_0(t) - C_1(t)\hat{x}(t)\right) dt \\ &+ \int_0^T \left(y_s(t) - C_2(t)\hat{x}(t)\right)^* \left(y_s(t) - C_2(t)\hat{x}(t)\right) dt \\ &- \gamma^{-2} \int_0^T (\check{z}(t) - L(t)\hat{x}(t))^* (\check{z}(t) - L(t)\hat{x}(t)) dt,\end{aligned} \tag{3.11}$$

where $\hat{x}(t)$ *is obtained from the Krein space projection of* $\mathbf{x}(t)$ *onto* $\mathcal{L}\{\{\mathbf{y}_z(j)\}|_{0\le\tau<t}\}$.

Remark 3.2. By analyzing the indefinite quadratic form $J_{\mathcal{F}}$ in (3.4) and using the Krein space linear estimation theory [1], it has been shown that the H_∞ filtering problem for linear systems with uncertain observations is equivalent to the H_2 estimation problem with respect to a Krein space stochastic system, which is new as far as we know. In this case, Krein space projection method can be applied to derive an H_∞ estimator for linear systems with uncertain observations, which is more simple and intuitive than previous versions.

3.2. Solution of the Finite-Time H_∞ Filtering Problem

By applying the standard Kalman filter formula to system (3.6), we have the following lemma.

Lemma 3.3. *Consider the Krein space stochastic system* (3.6), *the prediction* $\hat{\mathbf{x}}(t)$ *is calculated by*

$$\dot{\hat{\mathbf{x}}}(t) = A(t)\hat{\mathbf{x}}(t) + K(t)\tilde{\mathbf{y}}_z(t), \tag{3.12}$$

where

$$\begin{aligned}\tilde{\mathbf{y}}_z(t) &= \mathbf{y}_z(t) - H(t)\hat{\mathbf{x}}(t), \\ H(t) &= \left[C_1^*(t) \ \ C_2^*(t) \ \ L^*(t)\right]^*, \\ K(t) &= P(t)H^*(t)R_{\tilde{y}z}^{-1}(t), \\ R_{\tilde{y}z}(t) &= \operatorname{diag}\left\{I, I, -\gamma^2 I\right\},\end{aligned} \tag{3.13}$$

and $P(t)$ *is computed by*

$$\begin{aligned}\dot{P}(t) &= A(t)P(t) + P(t)A^*(t) + B(t)B^*(t) - K(t)R_{\tilde{y}_z}(t)K^*(t), \\ P(0) &= P_0.\end{aligned} \tag{3.14}$$

Now we are in the position to present the main results of this subsection.

Theorem 3.4. *Consider system* (2.1), *given a scalar* $\gamma > 0$ *and a matrix* $P_0 > 0$, *and suppose* $P(t)$ *is the bounded positive definite solution to Riccati differential equation* (3.14). *Then, one possible level-*γ *finite-time* H_∞ *filter that achieves* (2.3) *is given by*

$$\check{z}(t) = L(t)\hat{x}(t), \quad 0 \leq t \leq T, \tag{3.15}$$

where

$$\begin{gathered}\dot{\hat{x}}(t) = A(t)\hat{x}(t) + P(t)C_f^*(t)\left(y_f(t) - C_f(t)\hat{x}(t)\right), \\ \hat{x}(0) = 0\end{gathered} \tag{3.16}$$

with $y_f(t) = [y_0^*(t) \;\; y_s^*(t)]^*$, $C_f(t) = [C_1^*(t) \;\; C_2^*(t)]^*$.

Proof. It follows from Lemma 3.3 that if $P(t)$ is a bounded positive definite solution to Riccati differential equation (3.14), then the projection $\hat{\mathbf{x}}(t)$ exists. According to Lemma 3.1, it is obvious that the H_∞ filter that achieves (2.3) exists. If this is the case, the minimum value of $J_{\mathcal{F}}$ is given by (3.11). In order to achieve min $J_{\mathcal{F}} > 0$, one natural choice is to set

$$\check{z}(t) - L(t)\hat{x}(t) = 0 \tag{3.17}$$

thus the finite-time H_∞ filter can be given by (3.15).

On the other hand, from (3.12) and (3.15), It is easy to verify that (3.16) holds. □

Remark 3.5. Let

$$e(t) = x(t) - \hat{x}(t), \tag{3.18}$$

it follows from (2.1) and (3.16) that

$$\dot{e}(t) = (A(t) - \Gamma(t)C(t))e(t) + B(t)w(t) - P(t)C_f^*(t)v_z(t), \tag{3.19}$$

where

$$\Gamma(t) = P(t)C_f^*(t)\begin{bmatrix} p(t)I \\ \sqrt{p(t)(1-p(t))}I \end{bmatrix}, \qquad v_z(t) = \begin{bmatrix} v(t) \\ v_s(t) \end{bmatrix}. \tag{3.20}$$

Unlike [14–16], the parameter matrices in the filtering error equation (3.19) do not contain the stochastic variable $r(t)$, which is an interesting phenomenon. As mentioned in Definition 1 in [19], it is obvious that, if $(C(t), A(t))$ is detectable, then the filtering error equation (3.19) is exponentially stable. Based on the above analysis, it can be concluded that the following assumptions are necessary for the finite-time H_∞ filter design in this paper:

(i) $(C(t), A(t))$ is detectable,

(ii) $w(t), v(t) \in \mathcal{L}_2[0, T]$.

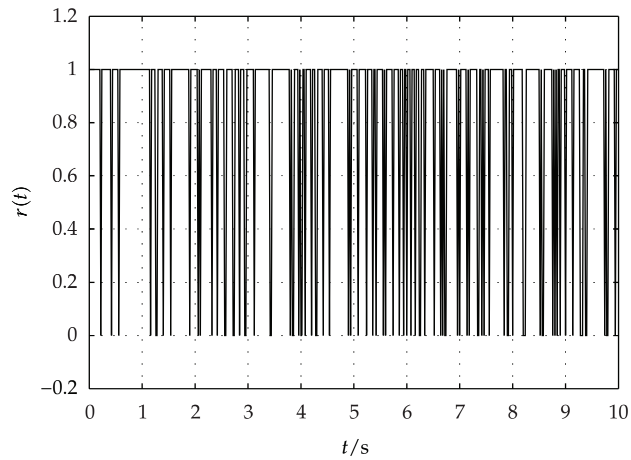

Figure 1: Stochastic variable $r(t)$.

4. A Numerical Example

We consider system (2.1) with the following parameters:

$$A(t) = \begin{bmatrix} -10 & 6 \\ 2 & -5 \end{bmatrix}, \qquad B(t) = \begin{bmatrix} 2.8 \\ 1.6 \end{bmatrix}, \qquad C(t) = [18 \;\; 9.5], \qquad L(t) = [1 \;\; 1] \tag{4.1}$$

and set $\gamma = 1.1$, $x(0) = [0 \;\; 0]^*$, $p(t) = 0.8$, and $P_0 = I$. In addition, we suppose that the noises $w(t)$ and $v(t)$ are generated by Gaussian with zero means and covariances $Q_w = 1$, $Q_v = 0.02$, the sampling time is 0.02 s, and the stochastic variable $r(t)$ is simulated as in Figure 1. Based on Theorem 3.4, we design the finite-time H_∞ filter. Figure 2 shows the true value of signal $z(t)$ and its H_∞ filtering estimate, and Figure 3 shows the estimation error $\tilde{z}(t) = z(t) - \check{z}(t)$. It can be observed from the simulation results that the finite-time H_∞ filter has good tracking performance.

5. Conclusions

In this paper, we have proposed a new finite-time H_∞ filtering technique for linear continuous time-varying systems with uncertain observations. By introducing a Krein state-space model, it is shown that the H_∞ filtering problem can be partially equivalent to a Krein space H_2 filtering problem. A sufficient condition for the existence of the finite-time H_∞ filter is given, and the filter is derived in terms of a differential Riccati equation.

Future research work will extend the proposed method to investigate the H_∞ multistep prediction and fixed-lag smoothing problem for linear continuous time-varying systems with uncertain observations.

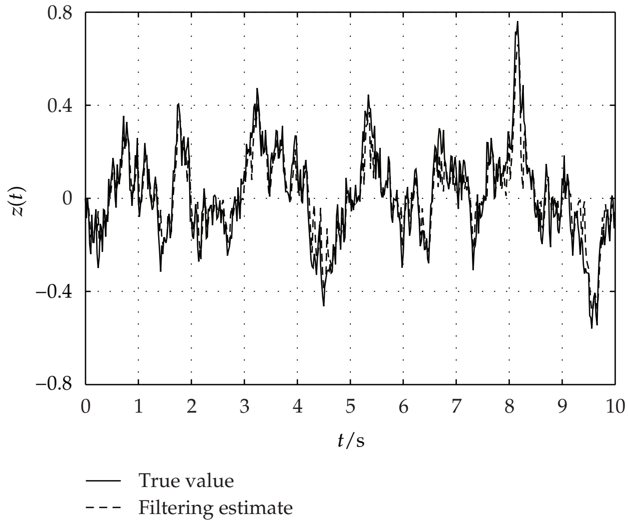

Figure 2: True value of signal $z(t)$ (solid line) and its H_∞ filtering estimate (dashed line).

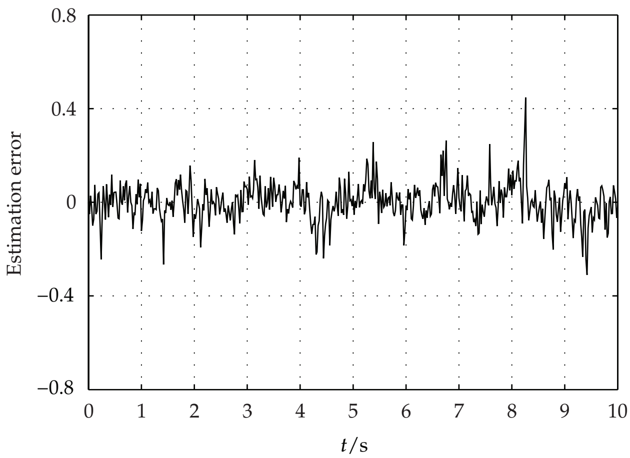

Figure 3: Estimation error $\tilde{z}(t)$.

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