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Research Article

On Fuzzy Corsini's Hyperoperations

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We generalize the concept of C-hyperoperation and introduce the concept of F-C-hyperoperation. We list some basic properties of F-C-hyperoperation and the relationship between the concept of C-hyperoperation and the concept of F-C-hyperoperation. We also research F-C-hyperoperations associated with special fuzzy relations.

1. Introduction and Preliminaries

Hyperstructures and binary relations have been studied by many researchers, for instance, Chvalina [1, 2], Corsini and Leoreanu [3], Feng [4], Hort [5], Rosenberg [6], Spartalis [7], and so on.

A partial hypergroupoid $\langle H, * \rangle$ is a nonempty set H with a function from $H \times H$ to the set of subsets of H.

A hypergroupoid is a nonempty set H, endowed with a hyperoperation, that is, a function from $H \times H$ to $P(H)$, the set of nonempty subsets of H.

If $A, B \in \mathbf{P}(H) - \{\emptyset\}$, then we define $A * B = \cup\{a * b \mid a \in A, b \in B\}$, $x * B = \{x\} * B$ and $A * y = A * \{y\}$.

A Corsini's hyperoperation was first introduced by Corsini [8] and studied by many researchers; for example, see [3, 8–15].

Definition 1.1 (see [8]). Let $\langle H, R \rangle$ be a a pair of sets where H is a nonempty set and R is a binary relation on H. Corsini's hyperoperation (briefly, *C-hyperoperation*) $*_R$ associated with

R is defined in the following way:

$$*_R : H \times H \longrightarrow P(H) : x *_R y = \{z \in H \mid xRz, zRy\}, \tag{1.1}$$

where $P(H)$ denotes the family of all the subsets of H.

A fuzzy subset A of a nonempty set H is a function $A : H \to [0,1]$. The family of all the fuzzy subsets of H is denoted by $F(H)$.

We use $\emptyset$ to denote a special fuzzy subset of H which is defined by $\emptyset(x) = 0$, for all $x \in H$.

For a fuzzy subset A of a nonempty set H, the *p-cut* of A is denoted A_p, for any $p \in (0,1]$, and defined by $A_p \doteq \{x \in H \mid A(x) \geq p\}$.

A fuzzy binary relation R on a nonempty set H is a function $R : H \times H \to [0,1]$. In the following, sometimes we use fuzzy relation to refer to fuzzy binary relation.

For any $a, b \in [0,1]$, we use $a \wedge b$ to stand for the minimum of a and b and $a \vee b$ to denote the maximum of a and b.

Given $A, B \in F(H)$, we will use the following definitions:

$$\begin{gathered} A \subseteq B \doteq A(x) \leq B(x), \quad \forall x \in H, \\ A = B \doteq A(x) = B(x), \quad \forall x \in H, \\ (A \cup B)(x) \doteq A(x) \vee B(x), \quad \forall x \in H, \\ (A \cap B)(x) \doteq A(x) \wedge B(x), \quad \forall x \in H. \end{gathered} \tag{1.2}$$

A partial fuzzy hypergroupoid $\langle H, * \rangle$ is a nonempty set endowed with a fuzzy hyperoperation $* : H \times H \to F(H)$. Moreover, $\langle H, * \rangle$ is called a fuzzy hypergroupoid if for all $x, y \in H$, there exists at least one $z \in H$, such that $(x * y)(z) \neq 0$ holds.

Given a fuzzy hyperoperation $* : H \times H \to F(H)$, for all $a \in H$, $B \in F(H)$, the fuzzy subset $a * B$ of H is defined by

$$(a * B)(x) \doteq \vee_{B(b)>0} (a * b)(x). \tag{1.3}$$

$B * a$, $A * B$ can be defined similarly. When B is a *crisp* subset of H, we treat B as a fuzzy subset by treating it as $B(x) = 1$, for all $x \in B$ and $B(x) = 0$, for all $x \in H - B$.

2. Fuzzy Corsini's Hyperoperation

In this section, we will generalize the concept of Corsini's hyperoperation and introduce the fuzzy version of Corsini's hyperoperation.

Definition 2.1. Let $\langle H, R \rangle$ be a pair of sets where H is a non-empty set and R is a fuzzy relation on H. We define a fuzzy hyperoperation $*_R : H \times H \to F(H)$, for any $x, y, z \in H$, as follows:

$$(x *_R y)(z) \doteq R(x,z) \wedge R(z,y). \tag{2.1}$$

Table 1

R	a	b
a	0.1	0.2
b	0.3	0.4

Table 2

$*_R$	a	b
a	$0.1/a+0.2/b$	$0.1/a+0.2/b$
b	$0.1/a+0.3/b$	$0.2/a+0.4/b$

$*_R$ is called a *fuzzy Corsini's hyperoperation* (briefly, *F-C-hyperoperation*) associated with R. The fuzzy hyperstructure $\langle H, *_R\rangle$ is called a partial F-C-hypergroupoid.

Remark 2.2. It is obvious that the concept of F-C-hyperoperation is a generalization of the concept of C-hyperoperation.

Example 2.3. Letting $H = \{a, b\}$ be a non-empty set, R is a fuzzy relation on H as described in Table 1.

From the previous definition, by calculating, for example, $(a *_R a)(a) = R(a,a) \wedge R(a,a) = 0.1 \wedge 0.1 = 0.1$, $R(a * b)(a) = R(a,a) \wedge R(a,b) = 0.1 \wedge 0.2 = 0.1$, we can obtain Table 2 which is a partial F-C-hypergroupoid.

Definition 2.4. Supposing R, S are two fuzzy relations on a non-empty set H, the composition of R and S is a fuzzy relation on H and is defined by $(R \circ S)(x,y) \doteq \bigvee_{z\in H}(R(x,z) \wedge S(z,y))$, for all $x, y \in H$.

Proposition 2.5. *A partial F-C-hypergroupoid $\langle H, *_R\rangle$ is a F-C-hypergroupoid if and only if* $\operatorname{supp}(R \circ R) = H \times H$, *where* $\operatorname{supp}(R \circ R) = \{(x,y) \mid (R \circ R)(x,y) \neq 0\}$.

Proof. Suppose that $\langle H, *_R\rangle$ is a hypergroupoid. For any $x, y \in H$, there exists at least one $z \in H$, such that $(x *_R y)(z) \neq 0$ holds.

So $(R \circ R)(x,y) = \bigvee_{z\in H}(R(x,z) \wedge R(z,y)) \neq 0$. Thus $(x,y) \in \operatorname{supp}(R \circ R)$. And we conclude that $H \times \mathrm{H} \subseteq \operatorname{supp}(R \circ R)$.

$\operatorname{supp}(R \circ R) \subseteq H \times H$ is obvious. And so $\operatorname{supp}(R \circ R) = H \times H$.

Conversely, if $\operatorname{supp}(R\circ R) = H\times H$, then for any $x, y \in H$, $(x,y) \in H\times H = \operatorname{supp}(R\circ R)$. So $(R \circ R)(x,y) = \bigvee_{z\in H}(R(x,z) \wedge R(z,y)) \neq 0$. That is, there exists at least one $z \in H$ such that $(x *_R y)(z) \neq 0$ holds. And so $\langle H, *_R\rangle$ is a hypergroupoid.

Thus we complete the proof. □

Definition 2.6. Letting H be a non-empty set, $*$ is a fuzzy hyperoperation of H, the hyperoperation $*_p$ is defined by $x *_p y = (x * y)_p$, for all $x, y \in H$, $p \in [0,1]$. $*_p$ is called the p-cut of $*$.

Definition 2.7. Letting R be a fuzzy relation on a non-empty set H, we define a binary relation R_p on H, for all $p \in (0,1]$, as follows:

$$xR_p y \doteq R(x,y) \geq p. \tag{2.2}$$

R_p is called the p-cut of the fuzzy relation R.

Proposition 2.8. *Let $\langle H, *_R \rangle$ be a partial F-C-hypergroupoid. Then $(*_R)_p$ is a C-hyperoperation associated with R_p, for all $0 < p \leq 1$.*

Proof. For any $0 < p \leq 1$ and for any $x, y \in H$, we have

$$\begin{aligned} x(*_R)_p y &= (x *_R y)_p = \{z \in H \mid (x *_R y)(z) \geq p\} = \{z \in H \mid R(x,z) \wedge R(z,y) \geq p\} \\ &= \{z \in H \mid R(x,z) \geq p, R(z,y) \geq p\} = \{z \in H \mid xR_p z, zR_p y\}. \end{aligned} \tag{2.3}$$

From the definition of C-hyperoperation, we conclude that $(*_R)_p$ is a C-hyperoperation associated with R_p.

Thus we complete the proof. □

From the previous proposition and the construction of the F-C-hyperoperation, we can easily conclude that a fuzzy hyperoperation is a F-C-hyperoperation if and only if every p-cut of the F-C-hyperoperation is a C-hyperoperation. That is, consider the following.

Proposition 2.9. *Let H be a non-empty set and let $*$ be a fuzzy hyperoperation of H, then the fuzzy hyperoperation $*$ is an F-C-hyperoperation associated with a fuzzy relation R on H if and only if $*_p$ is a C-hyperoperation associated with R_p, for any $0 < p \leq 1$.*

3. Basic Properties of F-C-Hyperoperations

In this section, we list some basic properties of F-C-hyperoperations.

Proposition 3.1. *Let $\langle H, *_R \rangle$ be a partial or nonpartial F-C-hypergroupoid defined on $H \neq \emptyset$. Then, for all $x, y, a, b \in H$, we have*

$$x *_R y \cap a *_R b = x *_R b \cap a *_R y. \tag{3.1}$$

Proof. For any $x, y, a, b, z \in H$, we have that $(x *_R y \cap a *_R b)(z) = (x *_R y)(z) \wedge (a *_R b)(z) = R(x,z) \wedge R(z,y) \wedge R(a,z) \wedge R(z,b) = R(x,z) \wedge R(z,b) \wedge R(a,z) \wedge R(z,y) = (x *_R b \cap a *_R y)(z)$.

So

$$x *_R y \cap a *_R b = x *_R b \cap a *_R y, \tag{3.2}$$

for all $x, y, a, b \in H$. □

Proposition 3.2. *Let $\langle H, *_R \rangle$ be a partial F-C-hypergroupoid and $x, y \in H$, $x *_R y = \emptyset$. Then,*

(1) $x *_R H \cap H *_R y = \emptyset$;

(2) *If* $H = x *_R H$ *then* $H *_R y = \emptyset$;

(3) *If* $H = H *_R x$ *then* $y *_R H = \emptyset$.

Proof. (1) Supposing $x *_R H \cap H *_R y \neq \emptyset$, then there exist $a, b \in H$, such that $x *_R a \cap b *_R y \neq \emptyset$. So from the previous proposition, we have $x *_R y \cap b *_R a \neq \emptyset$. This is a contradiction.

(2) From $H = x *_R H$ and $x *_R H \cap H *_R y = \emptyset$, we have that $H \cap H *_R y = \emptyset$, and so, $H *_R y = \emptyset$.

(3) is proved similar to (2). □

Proposition 3.3. *Letting $*_R$ be the F-C-hyperoperation defined on the non-empty set H, $p \in (0, 1]$, then the following are equivalent:*

(1) *for some* $a \in H$, $(a *_R a)_p = H$;

(2) *for all* $x, y \in H$, $a \in (x *_R y)_p$.

Proof. Let $(a *_R a)_p = H$. Then, for all $x, y \in H$, we have that $(a *_R a)(x) \geq p, (a *_R a)(y) \geq p$, that is $R(a, x) \geq p, R(x, a) \geq p, R(a, y) \geq p, R(y, a) \geq p$ and so $R(x, a) \wedge R(a, y) \geq p$. Thus $a \in (x *_R y)_p$, for all $x, y \in H$.

Conversely, let $a \in (x *_R y)_p$, for all $x, y \in H$. Specially, we have $a \in (a *_R x)_p$ and $a \in (x *_R a)_p$. Thus, $R(a, x) \geq p$ and $R(x, a) \geq p$. And so $x \in (a *_R a)_p$. □

Proposition 3.4. *Let $\langle H, *_R \rangle$ be a partial or nonpartial F-C-hypergroupoid defined on $H \neq \emptyset$. Then, for all $a, b \in H$, $p \in (0, 1]$, we have*

$$a \in (b *_R b)_p \Longleftrightarrow b \in (a *_R a)_p. \tag{3.3}$$

Proof. For any $a, b \in H$, we have that

$$\begin{aligned} a \in (b *_R b)_p &\Longrightarrow (b *_R b)(a) \geq p \Longrightarrow R(b, a) \wedge R(a, b) \geq p \\ &\Longrightarrow R(a, b) \wedge R(b, a) \geq p \Longrightarrow (a *_R a)(b) \geq p \Longrightarrow b \in (a *_R a)_p. \end{aligned} \tag{3.4}$$

The remaining part can be proved similarly. □

4. F-C-Hyperoperations Associated with p-Fuzzy Reflexive Relations

In this section, we will assume that R is a p-fuzzy reflexive relation on a non-empty set.

Definition 4.1. A fuzzy relation R on a non-empty set H is called *p-fuzzy reflexive* if for any $x \in H$,

$$R(x, x) \geq p. \tag{4.1}$$

Example 4.2. The fuzzy relation R introduced in Example 2.3 is 0.1-fuzzy reflexive. Of course, it is p-fuzzy reflexive, where $0 \leq p \leq 0.1$.

Proposition 4.3. *Letting* $\langle H, *_R\rangle$ *be a partial F-C-hypergroupoid defined on* $H \neq \emptyset$, *R is p-fuzzy reflexive. Then, for all* $a, b \in H$, $p \in (0,1]$, *the following are equivalent:*

(1) $R(a,b) \geq p$;

(2) $a \in (a *_R b)_p$;

(3) $b \in (a *_R b)_p$.

Proof. "(1)⇒(2)"

From $R(a,a) \geq p$ and $R(a,b) \geq p$ we have that $R(a,a) \wedge R(a,b) \geq p$ which shows that $a \in (a *_R b)_p$.

"(2)⇒(3)"

From $a \in (a *_R b)_p$ we have that $R(a,b) \geq p$. Since $R(b,b) \geq p$, so $R(a,b) \wedge R(b,b) \geq p$ which implies that $b \in (a *_R b)_p$.

"(3)⇒(1)"

It is obvious. □

Proposition 4.4. *Letting* $\langle H, *_R\rangle$ *be a partial F-C-hypergroupoid defined on* $H \neq \emptyset$, *R is p-fuzzy reflexive. Then, for any* $a \in H$, *we have that*

$$a \in (a *_R a)_p. \tag{4.2}$$

Proof. From $R(a,a) \geq p$ we have $R(a,a) \wedge R(a,a) \geq p$. That is $a \in (a *_R a)_p$. □

Proposition 4.5. *Letting* $\langle H, *_R\rangle$ *be a partial F-C-hypergroupoid defined on* $H \neq \emptyset$, *R is p-fuzzy reflexive. Then, for any* $a, b \in H$, $p \in (0,1]$, *we have that*

$$b \in (a *_R a)_p \Longleftrightarrow a \in (a *_R b \cap b *_R a)_p. \tag{4.3}$$

Proof. From $b \in (a *_R a)_p$ we have that $R(a,b) \wedge R(b,a) \geq p$. So $R(a,b) \geq p$ and $R(b,a) \geq p$. Thus $R(a,a) \wedge R(a,b) \geq p$ and $R(b,a) \wedge R(a,a) \geq p$. That is $(a *_R b)(a) \geq p$ and $(b *_R a)(a) \geq p$. So $(a *_R b \cap b *_R a)(a) \geq p$. Thus $a \in (a *_R b \cap b *_R a)_p$.

Conversely, suppose that $a \in (a *_R b \cap b *_R a)_p$. Then $(a *_R b)(a) \wedge (b *_R a)(a) \geq p$. Thus $R(a,a) \wedge R(a,b) \wedge R(b,a) \wedge R(a,a) \geq p$. So $R(a,b) \wedge R(b,a) \geq p$. That is $b \in (a *_R a)_p$. □

Corollary 4.6. *Letting* $\langle H, *_R\rangle$ *be a partial F-C-hypergroupoid defined on* $H \neq \emptyset$, *R is p-fuzzy reflexive. Then, for any* $a, b \in H$, $p \in (0,1]$, *we have that*

$$b \in (a *_R a)_p \Longleftrightarrow a \in (b *_R b)_p \Longleftrightarrow a \in (a *_R b \cap b *_R a)_p \Longleftrightarrow b \in (a *_R b \cap b *_R a)_p. \tag{4.4}$$

Proposition 4.7. *Letting* $\langle H, *_R\rangle$ *be a partial F-C-hypergroupoid defined on* $H \neq \emptyset$, *R is p-fuzzy reflexive. Then, for any* $a, b \in H$, *we have that*

$$c \in (a *_R b)_p \Longleftrightarrow c \in (a *_R c \cap c *_R b)_p. \tag{4.5}$$

Proof. If $c \in (a *_R b)_p$, then $R(a,c) \geq p$ and $R(c,b) \geq p$. Thus $c \in (a *_R c)_p$ and $c \in (c *_R b)_p$. So $c \in (a *_R c \cap c *_R b)_p$.

Conversely, if $c \in (a *_R c \cap c *_R b)_p$, then $(a *_R c)(c) \wedge (c *_R b)(c) \geq p$. Thus $R(a,c) \wedge R(c,c) \wedge R(c,c) \wedge R(c,b) \geq p$. And so $R(a,c) \wedge R(c,b) \geq p$. Thus $c \in (a *_R b)_p$. □

Proposition 4.8. *Letting $\langle H, *_R \rangle$ be a partial F-C-hypergroupoid defined on $H \neq \emptyset$, R is p-fuzzy reflexive. Then, for any $a, b, c \in H$, $p \in (0,1]$, the following are equivalent:*

(1) $c \in (a *_R b)_p$;

(2) $a \in (a *_R c)_p$ *and* $b \in (c *_R b)_p$;

(3) $a \in (a *_R c)_p$ *and* $c \in (c *_R b)_p$.

Proof. "(1)⇒(2)"

Suppose that $c \in (a *_R b)_p$. Then $R(a,c) \geq p$ and $R(c,b) \geq p$. So $R(a,a) \wedge R(a,c) \geq p$ and $R(c,b) \wedge R(b,b) \geq p$. Thus $a \in (a *_R c)_p$ and $b \in (c *_R b)_p$.

"(2)⇒(3)"

Suppose that $b \in (c *_R b)_p$. Then $R(c,b) \geq p$. Thus $R(c,c) \wedge R(c,b) \geq p$. And so $c \in (c *_R b)_p$.

"(3)⇒(1)"

From $a \in (a *_R c)_p$ and $c \in (c *_R b)_p$, we have that $R(a,c) \geq p$ and $R(c,b) \geq p$. Thus $R(a,c) \wedge R(c,b) \geq p$. So $c \in (a *_R b)_p$. □

5. F-C-Hyperoperations Associated with p-Fuzzy Symmetric Relations

In this section, we will assume that R is a p-fuzzy symmetric relation on a non-empty set.

Definition 5.1. A fuzzy binary relation R on a non-empty set H is called *p-fuzzy symmetric* if for any $x, y \in H$,

$$R(x,y) \geq p \Longrightarrow R(y,x) \geq p. \tag{5.1}$$

Example 5.2. The fuzzy relation R introduced in Example 2.3 is 0.2-fuzzy symmetric. Of course, it is p-fuzzy reflexive, where $0 \leq p \leq 0.2$.

Proposition 5.3. *Letting $\langle H, *_R \rangle$ be a partial F-C-hypergroupoid defined on $H \neq \emptyset$, R is p-fuzzy symmetric relation. Then, for all $a, b \in H$, we have that*

$$(a *_R b)_p = (b *_R a)_p. \tag{5.2}$$

Proof. For all $a, b \in H$, two cases are possible.

(1) If $(a *_R b)_p = \emptyset$, then $(a *_R b)_p \subseteq (b *_R a)_p$.

(2) If $(a *_R b)_p \neq \emptyset$, let $x \in (a *_R b)_p$. Then $R(a,x) \geq p$ and $R(x,b) \geq p$.

Since R is p-fuzzy symmetric, so $R(x,a) \geq p$ and $R(b,x) \geq p$. Thus $(b *_R a)(x) = R(b,x) \wedge R(x,a) \geq p$. So $x \in (b *_R a)_p$. And in this case, we also have that $(a *_R b)_p \subseteq (b *_R a)_p$.

The remaining part can be proved by exchanging a and b. □

Proposition 5.4. *Let $\langle H, *_R \rangle$ be a partial F-C-hypergroupoid defined on $H \neq \emptyset$, $p \in (0,1]$, if*

(1) *for all $a, b \in H$, $(a *_R b)_p = (b *_R a)_p$,*

(2) *for any $x \in H$, there exists a $y \in H$, such that $R(x,y) \geq p$.*

Then R is a p-fuzzy symmetric binary relation on H.

Proof. For all $a, b \in H$, suppose that $R(a,b) \geq p$. We need to show that $R(b,a) \geq p$.

Since for $b \in H$, there exists a $x \in H$, such that $R(b,x) \geq p$. So $R(a,b) \wedge R(b,x) \geq p$. That is, $b \in (a *_R x)_p = (x *_R a)_p$. And so $R(x,b) \wedge R(b,a) \geq p$. And finally we have that $R(b,a) \geq p$. □

6. F-C-Hyperoperations Associated with p-Fuzzy Transitive Relations

In this section, we will assume that R is a p-fuzzy transitive relation on a non-empty set.

Definition 6.1. A fuzzy binary relation R on a non-empty set H is called p-fuzzy transitive if for any $x, y, z \in H$,

$$R(x,y) \geq p, R(y,z) \geq p \Longrightarrow R(x,z) \geq p. \tag{6.1}$$

Example 6.2. The fuzzy relation R introduced in Example 2.3 is 0.1-fuzzy transitive. Of course, it is p-fuzzy transitive, where $0 \leq p \leq 0.1$.

Proposition 6.3. *Letting $\langle H, *_R \rangle$ be a partial F-C-hypergroupoid defined on $H \neq \emptyset$, R is a p-fuzzy transitive relation on H, $p \in (0,1]$. Then for all $x, y \in H$, we have that*

$$R(x,y) \geq p \Longrightarrow (x *_R x \cup y *_R y)_p \subseteq (x *_R y)_p. \tag{6.2}$$

Proof. (1) If $(x *_R x)_p = \emptyset$, then obviously $(x *_R x)_p \subseteq (x *_R y)_p$.

Supposing that $(x *_R x)_p \neq \emptyset$, then for any $w \in (x *_R x)_p$, we have that $R(x,w) \wedge R(w,x) \geq p$, that is, $R(x,w) \geq p$ and $R(w,x) \geq p$. From $R(w,x) \geq p$ and $R(x,y) \geq p$ we have that $R(w,y) \geq p$. From $R(x,w) \geq p$ and $R(w,y) \geq p$ we conclude that $w \in (x *_R y)_p$.

So $(x *_R x)_p \subseteq (x *_R y)_p$.

(2) If $(y *_R y)_p = \emptyset$, then obviously $(y *_R y)_p \subseteq (x *_R y)_p$.

Supposing that $(y *_R y)_p \neq \emptyset$, then for any $w \in (y *_R y)_p$, we have that $R(y,w) \wedge R(w,y) \geq p$, that is, $R(y,w) \geq p$ and $R(w,y) \geq p$. From $R(y,w) \geq p$ and $R(x,y) \geq p$ we have that $R(x,w) \geq p$. From $R(x,w) \geq p$ and $R(w,y) \geq p$ we conclude that $w \in (x *_R y)_p$.

So $(y *_R y)_p \subseteq (x *_R y)_p$. □

Proposition 6.4. *Letting $\langle H, *_R \rangle$ be a partial F-C-hypergroupoid defined on $H \neq \emptyset$, R is a p-fuzzy transitive binary relation. For any $a, b, c \in H$, we have that*

(1) $((a *_R b)_p *_R c)_p \subseteq (a *_R c)_p$;

(2) $(a *_R (b *_R c)_p)_p \subseteq (a *_R c)_p$.

Proof. (1) If $((a *_R b)_p *_R c)_p = \emptyset$, then it is obvious that $((a *_R b)_p *_R c)_p \subseteq (a *_R c)_p$.

Suppose that $((a *_R b)_p *_R c)_p \neq \emptyset$. Then for any $w \in ((a *_R b)_p *_R c)_p$, there exists a $w_1 \in (a *_R b)_p$ such that $w \in (w_1 *_R c)_p$. That is $R(a, w_1) \geq p$, $R(w_1, b) \geq p$, $R(w_1, w) \geq p$ and $R(w, c) \geq p$. From $R(a, w_1) \geq p$ and $R(w_1, w) \geq p$, we have that $R(a, w) \geq p$. Thus $R(a, w) \wedge R(w, c) \geq p \wedge p = p$. That is, $w \in (a *_R c)_p$. So $((a *_R b)_p *_R c)_p \subseteq (a *_R c)_p$.

(2) Can be proved similarly. □

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