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Research Article

Impulsive Control for the Synchronization of Chaotic Systems with Time Delay

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This paper considers impulsive control for the synchronization of chaotic systems with time delays. Based on the Lyapunov functions and the Razumikhin technique, some new synchronization criteria with an exponential convergence rate are derived. Our results show that impulses do contribute to globally exponential synchronization of dynamical systems. Besides, the impulsive moments are independent of the upper bound of time delays. Furthermore, a bigger upper bound of impulsive intervals for the synchronization of chaotic systems can be obtained when compared with many previous studies. Hence, our results are less conservative and more effective for the synchronization analysis. A numerical example is given to show the validity and potential of the developed results.

1. Introduction

In the last two decades, control and synchronization problems of chaotic systems have been extensively studied, due to their potential applications in many areas. For instance, they are used to understand self-organizational behavior in the brain as well as in ecological systems, and they also have been applied to produce secure message communication between a sender and a receiver. So far, different synchronization techniques have been proposed and implemented in practice, such as PC (Pecora and Carroll) method [1], active control [2], adaptive control [3, 4], sliding mode control [5], and impulsive method [6–12].

Impulsive phenomena exist in many biological systems and mechanics fields. Impulsive control has been widely studied and gradually become an interesting and useful synchronization approach [13–16]. Impulsive control can provide an efficient method for some cases in which the systems cannot endure continuous disturbance. The main idea of the impulsive control strategy is to change the states of a system by some sudden jumps instantaneously. Furthermore, using the impulsive control method, the response system needs to receive the information from the drive system only at some discrete instants which means that impulsive control is easier to be implemented to some extent. The asymptotical synchronization of chaotic systems without time delay [17–20] and with time delay [21, 22] has been widely investigated by using impulsive control. However, most of the previous impulsive stability criteria are only valid for some specific systems with small delays due to the restrictive requirement that the time delays are required to be smaller than the length of impulsive interval [23–25].

In this paper, we will investigate globally exponential synchronization of coupled chaotic systems with time delay by using impulsive control. The main contributions of this paper include the following. (i) Our results show that impulses do contribute to globally exponential synchronization of dynamical systems and the time delays in systems not to be smaller than the length of impulsive intervals. Therefore, they can be usually used as an effective control strategy to synchronize the underlying delayed dynamical systems in more practical application. (ii) A new approach for the exponential synchronization of impulsive chaotic systems with any finite delay is given. (iii) In order to deal with uncertainty and/or measurement noise effectively in nominal case for identical chaotic and hyperchaotic systems, the impulse distance should increases which in turn decreases the control cost in accordance with [19]. Thus, having a minimum level of synchronization error with the largest impulsive intervals is generally desired [26]. In our proposed results, a bigger upper bound of impulsive intervals for the synchronization of chaotic systems can be obtained by comparing with the results in [13–16]. It should also be noticed that our paper was inspired in part by the work of Zhou and Wu in [27] for delayed linear differential equations.

The rest of this paper is organized as follows. In Section 2, the problem and some preliminaries are presented. The main result on exponential synchronization is given in Section 3. Section 4 gives an example for illustration, and some conclusions are finally drawn in Section 5.

2. Preliminaries

Denote that R is the set of real numbers, R^+ is the set of nonnegative real numbers, R^n is the n-dimensional real space, and N is the set of positive integers. Let $\Psi(t^+) = \lim_{s\to t^+}\Psi(s)$ and $\Psi(t^-) = \lim_{s\to t^-}\Psi(s)$. For $a, b \in R$ with $a < b$ and for $S \subset R^n$, we define the following function: $\mathrm{PC}([a,b],S) = \{\Psi : [a,b] \to S \mid \Psi(t) = \Psi(t^+)$, for all $t \in [a,b)$; $\Psi(t^-)$ exists in S, for all $t \in (a,b]$ and $\Psi(t^-) = \Psi(t)$, for all but at most a finite number of points $t \in (a,b]\}$. For $\Psi \in \mathrm{PC}([-\tau,0],R^n)$, the norm of Ψ is, respectively, defined by $\|\Psi\| = \sup_{-\tau\le s\le 0}\|\Psi(s)\|$. $e_t, e_{t^-} \in \mathrm{PC}([-\tau,0],R^n)$ are defined by $e_t(s) = e(t+s)$ and $e_{t^-}(s) = e(t^- + s)$ for $s \in [-\tau,0]$.

Now we introduce the coupled chaotic systems that will be studied. It usually consists of two chaotic systems at the transmitter and the receiver ends.

At the transmitter end, we have

$$\dot{x}(t) = Ax(t) + \Phi_1(x(t)), \tag{2.1}$$

and, at the receiver end, we have

$$\begin{gathered}\dot{y}(t) = Ay(t) + \Phi_2\big(y(t), x(t-\tau), y(t-\tau)\big), \quad t \neq t_k,\\ \Delta y(t) = -B_k e(t), \quad t = t_k,\\ y(t_0 - s) = \varphi(s), \quad s \in [0,\tau],\end{gathered} \tag{2.2}$$

where the matrices $A, B_k \in R^{n\times n}$, $e(t) = x(t) - y(t)$ are the synchronization error states between the states of system (2.1) and system (2.2), and the functions Φ_1, Φ_2 are the continuous functions in their respective domain of definition. The time sequence $\{t_k\}_{k=1}^{+\infty}$ satisfies $0 = t_0 < t_1 < t_2 < \cdots < t_k < \cdots$, $\lim_{k\to\infty} t_k = +\infty$, the time delay $\tau > 0$, and $\Delta y(t_k) = y(t_k^+) - y(t_k^-)$, in which $y(t_k^+) = \lim_{s\to 0^+} y(t_k + s)$, $y(t_k^-) = \lim_{s\to 0^-} y(t_k + s)$ for $k = 1, 2, \ldots$. A typical form of the function Φ_2 is given as follows:

$$\Phi_2(y(t), x(t-\tau), y(t-\tau)) = \Phi_1(y(t)) + FK(x(t-\tau) - y(t-\tau)), \tag{2.3}$$

where $F \in R^{n\times m}, K \in R^{m\times n}$ are constant matrices. The second term is known as the delayed feedback controller, which is applied to the input state and then influences the system function.

Then, we can get the following error dynamical system:

$$\begin{aligned} \dot{e}(t) &= Ae(t) + \Psi(x(t), y(t), x(t-\tau), y(t-\tau)), \quad t \neq t_k, \\ \Delta e(t) &= B_k e(t), \quad t = t_k,\ k = 1, 2, \ldots, \end{aligned} \tag{2.4}$$

where $\Psi(x(t), y(t), x(t-\tau), y(t-\tau)) = \Phi_1(x(t)) - \Phi_2(y(t), x(t-\tau), y(t-\tau))$.

Obviously, $e(t) = 0$ is a trivial solution of system (2.4). We shall analyze the dynamics of system (2.4) and drive criteria under which its trivial solution is globally exponentially stable. It is clear that the globally and exponential stability of trivial solution of (2.4) implies the global synchronization of systems (2.1) and (2.2).

For a given $t > t_0$ and $\varphi \in \mathrm{PC}([-\tau, 0], R^n)$, the initial value problem of (2.4) is

$$\begin{aligned} \dot{e}(t) &= Ae(t) + \Psi(x(t), y(t), x(t-\tau), y(t-\tau)), \quad t \neq t_k, \\ \Delta e(t) &= B_k e(t), \quad t = t_k,\ t \in N \\ e(t_0 - s) &= \varphi(s), \quad s \in (0, \tau). \end{aligned} \tag{2.5}$$

We assume that (2.4) has a unique solution with respect to initial conditions. Denote by $e(t) = e(t, t_0, \varphi)$ the solution of (2.4) such that $e(t_0 - s) = \varphi(s)$, $s \in (0, \tau)$. Also we assume that the function Ψ satisfies the following assumption: for certain positive definite matrix P, there exist constant matrices $D_i \in R^{n\times n}$, $i = 1, 2$, such that

$$(x-y)^T P\Psi(x, y, u, v) \le (x-y)^T PD_1(x-y) + (x-y)^T PD_2(u-v). \tag{2.6}$$

Now we have the following definitions.

Definition 2.1. The trivial solution of (2.4) is said to be globally and exponentially stable if for any initial data $e_{t_0} = \varphi \in \mathrm{PC}([-\tau, 0], R^n)$ and $\|\Psi\|_\tau = \sup_{-\tau\le s\le 0}\|\Psi(s)\|$, there exist some constants $\lambda > 0$ and $M \ge 1$ such that

$$\|e(t, t_0, \varphi)\| \le M\|\varphi\|_\tau e^{-\lambda(t-t_0)}, \quad t \ge t_0. \tag{2.7}$$

Definition 2.2. Given a function $V : R^+ \times R^n \rightarrow R^+$, the upper right-hand derivative of V with respect to system (2.4) is defined by

$$D^+V(t, e(t)) = \lim_{\delta \to 0^+} \sup \frac{1}{\delta}[V(t+\delta, e(t+\delta)) - V(t, e(t))]. \tag{2.8}$$

3. Synchronization of Chaotic Systems

In the following, we shall address the exponential stability problem for impulsive delayed nonlinear differential equation (2.4), which implies the global synchronization of two chaotic systems. Our result shows that impulses play an important role in making the delayed nonlinear differential equations globally and exponentially stable.

Theorem 3.1. *Let $P \in R^{n\times n}$ be a positive definite matrix and λ_k $(0 < \lambda_k \leq 1, k \in N)$ be largest eigenvalue of $P^{-1}(I+B_k)^T P(I+B_k)$. Assume that there exist constants α_i, $i = 1, 2$ with $\alpha_2 \in R^+$ and $\lambda > 0$, $\sigma > 0$, such that for all $k \in N$, one has*

(i)

$$\begin{bmatrix} A^TP + PA + PD_1 - \alpha_1 P & PD_2 \\ D_2^T P & -\alpha_2 P \end{bmatrix} \leq 0, \tag{3.1}$$

(ii)

$$\sigma - \lambda - \left[\alpha_1 + \left(\frac{\alpha_2}{\lambda_{k-1}}\right)e^{\lambda\tau}\right] \geq 0, \tag{3.2}$$

where $0 < \lambda_0 \leq 1$,

(iii)

$$\ln \lambda_{k-1} < -(\sigma + \lambda)(t_k - t_{k-1}). \tag{3.3}$$

Then, the trivial solution of system (2.4) *is globally and exponentially stable with a convergence rate of $\lambda/2$ for any fixed delay $\tau \in (0, +\infty)$, that is, system* (2.1) *and system* (2.2) *are globally and exponentially synchronized.*

Proof. Let $e(t) = e(t, t_0, \varphi)$ be any solution of system (2.5), and consider a Lyapunov function as follows:

$$V(t, e(t)) = e^T(t)Pe(t). \tag{3.4}$$

Let $\mu_1 > 0$ and $\mu_2 > 0$ be the smallest and largest eigenvalues of positive definite matrix P, respectively. Then, we can obtain

$$\mu_1\|e(t)\|^2 \leq V(t, e(t)) \leq \mu_2\|e(t)\|^2. \tag{3.5}$$

Now, we are in a position to prove that, for any $t \in [t_{k-1}, t_k)$,

$$V(t, e(t)) \leq \mu_2 M \|\varphi\|_\tau^2 e^{-\lambda(t-t_0)}, \quad k \in N. \tag{3.6}$$

For $t \neq t_k$ $(k \in N)$, it follows from assumption (2.6) and condition (i) that

$$\begin{aligned}
D^+V(t, e(t)) &= \dot{e}^T P e(t) + e^T(t) P \dot{e}(t) \\
&= \left[e^T(t) A^T + \Psi^T\right] P e(t) + e^T(t) P [A e(t) + \Psi] \\
&= e^T \left[A^T P + P A\right] e(t) + e^T(t) P \Psi + \Psi^T P e(t) \\
&\leq e^T(t) \left[A^T P + P A + 2 P D_1\right] e(t) + e^T(t) P D_2 \\
&\quad \times e(t-\tau) + e^T(t-\tau) D_2^T P e(t) \\
&= \left[e^T(t) \;\; e^T(t-\tau)\right] \begin{bmatrix} A^T P + P A + 2 P D_1 & P D_2 \\ D_2^T P & 0 \end{bmatrix} \begin{bmatrix} e(t) \\ e(t-\tau) \end{bmatrix}.
\end{aligned} \tag{3.7}$$

It further follows from condition (i) that

$$\begin{aligned}
D^+V(t, e(t)) &\leq \left[e^T(t) \;\; e^T(t-\tau)\right] \begin{bmatrix} \alpha_1 P & 0 \\ 0 & \alpha_2 P \end{bmatrix} \begin{bmatrix} e(t) \\ e(t-\tau) \end{bmatrix} \\
&= \alpha_1 V(t, e(t)) + \alpha_2 V(t-\tau, e(t-\tau)).
\end{aligned} \tag{3.8}$$

Defining $\gamma = \sup_{k \in N} (1/\lambda_{k-1}) \geq 1$ in condition (iii), we have

$$\ln \gamma + \lambda \tau - (\sigma + \lambda)(t_k - t_{k-1}) > 0. \tag{3.9}$$

Then, in view of the inequality in (3.9), we can find constant $M \geq 1$ such that

$$1 < e^{(\sigma+\lambda)(t_1-t_0)} \leq M \leq \gamma e^{\lambda\tau - (\sigma+\lambda)(t_1-t_0)} e^{(\sigma+\lambda)(t_1-t_0)}, \tag{3.10}$$

which shows that

$$\|\varphi\|_\tau^2 < \|\varphi\|_\tau^2 e^{\sigma(t_1-t_0)} \leq M \|\varphi\|_\tau^2 e^{-\lambda(t_1-t_0)}. \tag{3.11}$$

Firstly, we prove that

$$V(t, e(t)) \leq \mu_2 M \|\varphi\|_\tau^2 e^{-\lambda(t-t_0)}, \quad t \in [t_0, t_1). \tag{3.12}$$

To do this, we only need to prove that

$$V(t,e(t)) \le \mu_2 M\|\varphi\|_\tau^2 e^{-\lambda(t_1-t_0)}, \quad t \in [t_0,t_1). \tag{3.13}$$

If the inequality (3.13) is not true, it follows from (3.11) that there must exist some $\bar{t} \in [t_0,t_1)$ such that

$$\begin{aligned} V\left(\bar{t},e\left(\bar{t}\right)\right) &> \mu_2 M\|\varphi\|_\tau^2 e^{-\lambda(t_1-t_0)} \ge \mu_2\|\varphi\|_\tau^2 e^{\sigma(t_1-t_0)} \\ &> \mu_2\|\varphi\|_\tau^2 \ge V(t_0+s,e(t_0+s)), \quad s \in [-\tau,0], \end{aligned} \tag{3.14}$$

which implies that there exists $t^* \in [t_0,\bar{t})$ such that

$$\begin{aligned} V(t^*,e(t^*)) &= \mu_2 M\|\varphi\|_\tau^2 e^{-\lambda(t_1-t_0)}, \\ V(t,e(t)) &\le V(t^*,e(t^*)), \quad t \in [t_0-\tau,t^*], \end{aligned} \tag{3.15}$$

and there exists $t^{**} \in [t_0,t^*)$ such that

$$\begin{aligned} V(t^{**},e(t^{**})) &= \lambda_2\|\varphi\|_\tau^2, \\ V(t^{**},e(t^{**})) &\le V(t,e(t)), \quad t \in [t^{**},t^*]. \end{aligned} \tag{3.16}$$

Hence, for any $s \in [-\tau,0]$, we can obtain that

$$\begin{aligned} V(t+s,e(t+s)) &\le \mu_2 M\|\varphi\|_\tau^2 e^{-\lambda(t_1-t_0)} \\ &\le \mu_2\gamma e^{\lambda\tau-(\sigma+\lambda)(t_1-t_0)} e^{(\sigma+\lambda)(t_1-t_0)}\|\varphi\|_\tau^2 \\ &= \gamma e^{\lambda\tau}\mu_2\|\varphi\|_\tau^2 = \gamma e^{\lambda\tau}V(t^{**},e(t^{**})) \\ &\le \gamma e^{\lambda\tau}V(t,e(t)), \quad t \in [t^{**},t^*]. \end{aligned} \tag{3.17}$$

By condition (ii), (3.8), and (3.17), we can obtain

$$\begin{aligned} D^+V(t,e(t)) &\le \left(\alpha_1+\gamma\alpha_2 e^{\lambda\tau}\right)V(t,e(t)) \\ &\le \left(\alpha_1+\frac{\alpha_2}{\lambda_0}e^{\lambda\tau}\right)V(t,e(t)) \\ &\le (\sigma-\lambda)V(t,e(t)), \quad t \in [t^{**},t^*], \end{aligned} \tag{3.18}$$

which shows that

$$\begin{aligned} V(t^*, e(t^*)) &\leq V(t^{**}, e(t^{**}))e^{(\sigma-\lambda)(t^*-t^{**})} \\ &= \mu_2 \|\varphi\|_\tau^2 e^{(\sigma-\lambda)(t^*-t^{**})} \\ &< \mu_2 \|\varphi\|_\tau^2 e^{\sigma(t_1-t_0)} \\ &\leq \mu_2 M \|\varphi\|_\tau^2 e^{-\lambda(t_1-t_0)} \\ &= V(t^*, e(t^*)). \end{aligned} \tag{3.19}$$

It is obviously a contradiction. Hence, (3.12) holds, and then (3.6) is true for $k = 1$. Now, we suppose that (3.6) holds for $k = 1, 2, \dots, m$ $(m \in N, m \geq 1)$, that is,

$$V(t, e(t)) \leq \mu_2 M \|\varphi\|_\tau^2 e^{-\lambda(t-t_0)}, \quad t \in [t_{k-1}, t_k). \tag{3.20}$$

Next, we will prove that (3.6) holds for $k = m + 1$, that is,

$$V(t, e(t)) \leq \mu_2 M \|\varphi\|_\tau^2 e^{-\lambda(t-t_0)}, \quad t \in [t_m, t_{m+1}). \tag{3.21}$$

For the sake of contradiction, we suppose that (3.21) is not true. Then, we define

$$\bar{t} = \inf\left\{ t \in [t_m, t_{m+1}) \mid V(t, e(t)) > \mu_2 M \|\varphi\|_\tau^2 e^{-\lambda(t-t_0)} \right\}. \tag{3.22}$$

From condition (iii) and (3.20), we can get

$$\begin{aligned} V(t_m, e(t_m)) &= e^T(t_m^-)(I + B_m)^T P(I + B_m)e(t_m^-) \\ &\leq \lambda_m V(t_m^-, e(t_m^-)) \\ &\leq \lambda_m \mu_2 M \|\varphi\|_\tau^2 e^{-\lambda(t_m-t_0)} \\ &= \lambda_m \mu_2 M \|\varphi\|_\tau^2 e^{\lambda(\bar{t}-t_m)} e^{-\lambda(\bar{t}-t_0)} \\ &< \lambda_m \mu_2 e^{\lambda(t_{m+1}-t_m)} M \|\varphi\|_\tau^2 e^{-\lambda(\bar{t}-t_0)} \\ &< \mu_2 M \|\varphi\|_\tau^2 e^{-\lambda(\bar{t}-t_0)}, \end{aligned} \tag{3.23}$$

and so $\bar{t} \neq t_m$. By the continuity of $V(t, e(t))$ in the interval $[t_m, t_{m+1})$, we obtain

$$\begin{gathered} V(t^*, e(t^*)) = \mu_2 M \|\varphi\|_\tau^2 e^{-\lambda(\bar{t}-t_0)}, \\ V(t, e(t)) \leq V(t^*, e(t^*)), \quad t \in [t_m, t^*]. \end{gathered} \tag{3.24}$$

From (3.23), we can derive that there exists $t^{**} \in [t_m, t^*)$ such that

$$V(t^{**}, e(t^{**})) = \lambda_m \mu_2 e^{\lambda(t_{m+1}-t_m)} M\|\varphi\|_\tau^2 e^{-\lambda(\bar{t}-t_0)},$$
$$V(t^{**}, e(t^{**})) \le V(t, e(t)) \le V(t^*, e(t^*)), \quad t \in [t^{**}, t^*]. \tag{3.25}$$

As for any $t \in [t^{**}, t^*]$, $s \in [-\tau, 0]$, then either $t+s \in [t_0 - \tau, t_m)$ or $t + s \in [t_m, t^*]$. These two cases will be discussed in the following.

If $t + s \in [t_0 - \tau, t_m)$, from (3.20), we have

$$\begin{aligned} V(t+s, e(t+s)) &\le \lambda_2 M\|\varphi\|_\tau^2 e^{-\lambda(t-t_0)} e^{-\lambda s} \\ &\le \mu_2 M\|\varphi\|_\tau^2 e^{-\lambda(\bar{t}-t_0)} e^{\lambda(\bar{t}-t)} e^{\lambda\tau} \\ &\le \mu_2 e^{\lambda\tau} e^{\lambda(t_{m+1}-t_m)} M\|\varphi\|_\tau^2 e^{-\lambda(\bar{t}-t_0)}, \end{aligned} \tag{3.26}$$

whereas if $t + s \in [t_m, t^*]$, from (3.24), then

$$\begin{aligned} V(t+s, e(t+s)) &\le \mu_2 M\|\varphi\|_\tau^2 e^{-\lambda(\bar{t}-t_0)} \\ &\le \mu_2 e^{\lambda\tau} e^{\lambda(t_{m+1}-t_m)} M\|\varphi\|_\tau^2 e^{-\lambda(\bar{t}-t_0)}. \end{aligned} \tag{3.27}$$

Above all, from (3.25)–(3.27), we can obtain, for any $s \in [-\tau, 0]$,

$$\begin{aligned} V(t+s, e(t+s)) &= \frac{e^{\lambda\tau}}{\lambda_m} V(t^{**}, e(t^{**})) \\ &\le \frac{e^{\lambda\tau}}{\lambda_m} V(t, e(t)), \quad t \in [t^{**}, t^*]. \end{aligned} \tag{3.28}$$

Hence, by condition (ii), (3.8), and (3.28), we can conclude that

$$\begin{aligned} D^+ V(t, e(t)) &\le \left(\alpha_1 + \frac{\alpha_2}{\lambda_m} e^{\lambda\tau}\right) V(t, e(t)) \\ &\le (\sigma - \lambda) V(t, e(t)). \end{aligned} \tag{3.29}$$

Thus, in view of condition (iii), we have

$$
\begin{aligned}
V(t^*, e^*) &\le V(t^{**}, e^{**}) e^{(\sigma-\lambda)(t^*-t^{**})} \\
&= \lambda_m \mu_2 e^{\lambda(t_{m+1}-t_m)} M \|\varphi\|_\tau^2 e^{-\lambda(\bar{t}-t_0)} e^{(\sigma-\lambda)(t^*-t^{**})} \\
&< \mu_2 e^{-(\sigma+\lambda)(t_{m+1}-t_m)} e^{\lambda(t_{m+1}-t_m)} \\
&\quad \times M \|\varphi\|_\tau^2 e^{-\lambda(\bar{t}-t_0)} e^{(\sigma-\lambda)(t^*-t^{**})} \\
&= \mu_2 M \|\varphi\|_\tau^2 e^{-\sigma(t_{m+1}-t_m)} e^{(\sigma-\lambda)(t^*-t^{**})} e^{-\lambda(\bar{t}-t_0)} \\
&< \mu_2 M \|\varphi\|_\tau^2 e^{-\lambda(\bar{t}-t_0)} \\
&= V(t^*, e^*),
\end{aligned}
\tag{3.30}
$$

which is a contradiction. It implies that the supposition is not true. Hence, (3.6) holds for $k = m+1$, then, we can conclude by some induction that (3.6) holds for any $k \in N$. It immediately follows from (3.6) that

$$
\|e(t)\| \le \sqrt{\frac{\mu_2}{\mu_1}} M^{1/2} \|\varphi\|_\tau e^{-(\lambda/2)(t-t_0)}, \quad t \ge t_0,
\tag{3.31}
$$

that is, the trivial solution of the impulsive delayed system (2.5) is globally and exponentially stable with a convergence rate of $\lambda/2$ for any fixed delays $\tau \in (0, \infty)$. Then, it implies that system (2.1) and system (2.2) are globally synchronized. The proof is thus complete. □

Remark 3.2. In LMI (3.1) of Theorem 3.1, the constant α_1 (if $\alpha_1 > 0$) is used to measure the level of instability for delay-free system and is determined by the matrix $A + PD_1$, while α_2 is decided by the matrix D_2.

Remark 3.3. Compared with Theorem 3.1 in [28], a distinct feature of Theorem 3.1 in this paper is to eliminate the restriction that the impulsive interval can not be too large, that is, the additional assumption (iv) $t_k - t_{k-1} \ge \tau$ in Theorem 3.1 of [28] is indeed deleted here. Moreover, a controlled parameter σ is introduced to adjust the degree of convergence rate $\lambda/2$ on globally exponential stability of the error system (2.4). It should be mentioned that our results allow us to develop an effective impulse control strategy to exponentially synchronize chaotic systems, and it is particularly meaningful for some practical applications.

Similarly to Theorem 3.1, we can obtain the following result.

Corollary 3.4. *Let $P \in R^{n\times n}$ be a symmetric and positive definite matrix, $0 < \beta = \sup_{k\in N} \lambda_k$, where λ_k is given by Theorem 3.1 for each $k \in N$. Assume that there exist constants α_i, $i = 1, 2$ with $\alpha_2 \in R^+$, such that for all $k \in N$ the following are satisfied:*

(i)

$$
\begin{bmatrix} A^T P + PA + 2PD_1 - \alpha_1 P & PD_2 \\ D_2^T P & -\alpha_2 P \end{bmatrix} \le 0,
\tag{3.32}
$$

(ii)

$$\left(\alpha_1 + \frac{\alpha_2}{\beta}\right)(t_k - t_{k-1}) < -\ln \beta. \tag{3.33}$$

Then, the trivial solution of system (2.4) *is globally and exponentially stable for any fixed delay* $\tau \in (0, +\infty)$*, that is, system* (2.1) *and system* (2.2) *are globally and exponentially synchronized.*

Proof. Condition (ii) in Theorem 3.1 can be reduced to the following form:

$$\sigma - \lambda + \left[\alpha_1 + \left(\frac{\alpha_2}{\beta}\right)e^{\lambda\tau}\right] \geq 0. \tag{3.34}$$

Then, it is easy to get the following inequality, which is equivalent to (3.34) and condition (iii) in Theorem 3.1:

$$\left[2\lambda + \alpha_1 + \left(\frac{\alpha_2}{\beta}\right)e^{\lambda\tau}\right](t_k - t_{k-1}) < -\ln \beta. \tag{3.35}$$

Hence, there must exist a small-enough real number $\lambda > 0$ such that (3.35) holds. And if λ is extremely tiny, we can obtain that condition (ii) in Corollary 3.4 holds. From Theorem 3.1, the proof is completed. □

Remark 3.5. As Corollary 3.4 can be applied to deal with globally exponential stability of impulsive differential equations with any time delays $\tau \in (0, +\infty)$, it is obviously more applicable than those existing in the works in the recent literature [23–25, 28] where the time delays need to be assumed not bigger than the length of impulsive interval. In view of Corollary 3.4, it is clear that we have removed the restriction of time delay indeed.

Remark 3.6. Since the amount of transmitted information decreases leading to reduced control cost, the cost of impulsive synchronization of chaotic systems is closely related to the impulse distances and having a minimum level of synchronization error with the largest impulsive intervals as generally a desired [26]. From the proposed result in Corollary 3.4, the upper bound of impulsive intervals for the globally and exponentially synchronization can be given by $t_k - t_{k-1} < -\ln \beta/(\alpha_1 + \alpha_2/\beta)$ for each $k \in N$. From the following numerical example, we will show that our upper bound of impulsive intervals is bigger than some existing results.

4. Numerical Simulation

In this section, an example is presented here to illustrate our main results.

Example 4.1. Consider the Lorenz system in [13] as follows:

$$\begin{aligned}
\dot{x}_1 &= -\sigma x_1 + \sigma x_2, \\
\dot{x}_2 &= r x_1 - x_2 - x_1 x_3, \\
\dot{x}_3 &= x_1 x_2 - b x_3,
\end{aligned} \tag{4.1}$$

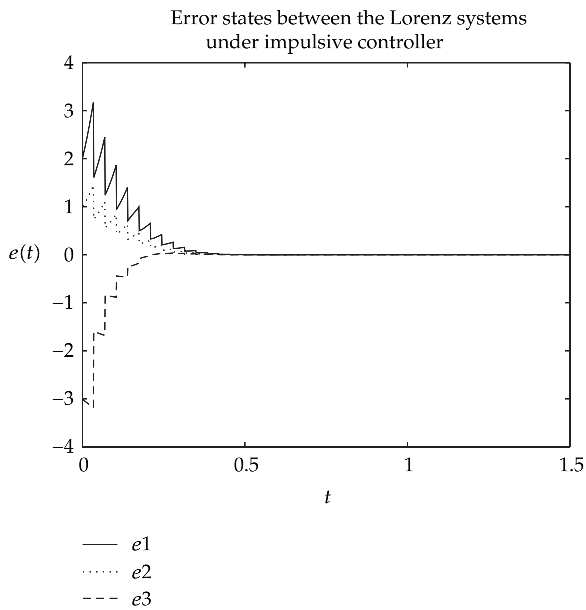

Figure 1: The trajectories of the error Lorenz system under impulsive controller with $t_k - t_{k-1} = 0.035$.

where $\sigma, r, b \in R^+$, $\sigma = 10$, $r = 28$, and $b = 8/3$. Let $x = [x_1, x_2, x_3]^T$, then we can rewrite the Lorenz system in the form of (2.1) as follows:

$$\dot{x}(t) = Ax(t) + \Phi_1(x(t)), \tag{4.2}$$

where

$$A = \begin{bmatrix} -10 & 10 & 0 \\ 28 & -1 & 0 \\ 0 & 0 & -\dfrac{8}{3} \end{bmatrix}, \qquad \Phi_1 = \begin{bmatrix} 0 \\ -x_1x_3 \\ x_1x_2 \end{bmatrix}. \tag{4.3}$$

We also choose Φ_2 defined by (2.3) with $FK = I$. Set the delay $\tau = 1$ and the impulsive matrices $B_k = -0.5I$ for all $k = 1, 2, \ldots$. Then, it can be observed that inequality (2.6) holds with $D_1 = I$ and $D_2 = -FK = -I$.

Solving (3.1) in Theorem 3.1, we get that $\alpha_1 = 31$, $\alpha_2 = 1$, and $P = I$. Since $B_k = -0.5I$, we can get $\beta = 0.25$. According to condition (ii) in Corollary 3.4, when

$$t_k - t_{k-1} < -\frac{\ln \beta}{\alpha_1 + \alpha_2/\beta} = 0.0396, \tag{4.4}$$

the trivial solution of error system based on impulsive delayed differential equation (4.1) is globally exponentially stable for any fixed delay $\tau \in (0, +\infty)$. With the same parameter, a comparison of the upper bound of impulsive intervals with [13–16] is presented in Table 1, which shows that our upper bound is much bigger. Choose $t_k - t_{k-1} = 0.035$, and the simulation result of error system $e(t)$ is shown in Figure 1.

Table 1: A comparison of the upper bound of impulsive intervals of (4.1).

Corollary 3.4	[13]	[14]	[15, 16]
0.0396	0.0072	0.0108	0.0178

5. Conclusion

In this paper, we have investigated the synchronization of coupled chaotic systems with time delay by using impulsive control. The time delays in systems need not to be smaller than the length of impulsive interval as many works existing in the literature assumed. A numerical example has been given to demonstrate the effectiveness of the theoretical results, and the estimation of the stable region of the impulsive intervals has also been presented. By comparison, the upper bound of impulsive intervals is greater than it was in some existing results. The obtained results can be easily used to control many systems, especially to stabilize and synchronize chaotic systems.

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