

Review Article

The Existence of Solutions to a System of Discrete Fractional Boundary Value Problems

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We study the existence of solutions for the boundary value problem $-\Delta^\nu y_1(t) = f(y_1(t+\nu-1), y_2(t+\mu-1))$, $-\Delta^\mu y_2(t) = g(y_1(t+\nu-1), y_2(t+\mu-1))$, $y_1(\nu-2) = \Delta y_1(\nu+b) = 0$, $y_2(\mu-2) = \Delta y_2(\mu+b) = 0$, where $1 < \mu, \nu \leq 2$, $f, g : \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}$ are continuous functions, $b \in \mathbb{N}_0$. The existence of solutions to this problem is established by the Guo-Krasnosel'kii theorem and the Schauder fixed-point theorem, and some examples are given to illustrate the main results.

1. Introduction

In recent years, fractional differential equations have been of great interest. It is caused both by the intensive development of the theory of fractional calculus itself and by the applications of such constructions in various sciences such as physics, mechanics, chemistry, and engineering. The continuous fractional calculus has seen tremendous growth within the last ten years or so. Some of the recent progress in the continuous fractional calculus has included a paper [1] in which the authors explored a continuous fractional boundary value problem of conjugate type. Using cone theory, they then deduced the existence of one or more positive solutions. Other recent work in the direction of those articles may be found, see [2–14].

Recently, there appeared a number of papers on the discrete fractional calculus, such as [15–32], which has helped to build up some of the basic theories of this area. For example, Atici and Elloe discussed properties of the generalized falling function, a corresponding power rule for fractional delta-operators, and the commutivity of fractional sums in [15]. And Goodrich studied a two-point fractional boundary value problem in [24], which gave the existence results for a certain two-point boundary value problem of right-focal type for a fractional difference equation.

From the above works, we can see a fact, although the discrete fractional boundary value problem has been studied by some authors, to the best of our knowledge, systems of discrete fractional boundary value problem are seldom considered.

Goodrich in [24] considered a discrete fractional boundary value problem

$$\begin{aligned} -\Delta^\nu y(t) &= f(t + \nu - 1, y(t + \nu - 1)), \\ y(\nu - 2) &= 0 = \Delta y(\nu + b), \end{aligned} \quad (1.1)$$

where $t \in [0, b + 1]_{\mathbb{N}_0}$, $f : [\nu - 1, \nu + b - 1]_{\mathbb{N}_{\nu-1}} \times \mathbb{R} \rightarrow \mathbb{R}$, and $1 < \nu \leq 2$, which gave a representation for the solution to this problem and established the existence and uniqueness of solution to this problem by the Guo-Krasnosel'kii theorem.

Goodrich in [25] examined a system of discrete fractional difference equations subject to nonlocal boundary conditions

$$\begin{aligned} -\Delta^{\nu_1} y_1(t) &= \lambda_1 a_1(t + \nu_1 - 1) f_1(y_1(t + \nu_1 - 1), y_2(t + \nu_2 - 1)), \\ -\Delta^{\nu_2} y_2(t) &= \lambda_2 a_2(t + \nu_2 - 1) f_2(y_1(t + \nu_1 - 1), y_2(t + \nu_2 - 1)), \end{aligned} \quad (1.2)$$

for $t \in [0, b]_{\mathbb{N}_0}$, subject to the conditions

$$\begin{aligned} y_1(\nu_1 - 2) &= \psi_1(y_1), & y_2(\nu_2 - 2) &= \psi_2(y_2), \\ y_1(\nu_1 + b) &= \phi_1(y_1), & y_2(\nu_2 + b) &= \phi_2(y_2), \end{aligned} \quad (1.3)$$

where $\lambda_i > 0$, $a_i : \mathbb{R} \rightarrow [0, +\infty)$, $\nu_i \in (1, 2]$, and for each $i = 1, 2$, $\psi_i, \phi_i : \mathbb{R}^{b+3} \rightarrow \mathbb{R}$ are given functionals, and $f_i : [0, +\infty) \times [0, +\infty) \rightarrow [0, +\infty)$ are continuous for each admissible i . This paper gives the existence of a positive solution on discrete fractional boundary value problems.

Motivated by all the works above, in this paper, we discuss the existence of solutions to a system of discrete fractional boundary value problem

$$\begin{aligned} -\Delta^\nu y_1(t) &= f(y_1(t + \nu - 1), y_2(t + \mu - 1)), & \text{for } t \in [0, b + 1]_{\mathbb{N}_0}, \\ -\Delta^\mu y_2(t) &= g(y_1(t + \nu - 1), y_2(t + \mu - 1)), & \text{for } t \in [0, b + 1]_{\mathbb{N}_0}, \\ y_1(\nu - 2) &= \Delta y_1(\nu + b) = 0, \\ y_2(\mu - 2) &= \Delta y_2(\mu + b) = 0, \end{aligned} \quad (1.4)$$

where $1 < \mu, \nu \leq 2$, $f, g : \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}$ are continuous functions, $b \in \mathbb{N}_0$.

The plan of the paper is as follows. In Section 2, we will present some lemmas in order to prove our main results. Section 3, establishes and proves our main results. In Section 4 we give some examples to illustrate the main results.

2. Preliminaries

For the convenience of the reader, we give some background materials from fractional difference theory to facilitate the analysis of problem (1.4). These and other related results and their proof can be found in [15–18].

Definition 2.1. One defines $t^\nu := \Gamma(t + 1)/\Gamma(t + 1 - \nu)$, for any t and ν for which the right-hand side is defined. One also appeals to the convention that if $t + 1 - \nu$ is a pole of the Gamma function and $t + 1$ is not a pole, then $t^\nu = 0$.

Definition 2.2. The ν th fractional sum of a function f , for $\nu > 0$, is defined by

$$\Delta^{-\nu} f(t; a) := \frac{1}{\Gamma(\nu)} \sum_{s=a}^{t-\nu} (t-s-1)^{\nu-1} f(s), \tag{2.1}$$

for $t \in \{a + \nu, a + \nu + 1, \dots\} := \mathbb{N}_{a+\nu}$. One also defines the ν th fractional difference for $\nu > 0$ by $\Delta^\nu f(t) := \Delta^N \Delta^{\nu-N} f(t)$, where $t \in \mathbb{N}_{a+\nu}$ and $\nu \in \mathbb{N}$ is chosen so that $0 \leq N - 1 < \nu \leq N$.

Lemma 2.3. *Let t and ν be any numbers for which t^ν and $t^{\nu-1}$ are defined, then*

$$\Delta t^\nu = \nu t^{\nu-1}. \tag{2.2}$$

Lemma 2.4. *Let $0 \leq N - 1 < \nu \leq N$, where $N \in \mathbb{N}$ and $N - 1 \geq 0$, then*

$$\Delta^{-\nu} \Delta^\nu y(t) = y(t) + C_1 t^{\nu-1} + C_2 t^{\nu-2} + \dots + C_N t^{\nu-N}, \tag{2.3}$$

for some $C_i \in \mathbb{R}$, with $1 \leq i \leq N$.

Before stating the next useful lemma, let us introduce the following notation, which will be important in the sequel:

$$\begin{aligned} T_1 &:= \{(t, s) \in [\nu - 1, \nu + b + 1]_{\mathbb{N}_{\nu-1}} \times [0, b + 1]_{\mathbb{N}_0} : 0 \leq s < t - \nu + 1 \leq b + 2\}, \\ T_2 &:= \{(t, s) \in [\nu - 1, \nu + b + 1]_{\mathbb{N}_{\nu-1}} \times [0, b + 1]_{\mathbb{N}_0} : 0 \leq t - \nu + 1 \leq s \leq b + 2\}, \\ L_1 &:= \{(t, s) \in [\mu - 1, \mu + b + 1]_{\mathbb{N}_{\mu-1}} \times [0, b + 1]_{\mathbb{N}_0} : 0 \leq s < t - \mu + 1 \leq b + 2\}, \\ L_2 &:= \{(t, s) \in [\mu - 1, \mu + b + 1]_{\mathbb{N}_{\mu-1}} \times [0, b + 1]_{\mathbb{N}_0} : 0 \leq t - \mu + 1 \leq s \leq b + 2\}. \end{aligned} \tag{2.4}$$

Lemma 2.5. ([24]) *The unique solution of the discrete fractional boundary value problem*

$$-\Delta^\nu y(t) = h(t + \nu - 1), \quad y(\nu - 2) = 0 = \Delta y(\nu + b) \tag{2.5}$$

is given by

$$y(t) := \sum_{s=0}^{b+1} G_1(t, s) h(s + \nu - 1), \tag{2.6}$$

where $1 < \nu \leq 2$, $G_1(t, s)$ is given by

$$G_1(t, s) := \frac{1}{\Gamma(\nu)} \begin{cases} \frac{\Gamma(b+3)t^{\nu-1}}{\Gamma(\nu+b+1)} (\nu+b-s-1)^{\nu-2} - (t-s-1)^{\nu-1}, & (t, s) \in T_1, \\ \frac{\Gamma(b+3)t^{\nu-1}}{\Gamma(\nu+b+1)} (\nu+b-s-1)^{\nu-2}, & (t, s) \in T_2. \end{cases} \quad (2.7)$$

Remark 2.6. Let us note that in case we put μ replacing ν in Lemma 2.5, it follows that

$$y(t) := \sum_{s=0}^{b+1} G_2(t, s) h(s + \mu - 1), \quad (2.8)$$

where $1 < \mu \leq 2$, $G_2(t, s)$ is given by

$$G_2(t, s) := \frac{1}{\Gamma(\mu)} \begin{cases} \frac{\Gamma(b+3)t^{\mu-1}}{\Gamma(\mu+b+1)} (\mu+b-s-1)^{\mu-2} - (t-s-1)^{\mu-1}, & (t, s) \in L_1, \\ \frac{\Gamma(b+3)t^{\mu-1}}{\Gamma(\mu+b+1)} (\mu+b-s-1)^{\mu-2}, & (t, s) \in L_2. \end{cases} \quad (2.9)$$

Lemma 2.7. ([24]) *The Green function $G_1(t, s)$ given in Lemma 2.5 satisfies*

- (i) $G_1(t, s) \geq 0$ for $(t, s) \in [\nu-1, \nu+b+1]_{\mathbb{N}_{\nu-1}} \times [0, b+1]_{\mathbb{N}_0}$
- (ii) $\max_{t \in [\nu-1, \nu+b+1]_{\mathbb{N}_{\nu-1}}} G_1(t, s) = G_1(s + \nu - 1, s)$, for $s \in [0, b]_{\mathbb{N}_0}$
- (iii) there exists a number $\gamma_1 \in (0, 1)$ such that

$$\min_{t \in [(b+\nu)/4, 3(b+\nu)/4]} G_1(t, s) \geq \gamma_1 \max_{t \in [\nu-1, \nu+b+1]_{\mathbb{N}_{\nu-1}}} G_1(t, s) = \gamma_1 G_1(s + \nu - 1, s), \text{ for } s \in [0, b+1]_{\mathbb{N}_0}. \quad (2.10)$$

Similarly, $G_2(t, s)$ satisfies

- (I) $G_2(t, s) \geq 0$ for $(t, s) \in [\mu-1, \mu+b+1]_{\mathbb{N}_{\mu-1}} \times [0, b+1]_{\mathbb{N}_0}$
- (II) $\max_{t \in [\mu-1, \mu+b+1]_{\mathbb{N}_{\mu-1}}} G_2(t, s) = G_2(s + \mu - 1, s)$, for $s \in [0, b]_{\mathbb{N}_0}$
- (III) there exists a number $\gamma_2 \in (0, 1)$ such that

$$\min_{t \in [(b+\mu)/4, 3(b+\mu)/4]} G_2(t, s) \geq \gamma_2 \max_{t \in [\mu-1, \mu+b+1]_{\mathbb{N}_{\mu-1}}} G_2(t, s) = \gamma_2 G_2(s + \mu - 1, s), \text{ for } s \in [0, b+1]_{\mathbb{N}_0}. \quad (2.11)$$

Let \mathcal{B}_1 and \mathcal{B}_2 represent the Banach space of all maps from $[\nu-1, \nu+b]_{\mathbb{N}_{\nu-1}}$ into \mathbb{R} and $[\mu-1, \mu+b]_{\mathbb{N}_{\mu-1}}$ into \mathbb{R} , respectively. And $\|\cdot\|$ is the usual maximum norm. Define

$$\mathcal{X} := \mathcal{B}_1 \times \mathcal{B}_2. \quad (2.12)$$

By equipping \mathcal{X} with the norm

$$\|(y_1, y_2)\| := \|y_1\| + \|y_2\|, \quad (2.13)$$

it follows that $(\mathcal{X}, \|\cdot\|)$ is a Banach space, see [26, 27].

Next, we wish to develop a representation for a solution of (1.4) as the fixed point of an appropriate operator on \mathcal{X} . To accomplish this, we present some straightforward adaptations of results from [24] that will be of use here. Since the proofs of these adaptations are evident, we do not include them.

Now, consider the operator $\mathcal{S} : \mathcal{X} \rightarrow \mathcal{X}$ defined by

$$\mathcal{S}(y_1, y_2)(t_1, t_2) := (\mathcal{S}_1(y_1, y_2)(t_1), \mathcal{S}_2(y_1, y_2)(t_2)), \quad (2.14)$$

where we define $\mathcal{S}_1 : \mathcal{X} \rightarrow \mathcal{B}_1$ by

$$\mathcal{S}_1(y_1, y_2)(t_1) := \sum_{s=0}^{b+1} G_1(t_1, s) f(y_1(s + \nu - 1), y_2(s + \mu - 1)), \quad (2.15)$$

and $\mathcal{S}_2 : \mathcal{X} \rightarrow \mathcal{B}_2$ by

$$\mathcal{S}_2(y_1, y_2)(t_2) := \sum_{s=0}^{b+1} G_2(t_2, s) f(y_1(s + \nu - 1), y_2(s + \mu - 1)). \quad (2.16)$$

We claim that whenever $(y_1, y_2) \in \mathcal{X}$ is a fixed point of the operator \mathcal{S} , it follows that the pair of functions $y_1(t)$ and $y_2(t)$ are a solution to problem (1.4).

Lemma 2.8. *Let $f, g : \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}$. If $(y_1, y_2) \in \mathcal{X}$ is a fixed point of \mathcal{S} , then (y_1, y_2) is a solution to problem (1.4).*

Proof. Suppose that the operator \mathcal{S} has a fixed point, say $(y_1, y_2) \in \mathcal{X}$. Let $(t_1, t_2) \in \mathbb{N}_{\nu-1} \times \mathbb{N}_{\mu-1}$, then we have

$$y_1 = \mathcal{S}_1(y_1, y_2)(t_1), \quad (2.17)$$

where \mathcal{S}_1 is defined as in (2.15). It is easy to check that $\mathcal{S}_1(y_1, y_2)(\nu - 2) = 0$ and

$$\begin{aligned}
\Delta \mathcal{S}_1(y_1, y_2)(\nu + b) &= \mathcal{S}_1(y_1, y_2)(\nu + b + 1) - \mathcal{S}_1(y_1, y_2)(\nu + b) \\
&= \sum_{s=0}^{b+1} \left(\frac{\Gamma(b+3)(\nu+b+1)^{\nu-1}}{\Gamma(\nu+b+1)} (\nu+b-s-1)^{\nu-2} - (\nu+b-s)^{\nu-1} \right) \\
&\quad \times f(y_1(s+\nu-1), y_2(s+\mu-1)) \\
&\quad - \sum_{s=0}^{b+1} \left(\frac{\Gamma(b+3)(\nu+b)^{\nu-1}}{\Gamma(\nu+b+1)} (\nu+b-s-1)^{\nu-2} - (\nu+b-s-1)^{\nu-1} \right) \\
&\quad \times f(y_1(s+\nu-1), y_2(s+\mu-1)) \\
&= \sum_{s=0}^{b+1} \left((\nu+b+1)(\nu+b-s-1)^{\nu-2} - (b+2)(\nu+b-s-1)^{\nu-2} \right) \\
&\quad \times f(y_1(s+\nu-1), y_2(s+\mu-1)) \\
&\quad - \sum_{s=0}^{b+1} \left(\frac{(\nu+b-s)\Gamma(\nu+b-s)}{(b-s+1)\Gamma(b-s+1)} - \frac{\Gamma(\nu+b-s)}{\Gamma(b-s+1)} \right) \\
&\quad \times f(y_1(s+\nu-1), y_2(s+\mu-1)) \\
&= \sum_{s=0}^{b+1} \left((\nu-1)(\nu+b-s-1)^{\nu-2} - \frac{(\nu-1)\Gamma(\nu+b-s)}{(b-s+1)\Gamma(b-s+1)} \right) \\
&\quad \times f(y_1(s+\nu-1), y_2(s+\mu-1)) \\
&= \sum_{s=0}^{b+1} \left((\nu-1) \frac{\Gamma(\nu+b-s)}{\Gamma(b-s+2)} - \frac{(\nu-1)\Gamma(\nu+b-s)}{\Gamma(b-s+2)} \right) \\
&\quad \times f(y_1(s+\nu-1), y_2(s+\mu-1)) = 0,
\end{aligned} \tag{2.18}$$

so the boundary conditions are satisfied. Furthermore, we can check that $\mathcal{S}_1(y_1, y_2)(t_1)$ satisfies the difference equation in (1.4). Since similar verifications may be made for $\mathcal{S}_2(y_1, y_2)(t_2)$, the claim follows, which completes the proof. \square

3. Main Results

In this section, we will show that under certain conditions, problem (1.4) has at least one solution.

Let us now present the conditions that we will assume henceforth. We note that conditions (F1)-(F2) are similar to the conditions given by Henderson et al. [26]

$$\begin{aligned}
\text{(F1)} : & f, g \in C([0, \infty), [0, \infty)) \\
\text{(F2)} : & \lim_{(y_1, y_2) \rightarrow (0^+, 0^+)} \frac{f(y_1, y_2)}{y_1 + y_2} = f^*, \quad \lim_{(y_1, y_2) \rightarrow (0^+, 0^+)} \frac{g(y_1, y_2)}{y_1 + y_2} = g^*, \\
& \lim_{(y_1, y_2) \rightarrow (+\infty, +\infty)} \frac{f(y_1, y_2)}{y_1 + y_2} = f^{**}, \quad \lim_{(y_1, y_2) \rightarrow (+\infty, +\infty)} \frac{g(y_1, y_2)}{y_1 + y_2} = g^{**},
\end{aligned} \tag{3.1}$$

satisfying

$$\begin{aligned} \sum_{s=0}^{b+1} G_1(s + \nu - 1, s)(f^* + \varepsilon) &\leq \frac{1}{2}, & \sum_{s=0}^{b+1} G_2(s + \mu - 1, s)(g^* + \varepsilon) &\leq \frac{1}{2}, \\ \sum_{s=0}^{b+1} \gamma G_1(s + \nu - 1, s)(f^{**} - \varepsilon) &\geq \frac{1}{2}, & \sum_{s=0}^{b+1} \gamma G_2(s + \mu - 1, s)(g^{**} - \varepsilon) &\geq \frac{1}{2}, \end{aligned} \quad (3.2)$$

for some $0 < \varepsilon < \min\{f^{**}, g^{**}\}$.

Define the cone by

$$\begin{aligned} \mathcal{K} := &\left\{ (y_1, y_2) \in \mathcal{X} : y_1, y_2 \geq 0, \right. \\ &\left. \min_{(t_1, t_2) \in [(\nu-1)/4, 3(\nu+b)/4] \times [(\mu-1)/4, 3(\mu+b)/4]} \right. \\ &\left. \times [y_1(t_1) + y_2(t_2)] \geq \gamma \|(y_1, y_2)\| \right\}, \end{aligned} \quad (3.3)$$

where $\gamma = \min\{\gamma_1, \gamma_2\}$, for $i = 1, 2$, γ_i is defined as in Lemma 2.7. Clearly, $\mathcal{K} \subseteq \mathcal{X}$. In order to show that \mathcal{S} has a fixed point in \mathcal{K} , we must first demonstrate that \mathcal{K} is invariant under \mathcal{S} , that is, $\mathcal{S} : \mathcal{K} \rightarrow \mathcal{K}$. We show this now.

Lemma 3.1. *Assume that f, g are nonnegative functions. Let $\mathcal{S} : \mathcal{X} \rightarrow \mathcal{X}$ be the operator defined as (2.14), then $\mathcal{S} : \mathcal{K} \rightarrow \mathcal{K}$.*

Proof. Suppose that $(y_1, y_2) \in \mathcal{K}$. We claim that

$$\min_{(t_1, t_2) \in [(\nu-1)/4, 3(\nu+b)/4] \times [(\mu-1)/4, 3(\mu+b)/4]} [\mathcal{S}_1(y_1, y_2)(t_1) + \mathcal{S}_2(y_1, y_2)(t_2)] \geq \gamma \|\mathcal{S}(y_1, y_2)\|. \quad (3.4)$$

In fact,

$$\begin{aligned} &\min_{t_1 \in [(\nu-1)/4, 3(\nu+b)/4]} \mathcal{S}_1(y_1, y_2)(t_1) \\ &\geq \min_{t_1 \in [(\nu-1)/4, 3(\nu+b)/4]} \sum_{s=0}^{b+1} G_1(t_1, s) f(y_1(s + \nu - 1), y_2(s + \mu - 1)) \\ &\geq \gamma_1 \sum_{s=0}^{b+1} G_1(s + \nu - 1, s) f(y_1(s + \nu - 1), y_2(s + \mu - 1)) \\ &= \gamma_1 \max_{t_1 \in [\nu-1, \nu+b]_{\mathbb{N}_{\nu-1}}} \sum_{s=0}^{b+1} G_1(t_1, s) f(y_1(s + \nu - 1), y_2(s + \mu - 1)) \\ &= \gamma_1 \|\mathcal{S}_1(y_1, y_2)\|. \end{aligned} \quad (3.5)$$

Similarly, we get

$$\min_{t_2 \in [(\mu-1)/4, 3(\mu+b)/4]} \mathcal{S}_2(y_1, y_2)(t_2) \geq \gamma_2 \|\mathcal{S}_2(y_1, y_2)\|. \quad (3.6)$$

Put $\gamma = \min\{\gamma_1, \gamma_2\}$. Consequently, from (3.5) and (3.6), we obtain

$$\begin{aligned}
 & \min_{(t_1, t_2) \in [(\nu-1)/4, 3(\nu+b)/4] \times [(\mu-1)/4, 3(\mu+b)/4]} [\mathcal{S}_1(y_1, y_2)(t_1) + \mathcal{S}_2(y_1, y_2)(t_2)] \\
 & \geq \min_{t_1 \in [(\nu-1)/4, 3(\nu+b)/4]} \mathcal{S}_1(y_1, y_2)(t_1) \\
 & \quad + \min_{t_2 \in [(\mu-1)/4, 3(\mu+b)/4]} \mathcal{S}_2(y_1, y_2)(t_2) \\
 & \geq \gamma_1 \|\mathcal{S}_1(y_1, y_2)\| + \gamma_2 \|\mathcal{S}_2(y_1, y_2)\| \\
 & \geq \gamma (\|\mathcal{S}_1(y_1, y_2)\| + \|\mathcal{S}_2(y_1, y_2)\|) \\
 & = \gamma \|\mathcal{S}(y_1, y_2)\|.
 \end{aligned} \tag{3.7}$$

So, for $(y_1, y_2) \in \mathcal{K}$, we find

$$\min_{(t_1, t_2) \in [(\nu-1)/4, 3(\nu+b)/4] \times [(\mu-1)/4, 3(\mu+b)/4]} [\mathcal{S}_1(y_1, y_2)(t_1) + \mathcal{S}_2(y_1, y_2)(t_2)] \geq \gamma \|\mathcal{S}(y_1, y_2)\|. \tag{3.8}$$

Therefore, from (3.8), whenever we get $(y_1, y_2) \in \mathcal{K}$, it follows that $\mathcal{S}(y_1, y_2) \in \mathcal{K}$. This completes the proof. \square

We next recall the Guo-Krasnosel'kii fixed-point theorem and Schauder fixed-point theorem, which we will use to prove the main results.

Lemma 3.2. ([21]) *Let \mathcal{B} be a Banach space, and let $\mathcal{K} \subseteq \mathcal{B}$ be a cone. Assume that Ω_1 and Ω_2 are open subsets contained in \mathcal{B} such that $0 \in \Omega_1$ and $\overline{\Omega_1} \subseteq \Omega_2$. Assume, further, that $T : \mathcal{K} \cap (\overline{\Omega_2} \setminus \Omega_1) \rightarrow \mathcal{K}$ is a completely continuous operator. If either*

- (1) $\|Ty\| \leq \|y\|$ for $y \in \mathcal{K} \cap \partial\Omega_1$ and $\|Ty\| \geq \|y\|$ for $y \in \mathcal{K} \cap \partial\Omega_2$,
- (2) or $\|Ty\| \geq \|y\|$ for $y \in \mathcal{K} \cap \partial\Omega_1$ and $\|Ty\| \leq \|y\|$ for $y \in \mathcal{K} \cap \partial\Omega_2$,

then T has at least one fixed point in $\mathcal{K} \cap (\overline{\Omega_2} \setminus \Omega_1)$.

Lemma 3.3. ([22]) (Schauder fixed-point theorem) *Suppose that \mathcal{X} is a Banach space. Let \mathcal{K} be a bounded closed-convex set of \mathcal{X} , and let $T : \mathcal{K} \rightarrow \mathcal{K}$ be a completely continuous operator, then T has at least one fixed point in \mathcal{K} .*

Theorem 3.4. *Suppose that conditions (F1)-(F2) hold, then problem (1.4) has at least one solution.*

Proof. From Lemma 3.1, we know that $\mathcal{S} : \mathcal{K} \rightarrow \mathcal{K}$. Note that \mathcal{S} is a summation operator on a discrete finite set. Hence, \mathcal{S} is a completely continuous operator.

By conditions (F1) and (F2), for ε given in (F2), there exists some constant $r_1 > 0$, such that

$$f(y_1, y_2) \leq (f^* + \varepsilon)(y_1 + y_2), \tag{3.9}$$

$$g(y_1, y_2) \leq (g^* + \varepsilon)(y_1 + y_2), \tag{3.10}$$

whenever $\|(y_1, y_2)\| < r_1$.

Setting

$$\Omega_1 := \{(y_1, y_2) \in \mathcal{K} : \|(y_1, y_2)\| < r_1\}, \quad (3.11)$$

Then, for $(y_1, y_2) \in \mathcal{K} \cap \partial\Omega_1$, by Lemma 2.7, (F2) and (3.9), we find that

$$\begin{aligned} \|\mathcal{S}_1(y_1, y_2)\| &= \max_{t_1 \in [\nu-1, \nu+b]_{\mathbb{N}_{\nu-1}}} \sum_{s=0}^{b+1} G_1(t_1, s) f(y_1(s + \nu - 1), y_2(s + \mu - 1)) \\ &\leq \sum_{s=0}^{b+1} G_1(s + \nu - 1, s) (f^* + \varepsilon) (y_1 + y_2) \\ &\leq \|(y_1, y_2)\| \sum_{s=0}^{b+1} G_1(s + \nu - 1, s) (f^* + \varepsilon) \\ &\leq \frac{1}{2} r_1 = \frac{1}{2} \|(y_1, y_2)\|. \end{aligned} \quad (3.12)$$

Similar, by (3.10), it can be shown that

$$\|\mathcal{S}_2(y_1, y_2)\| \leq \frac{1}{2} \|(y_1, y_2)\|. \quad (3.13)$$

Thus, putting (3.12) and (3.13) together, for $(y_1, y_2) \in \mathcal{K} \cap \partial\Omega_1$, we have

$$\|\mathcal{S}(y_1, y_2)\| \leq \frac{1}{2} \|(y_1, y_2)\| + \frac{1}{2} \|(y_1, y_2)\| = \|(y_1, y_2)\|. \quad (3.14)$$

Now, for the above ε , by (F2), we can find a constant $r^{**} > 0$ such that

$$f(y_1, y_2) \geq (f^{**} - \varepsilon)(y_1 + y_2), \quad (3.15)$$

$$g(y_1, y_2) \geq (g^{**} - \varepsilon)(y_1 + y_2), \quad (3.16)$$

whenever $\|y_1 + y_2\| \geq r^{**}$. Let

$$\frac{1}{2} r^{**} \leq r_2 \leq \min \left\{ \sum_{s=0}^{b+1} G_1(s + \nu - 1, s) (f^{**} - \varepsilon) \gamma r^{**}, \sum_{s=0}^{b+1} G_2(s + \mu - 1, s) (g^{**} - \varepsilon) \gamma r^{**} \right\}. \quad (3.17)$$

Moreover, put

$$\Omega_2 := \{(y_1, y_2) \in \mathcal{K} : \|(y_1, y_2)\| < 2r_2\}. \quad (3.18)$$

If $(y_1, y_2) \in \mathcal{K} \cap \partial\Omega_2$, then

$$y_1(t_1) + y_2(t_2) \geq \min_{(t_1, t_2) \in [(\nu-1)/4, 3(\nu+b)/4] \times [(\mu-1)/4, 3(\mu+b)/4]} [y_1(t_1) + y_2(t_2)] \geq \gamma \|(y_1, y_2)\|. \quad (3.19)$$

Thus, from (3.15) and (3.19), we obtain

$$\begin{aligned} \|S_1(y_1, y_2)\| &= \max_{t_1 \in [\nu-1, \nu+b]_{\mathbb{N}_{\nu-1}}} \sum_{s=0}^{b+1} G_1(t_1, s) f(y_1(s + \nu - 1), y_2(s + \mu - 1)) \\ &\geq \sum_{s=0}^{b+1} G_1(s + \nu - 1, s) (f^{**} - \varepsilon)(y_1 + y_2) \\ &\geq \sum_{s=0}^{b+1} G_1(s + \nu - 1, s) (f^{**} - \varepsilon) r \| (y_1, y_2) \| \\ &\geq r_2 = \frac{1}{2} \| (y_1, y_2) \|. \end{aligned} \quad (3.20)$$

Similarly, by (3.16) and (3.19), we have

$$\|S_2(y_1, y_2)\| \geq \frac{1}{2} \| (y_1, y_2) \|. \quad (3.21)$$

So, from (3.20) and (3.21), we get

$$\|S(y_1, y_2)\| \geq \| (y_1, y_2) \|, \quad (3.22)$$

whenever $(y_1, y_2) \in \mathcal{K} \cap \partial\Omega_2$.

Thus, by (3.14) and (3.22), we get that all of the hypotheses of Lemma 3.2 are satisfied. Consequently, we conclude that \mathcal{S} has a fixed point, say $(y_1^*, y_2^*) \in \mathcal{K}$. Then the theorem is proved. \square

Theorem 3.5. Let $f, g : \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}$ be continuous functions. Suppose that the following conditions are satisfied:

(i) there exist nonnegative functions $a, h, p, q \in C(\mathbb{R}, \mathbb{R}^+)$ such that

$$|f(y_1, y_2)| \leq a(y_1) + p(y_2), \quad |g(y_1, y_2)| \leq h(y_1) + q(y_2), \quad (3.23)$$

(ii) Consider the following:

$$\lim_{|y| \rightarrow +\infty} \frac{p(y)}{|y|} < A, \quad \lim_{|y| \rightarrow +\infty} \frac{q(y)}{|y|} < B, \quad (3.24)$$

where $A = (2 \max_{t \in [\nu-1, \nu+b+1]} \sum_{s=0}^{b+1} G_1(t, s))^{-1}$, $B = (2 \max_{t \in [\mu-1, \mu+b+1]} \sum_{s=0}^{b+1} G_2(t, s))^{-1}$, then boundary value problem (1.4) has at least one solution (y_1^{**}, y_2^{**}) .

Proof. Let $\varepsilon_1 = (1/2)(A - \lim_{|y| \rightarrow +\infty} p(y)/|y|)$. By (ii), there exists a $c_1 > 0$, such that $p(y_2) < (A - \varepsilon_1)|y_2|$, for $|y_2| \geq c_1$. Set $M = \max\{p(y_2) : |y_2| \leq c_1\}$, then there exists a $c_2 > c_1$, such that $M/c_2 \leq A - \varepsilon_1$, so we have $p(y_2) < (A - \varepsilon_1)c_2$, for $|y_2| \leq c_2$.

Let $\varepsilon_2 = (1/2)(B - \lim_{|y| \rightarrow +\infty} q(y)/|y|)$. In the same way, there exists a constant $c_3 > 0$, such that $q(y_2) < (B - \varepsilon_2)c_3$, $|y_2| \leq c_3$. Define

$$e_1 = \max_{t_1 \in [\nu-1, \nu+b+1]} \sum_{s=0}^{b+1} G_1(t_1, s) a(y_1), \quad e_2 = \max_{t_2 \in [\mu-1, \mu+b+1]} \sum_{s=0}^{b+1} G_2(t_2, s) h(y_1),$$

$$c = \max \left\{ c_2, c_3, \frac{2A}{\varepsilon_1} e_1, \frac{2B}{\varepsilon_2} e_2 \right\}, \tag{3.25}$$

$$U = \left\{ (y_1, y_2) \mid (y_1, y_2) \in \mathcal{X}, \|(y_1, y_2)\| \leq c, (t_1, t_2) \in [\nu-1, \nu+b+1]_{\mathbb{N}_{\nu-1}} \times [\mu-1, \mu+b+1]_{\mathbb{N}_{\mu-1}} \right\}.$$

Then U is a bounded closed-convex set of $\mathcal{X} \times \mathcal{X}$, and for any $(y_1, y_2) \in U$, we have $p(y_2) < (A - \varepsilon_1)c$, $q(y_2) < (B - \varepsilon_2)c$. Next, we prove $\mathcal{S} : U \rightarrow U$, where \mathcal{S} is defined as (2.14).

For $(y_1, y_2) \in U$, from (i) and (sii), we obtain

$$\begin{aligned} \mathcal{S}_1(y_1, y_2)(t_1) &= \sum_{s=0}^{b+1} G_1(t_1, s) f(y_1, y_2) \\ &\leq \sum_{s=0}^{b+1} G_1(t_1, s) a(y_1) + \sum_{s=0}^{b+1} G_1(t_1, s) p(y_2) \\ &\leq e_1 + (A - \varepsilon_1)c \sum_{s=0}^{b+1} G_1(t_1, s) \\ &\leq \frac{\varepsilon_1}{2A}c + \frac{1}{2} \left(1 - \frac{\varepsilon_1}{A}\right)c = \frac{1}{2}c, \end{aligned} \tag{3.26}$$

similarly,

$$\mathcal{S}_2(y_1, y_2)(t_2) \leq \frac{\varepsilon_2}{2B}c + \frac{1}{2} \left(1 - \frac{\varepsilon_2}{B}\right)c = \frac{1}{2}c. \tag{3.27}$$

Thus, from (3.26) and (3.27), we have $\|\mathcal{S}(y_1, y_2)\| \leq c$. Then, $\mathcal{S} : U \rightarrow U$.

Note that \mathcal{S} is a summation operator on a discrete finite set. Hence, \mathcal{S} is trivially completely continuous. By Schauder fixed-point theorem, the boundary problem value problem (1.4) has at least one solution, say (y_1^*, y_2^*) . This completes the proof. \square

4. Examples

In this section, we will present some examples to illustrate the main results.

Example 4.1. Suppose that $\nu = 3/2$, $\mu = 5/4$, $\eta_1 = 1/1000$, $\eta_2 = 10/\gamma$, $\gamma = \min\{\gamma_1, \gamma_2\}$, and γ_1, γ_2 are defined as in Lemma 2.7, $b = 2$. Let $f(y_1, y_2) := ((1/1000e^{-y_2}) + (1000/\gamma e^{1/y_1}))(y_1 + y_2)$ and $g(y) := ((1/2000e^{-y_1}) + (1000/\gamma e^{1/y_2}))(y_1 + y_2)$, then (1.4) becomes

$$\begin{aligned} -\Delta^{3/2}y_1(t) &= f\left(y_1\left(t + \frac{1}{2}\right), y_2\left(t + \frac{1}{4}\right)\right), \text{ for } t \in [0, 3]_{\mathbb{N}_0}, \\ -\Delta^{5/4}y_2(t) &= g\left(y_1\left(t + \frac{1}{2}\right), y_2\left(t + \frac{1}{4}\right)\right), \text{ for } t \in [0, 3]_{\mathbb{N}_0}, \\ y_1\left(-\frac{1}{2}\right) &= \Delta y_1\left(\frac{7}{2}\right) = 0, \\ y_2\left(-\frac{3}{4}\right) &= \Delta y_2\left(\frac{13}{4}\right) = 0. \end{aligned} \tag{4.1}$$

In this case,

$$\begin{aligned} \lim_{(y_1, y_2) \rightarrow (0^+, 0^+)} \frac{f(y_1, y_2)}{y_1 + y_2} &= \frac{1}{1000} = f^*, & \lim_{(y_1, y_2) \rightarrow (0^+, 0^+)} \frac{g(y_1, y_2)}{y_1 + y_2} &= \frac{1}{2000} = g^*, \\ \lim_{(y_1, y_2) \rightarrow (+\infty, +\infty)} \frac{f(y_1, y_2)}{y_1 + y_2} &= \frac{1000}{\gamma} = f^{**}, & \lim_{(y_1, y_2) \rightarrow (+\infty, +\infty)} \frac{g(y_1, y_2)}{y_1 + y_2} &= \frac{1000}{\gamma} = g^{**}, \end{aligned} \tag{4.2}$$

$$\begin{aligned} \sum_{s=0}^{b+1} G_1(s + \nu - 1, s) &= \sum_{s=0}^{b+1} \frac{\Gamma(b+3)(s + \nu - 1)^{\nu-1}}{\Gamma(\nu + b + 1)} (\nu + b - s - 1)^{\nu-2} \\ &= \sum_{s=0}^{b+1} \frac{\Gamma(b+3)\Gamma(s + \nu)\Gamma(\nu + b - s)}{\Gamma(\nu + b + 1)\Gamma(s + 1)\Gamma(b - s + 2)} \\ &= \sum_{s=0}^{b+1} \frac{(b+2)(b+1) \cdots (b-s+2)(s + \nu - 1) \cdots (s+1)}{(\nu + b)(\nu + b - 1) \cdots (\nu + b - s)} \\ &\leq \sum_{s=0}^{b+1} (s + \nu - 1) \cdots (s+1) \\ &\leq (b+2)(b + \nu)(b+2) = 4 \times \frac{7}{2} \times 4 = 56. \end{aligned} \tag{4.3}$$

Similarly, we get

$$\sum_{s=0}^{b+1} G_2(s + \mu - 1, s) \leq (b+2)(b + \mu)(b+2) = 52. \tag{4.4}$$

Thus,

$$\begin{aligned} \sum_{s=0}^{b+1} G_1(s + \nu - 1, s)(f^* + \eta_1) &\leq 56 \times \left(\frac{1}{1000} + \frac{1}{1000} \right) \leq \frac{1}{2}, \\ \sum_{s=0}^{b+1} G_2(s + \nu - 1, s)(g^* + \eta_1) &\leq 52 \times \left(\frac{1}{2000} + \frac{1}{1000} \right) \leq \frac{1}{2}. \end{aligned} \tag{4.5}$$

On the other hand,

$$\begin{aligned} \sum_{s=0}^{b+1} G_1(s + \nu - 1, s) &= \sum_{s=0}^{b+1} \frac{(b+2)(b+1) \cdots (b-s+2)(s+\nu-1) \cdots (s+1)}{(\nu+b)(\nu+b-1) \cdots (\nu+b-s)} \\ &\geq \sum_{s=0}^{b+1} (b+2)(b+1) \cdots (b-s+2)(s+\nu-1) \cdots (s+1) \\ &\geq (b+2)(b+2)(\nu-1) = 2 \times 2 \times \frac{1}{2} = 2, \end{aligned} \tag{4.6}$$

similarly,

$$\sum_{s=0}^{b+1} G_2(s + \mu - 1, s) \geq (b+2)(b+2)(\mu-1) \geq 4 \times 4 \times \frac{1}{4} = 1. \tag{4.7}$$

Then,

$$\begin{aligned} \sum_{s=0}^{b+1} \gamma G_1(s + \nu - 1, s)(f^{**} - \eta_2) &\geq 2\gamma \times \left(\frac{1000}{\gamma} - \frac{10}{\gamma} \right) = 1980 \geq \frac{1}{2}, \\ \sum_{s=0}^{b+1} \gamma G_2(s + \mu - 1, s)(g^{**} - \eta_2) &\geq \gamma \times \left(\frac{1000}{\gamma} - \frac{10}{\gamma} \right) = 990 \geq \frac{1}{2}. \end{aligned} \tag{4.8}$$

Thus, the conditions of Theorem 3.4 are satisfied. So, we obtain that the problem (4.1) has at least one solution.

Example 4.2. Suppose that $\nu = 5/3$, $\mu = 7/4$, and $b = 3$. Let $f(y_1, y_2) := y_1 + (A/4)y_2e^{-|y_2|}$, $g(y_1, y_2) := y_1 + (B/5)y_2e^{-|y_2|}$, $a(y_1) = |y_1|$, $h(y_1) = 2|y_1|$, $p(y_2) = (A/3)|y_2|(e^{-|y_2|} + 1)$, and $q(y_2) = (B/4)|y_2|(e^{-|y_2|} + 1)$, $y_1, y_2 \in \mathbb{R}^+$, where

$$A = \left(2 \max_{t \in [\nu-1, \nu+b+1]} \sum_{s=0}^{b+1} G_1(t, s) \right)^{-1}, \quad B = \left(2 \max_{t \in [\mu-1, \mu+b+1]} \sum_{s=0}^{b+1} G_2(t, s) \right)^{-1}. \tag{4.9}$$

Then problem (1.4) is

$$\begin{aligned}
 -\Delta^{5/3}y_1(t) &= y_1\left(t + \frac{2}{3}\right) + \frac{A}{4}y_2\left(t + \frac{3}{4}\right)e^{-|y_2(t+(3/4))|}, \quad \text{for } t \in [0, 4]_{\mathbb{N}_0}, \\
 -\Delta^{7/4}y_1(t) &= y_1\left(t + \frac{2}{3}\right) + \frac{B}{5}y_2\left(t + \frac{3}{4}\right)e^{-|y_2(t+(3/4))|}, \quad \text{for } t \in [0, 4]_{\mathbb{N}_0}, \\
 y_1\left(-\frac{1}{3}\right) &= \Delta y_1\left(\frac{14}{3}\right) = 0, \\
 y_2\left(-\frac{1}{4}\right) &= \Delta y_2\left(\frac{19}{4}\right) = 0.
 \end{aligned} \tag{4.10}$$

It is clear that $f, g, a, h, p,$ and q satisfy the conditions of Theorem 3.5. Thus, by Theorem 3.5, we deduce that problem (4.10) has at least one solution.

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