Research Article

The Permanence and Extinction of the Single Species with Contraception Control and Feedback Controls

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Population control has become a major problem in many wildlife species. Sterility control through contraception has been proposed as a method for reducing population size. In this paper, the single species with sterility control and feedback controls is considered. Sufficient conditions are obtained for the permanence and extinction of the system. The results show that the feedback controls do not influence the permanence of the species.

1. Introduction

Control of wildlife pest populations has usually relied on methods like chemical pesticides, biological pesticides, remote sensing and measure, computers, atomic energy, and so forth. Some brilliant achievements have been obtained. However, the warfare will never be over. Although a large variety of pesticide were used to control wildlife pest populations, the wildlife pests impairing crops are increasing especially because of resistance to the pesticide. So the pesticides are invalid. Moreover, wildlife pests will continue. On the other hand, the chemical pesticide kills not only wildlife pests but also their natural enemies. Therefore, wildlife pests are rampant. Now, sterile control to suppress wildlife pests is one of the most important measures in wildlife pest control. Sterile control [1–5] is especially for the purpose of suppressing the abundance of the pest in a new target region to a level at which it no longer causes economic damage. This can be achieved by releasing sterile insects into the environment in very large numbers in order to mate with the native insects that are present in the environment. A native female that mates with a sterile male will produce eggs, but

the eggs will not hatch (the same effect will occur for the reciprocal cross). If there is a sufficiently high number of sterile insects than most of the crosses are sterile, and as time goes on, the number of native insects decreases and the ratio of sterile to normal insects increases, thus driving the native population to extinction. Sterile male techniques were first used successfully in 1958 in Florida to control Screwworm fly (Cochliomya omnivorax). A number of mathematical models have been done to assist the effectiveness of the SIT (see, e.g., [2–4]). Recently, Liu and Li [6] considered the following contraception control model:

$$
\begin{aligned}
x_1'(t) &= x_1(t)\big(b - k(x_1(t) + x_2(t)) - \mu\big), \\
x_2'(t) &= \mu x_1(t) - [d + k(x_1(t) + x_2(t))]x_2(t),
\end{aligned}
\tag{1.1}
$$

where $x_1(t)$, $x_2(t)$ represent, respectively, the density of the fertile species and the sterile species at time t. The authors proved the equilibrium point of system (1.1) is stable under appropriate conditions. In view of the effects of a periodically changing environment, we consider the following nonautonomous contraception model:

$$
\begin{aligned}
x_1'(t) &= x_1(t)\big(b(t) - k(t)(x_1(t) + x_2(t)) - \mu(t)\big), \\
x_2'(t) &= \mu(t)x_1(t) - [d(t) + k(t)(x_1(t) + x_2(t))]x_2(t).
\end{aligned}
\tag{1.2}
$$

However, we note that ecosystem in the real world are continuously disturbed by unpredictable forces which can result in changes in the biological parameters such as survival rates. Of practical interest in ecosystem is the question of whether or not an ecosystem can withstand those unpredictable forces which persist for a finite period of time. In the language of control variables, we call the disturbance functions as control variables. So it is necessary to study models with control variables which are so-called disturbance functions, and to find some suitable conditions to prevent a particular species from dying out. In 1993, Gopalsamy and Weng [7] introduced a feedback control variable into the delay logistic model and discussed the asymptotic behavior of solution in logistic models with feedback controls, in which the control variables satisfy certain differential equation.

In recent years, the population dynamical systems with feedback controls have been studied in many articles, for example, see [7–12] and references cited therein. However, to the best of the authors knowledge, to this day, still less scholars consider the nonautonomous single species with contraception control and feedback controls.

Motivated by the above works, we focus our attention on the permanence of species for the following single species nonautonomous systems with delays and feedback control:

$$
\begin{aligned}
x_1'(t) &= x_1(t)\big(b(t) - k(t)(x_1(t) + x_2(t)) - \mu(t) - c_1(t)u(t - \sigma_1(t))\big), \\
x_2'(t) &= \mu(t)x_1(t) - [d(t) + k(t)(x_1(t) + x_2(t)) + c_2(t)u(t - \sigma_2(t))]x_2(t), \\
u'(t) &= -e(t)u(t) + f_1(t)x_1(t - \tau_1(t)) + f_2(t)x_2(t - \tau_2(t)),
\end{aligned}
\tag{1.3}
$$

where $x_1(t)$, $x_2(t)$ represent, respectively, the density of the fertile population and the sterile population at time t. $u(t)$ is the control variable at time t. $b(t)$, $k(t)$ represent, respectively, the intrinsic growth rate and density-dependent rate of the species at time t, respectively. $\mu(t)$ is the migration rates from the fertile population to the sterile population. The function $b(t)$ is bounded continuous defined on $R_+ = [0, \infty)$; functions $\mu(t)$, $k(t)$, $d(t)$, $e(t)$, $c_i(t)$,

$f_i(t)$, $\sigma_i(t)$, and $\tau_i(t)$ $(i = 1, 2)$ are continuous, bounded, and nonnegative defined on R_+. Let $\tau = \sup\{\tau_1(t), \tau_2(t), \sigma_1(t), \sigma_2(t) : t \geq 0\}$.

We will consider (1.3) together with initial conditions:

$$x_i(\theta) = \phi_i(\theta), \qquad u(\theta) = \psi(\theta), \tag{1.4}$$

where $\phi_i(\theta), \psi(\theta) \in BC^+[-\tau, 0]$ and

$$BC^+[-\tau, 0] = \{\phi \in C([-\tau, 0], [0, +\infty]) : \phi(0) > 0, \phi\ (\theta)\ \text{is bounded}\}. \tag{1.5}$$

By the fundamental theory of functional differential equations [12], it is not difficult to see that the solution $(x_1(t), x_2(t), u(t))$ of (1.3) is unique and positive if initial functions satisfy initial condition (1.4). So, in this paper, the solution of (1.3) satisfying initial conditions (1.4) is said to be positive.

The main purpose of this paper is to establish a new general criterion for the permanence and the extinction of (1.3), which is described by integral form and independent feedback controls. The paper is organized as follows. In the next section, we will give some assumptions and useful lemmas. In Section 3, some new sufficient conditions which guarantee the permanence of all positive solutions for (1.3) are obtained. In Section 4, we obtained some new sufficient conditions which guarantee the extinction of all positive solutions for (1.3).

2. Preliminaries

Throughout this paper, we will introduce the following assumptions:

(H$_1$) there exist constants $\omega_i > 0$ $(i = 1, 2)$ such that

$$\liminf_{t \to \infty} \int_t^{t+\omega_1} [b(s) - \mu(s)]\,ds > 0, \qquad \liminf_{t \to \infty} \int_t^{t+\omega_2} \mu(s)\,ds > 0; \tag{2.1}$$

(H$_2$) there exist constants $\lambda_i > 0$ $(i = 1, 2)$, such that

$$\liminf_{t \to \infty} \int_t^{t+\lambda_1} k(s)\,ds > 0, \qquad \liminf_{t \to \infty} \int_t^{t+\lambda_2} d(s)\,ds > 0; \tag{2.2}$$

(H$_3$) there exists a constant $\gamma > 0$ such that

$$\liminf_{t \to \infty} \int_t^{t+\gamma} e(s)\,ds > 0. \tag{2.3}$$

In addition, for a function $g(t)$ defined on set $I \subset R$, we denote

$$g^L = \inf_{t \in I} g(t), \qquad g^M = \sup_{t \in I} g(t). \tag{2.4}$$

Now, we state several lemmas which will be useful in the proving of main results in this paper.

First, we consider the following nonautonomous logistic equation:

$$x'(t) = x(t)(r(t) - a(t)x(t)), \tag{2.5}$$

where functions $a(t)$, $r(t)$ are bounded and continuous on R_+. Furthermore, $a(t) \geq 0$ for all $t \geq 0$. We have the following result which is given in [13] by Teng and Li.

Lemma 2.1. *Suppose that there exist constants* λ, σ *such that*

$$\liminf_{t \to \infty} \int_t^{t+\lambda} r(s)ds > 0, \qquad \liminf_{t \to \infty} \int_t^{t+\sigma} a(s)ds > 0. \tag{2.6}$$

Then,

(a) *there exist positive constants* m *and* M *such that*

$$m \leq \liminf_{t \to \infty} x(t) \leq \limsup_{t \to \infty} x(t) \leq M. \tag{2.7}$$

for any positive solution $x(t)$ *of* (2.5);

(b) $\lim_{t \to \infty}(x^{(1)}(t) - x^{(2)}(t)) = 0$ *for any two positive solutions* $x^{(1)}(t)$ *and* $x^{(2)}(t)$ *of* (2.5).

Further, consider the following nonautonomous linear equation:

$$u'(t) = r(t) - e(t)u(t), \tag{2.8}$$

where functions $r(t)$ and $e(t)$ are bounded continuous defined on R_+, and $r(t) \geq 0$ for all $t \geq 0$. One has the following result.

Lemma 2.2. *Suppose that* (H_3) *holds. Then,*

(a) *there exists a positive constant* U *such that* $\limsup_{t \to \infty} u(t) \leq U$ *for any positive solution* $u(t)$ *of* (2.8);

(b) $\lim_{t \to \infty}(u^{(1)}(t) - u^{(2)}(t)) = 0$ *for any two positive solutions* $u^{(1)}(t)$ *and* $u^{(2)}(t)$ *of* (2.8).

The proof of Lemma 2.2 is very simple by making a transformation with $u(t) = 1/x(t)$. This produces the calculations: $u'(t) = -(1/x^2(t))x'(t)$ and $x'(t) = x(t)(e(t) - r(t)x(t))$. Then, according to the Lemma 2.1 one can obtain Lemma 2.2.

Lemma 2.3. *Suppose that* (H_3) *holds. Then, for any constants* $\epsilon > 0$ *and* $M > 0$, *there exist constants* $\delta = \delta(\epsilon) > 0$ *and* $T = T(\epsilon, M) > 0$ *such that for any* $t_0 \in R_+$ *and* $u_0 \in R$ *with* $|u_0| \leq M$, *when* $|r(t)| < \delta$ *for all* $t \geq t_0$, *one has*

$$|u(t, t_0, u_0)| < \epsilon \quad \forall t \geq t_0 + T, \tag{2.9}$$

where $u(t, t_0, u_0)$ *is the solution of* (2.8) *with initial condition* $u(t_0) = u_0$.

The proof of Lemma 2.3 can be found as Lemma 2.4 in [11] by Wang et al.

3. Main Results

In this section, we study the permanence of species $x_1(t)$, $x_2(t)$ of (1.3). First, we have the theorem on the ultimate boundedness of all positive solutions of (1.3).

Theorem 3.1. *Suppose that assumptions* (H_1)–(H_3) *hold. Then, any positive solution of* (1.3) *is ultimate bounded, in the sense that there exists a positive constant* $M > 0$ *such that*

$$\limsup_{t \to \infty} x_1(t) < M, \qquad \limsup_{t \to \infty} x_2(t) < M, \qquad \limsup_{t \to \infty} u(t) < M, \tag{3.1}$$

for any positive solution $(x_1(t), x_2(t), u(t))$ *of* (1.3).

Proof. Let $(x_1(t), x_2(t), u(t))$ be any positive solution of (1.3). We first prove that the component $x_1(t)$ of (1.3) is ultimately bounded. From the first equation of (1.3), we have

$$\frac{dx_1(t)}{dt} \le x_1(t)\big(b(t) - \mu(t) - k(t)x_1(t)\big). \tag{3.2}$$

We consider the following auxiliary equation:

$$\frac{dy(t)}{dt} = y(t)\big(b(t) - \mu(t) - k(t)y(t)\big), \tag{3.3}$$

then by (H_1) and applying Lemma 2.1, there exists a constant M_1 such that

$$\limsup_{t \to \infty} y(t) < M_1 \tag{3.4}$$

for any positive solution $y(t)$ of (3.3). Let $y^*(t)$ be the solution of (3.3) satisfying initial condition $y^*(t_0) = x_1(t_0)$. Further, from comparison theorem, it follows that

$$x_1(t) < y^*(t) \quad \forall t > t_0. \tag{3.5}$$

Thus, we finally obtain that

$$\limsup_{t \to \infty} x_1(t) < M_1. \tag{3.6}$$

From inequality (3.6), we obtain that there exists a positive constant T_1 such that

$$x_1(t) < M_1 \quad \forall t \ge T_1. \tag{3.7}$$

Hence, from the second equation of (1.3), one has

$$\frac{dx_2(t)}{dt} \leq \mu(t)M_1 - d(t)x_2(t) \tag{3.8}$$

for all $t \geq T_1$. Further, consider the following auxiliary equation:

$$\frac{dv(t)}{dt} = \mu(t)M_1 - d(t)v(t), \tag{3.9}$$

from assumptions (H$_1$) and (H$_2$) and according to Lemma 2.2, there exists constant $M_2 > 0$ such that

$$\limsup_{t \to \infty} v(t) < M_2 \tag{3.10}$$

for the solution $v(t)$ of (3.9) with initial condition $v(T_1) = x_2(T_1)$. By the comparison theorem, we have

$$x_2(t) \leq v(t) \quad \forall t \geq T_1. \tag{3.11}$$

From this, we further obtain

$$\limsup_{t \to \infty} x_2(t) < M_2. \tag{3.12}$$

Then, we obtain that there exists constant $T_2 > T_1$ such that

$$x_2(t) < M_2 \quad \forall t \geq T_2. \tag{3.13}$$

From the third equation of (1.3), we have

$$\frac{du(t)}{dt} \leq -e(t)u(t) + M_1 f_1(t) + M_2 f_2(t) \tag{3.14}$$

for all $t \geq T_2 + \tau$. Consider the following auxiliary equation:

$$\frac{dv(t)}{dt} = -e(t)v(t) + M_1 f_1(t) + M_2 f_2(t). \tag{3.15}$$

By assumption (H$_3$) and conclusions of Lemma 2.2, we can get that there exists a constant $M_3 > 0$ such that

$$\limsup_{t \to \infty} v(t) < M_3 \tag{3.16}$$

for the solution $v(t)$ of (3.15) with initial condition $v(T_2 + \tau) = u(T_2 + \tau)$. By the comparison theorem, we have

$$u(t) \leq v(t) \quad \forall t \geq T_2 + \tau. \tag{3.17}$$

Hence, we further obtain

$$\limsup_{t \to \infty} u(t) < M_3. \tag{3.18}$$

Choose the constant $M = \max\{M_1, M_2, M_3\}$, then we finally obtain

$$\limsup_{t \to \infty} x_1(t) < M, \qquad \limsup_{t \to \infty} x_2(t) < M, \qquad \limsup_{t \to \infty} u(t) < M. \tag{3.19}$$

This completes the proof. $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\square$

Theorem 3.2. *Suppose that assumptions (H_1)–(H_3) hold. Then, there exists a constant $\eta > 0$, which is independent of any solution of (1.3), such that*

$$\liminf_{t \to \infty} x_1(t) > \eta, \qquad \liminf_{t \to \infty} x_2(t) > \eta, \tag{3.20}$$

for any positive solution $(x_1(t), x_2(t), u(t))$ of (1.3).

Proof. Let $(x_1(t), x_2(t), u(t))$ be a solution of (1.3) satisfying initial condition (1.4). In view of Theorem 3.1, there exists a T_0 such that for all $t > T_0$ we have $x_i(t) < M$, $u(t) < M$ $(i = 1, 2)$. According to (H_1), we can choose constants $\varepsilon_0 > 0$ and $T_1 > 0$ such that, for all $t \geq T_1$, we have

$$\int_t^{t+\omega_1} \left(b(s) - \mu(s) - 2k(s)\varepsilon_0 - c_1(s)\varepsilon_0\right) ds > \varepsilon_0. \tag{3.21}$$

Next, we consider the following equation:

$$\frac{du(t)}{dt} = -e(t)u(t) + f_1(t)\alpha_0 + f_2(t)\alpha_0, \tag{3.22}$$

where α_0 was given in the later. By (H_3), we have (3.22) satisfying all the conditions of Lemma 2.3. So, we can obtain that, for given constants $\varepsilon_0 > 0$ and $M > 0$ (M was given in Theorem 3.1), there exist constants $\delta_0 = \delta_0(\varepsilon_0) > 0$ and $T^* = T^*(\varepsilon_0, M) > 0$ such that for any $t_0 \in R_+$ and $0 \leq u_0 \leq M$, when $\alpha_0 f_1(t) + \alpha_0 f_2(t) < \delta_0$ for all $t \geq t_0$, we have

$$u(t, t_0, u_0) < \varepsilon_0 \quad \forall t \geq t_0 + T^*, \tag{3.23}$$

where $u(t, t_0, u_0)$ is the solution of (3.22) with initial condition $u(t_0) = u_0$.

Further, consider the following equation:

$$\frac{dz(t)}{dt} = \mu(t)\alpha_0 - d(t)z(t), \tag{3.24}$$

where α_0 was given in (3.22). By (H_2), we have (3.24) satisfying all the conditions of Lemma 2.3, so by Lemma 2.3, for given constants $\alpha_0 > 0$ and $M > 0$, there exist constants $\delta_1 = \delta_1(\alpha_0) > 0$ and $T^{**} = T^{**}(\alpha_0, M) > 0$ such that for any $t_0 \in R_+$ and $0 \le z_0 \le M$, when $\mu(t)\alpha_0 < \delta_1$ for all $t \ge t_0$, we have

$$z(t, t_0, z_0) < \alpha_0 \quad \forall t \ge t_0 + T^{**}, \tag{3.25}$$

where $z(t, t_0, z_0)$ is the solution of (3.24) with initial condition $z(t_0) = z_0$.

Let $\alpha_0 \le \min\{\varepsilon_0, (\delta_1/(\mu^M + 1)), (\delta_0/(f_1^M + f_2^M + 1))\}$ such that for all $t \ge T_1$, we have

$$\int_t^{t+\omega_1} (b(s) - \mu(s) - 2k(s)\alpha_0 - f_1(s)\varepsilon_0) ds > \varepsilon_0. \tag{3.26}$$

We first prove that

$$\limsup_{t \to \infty} x_1(t) \ge \alpha_0. \tag{3.27}$$

In fact, if (3.27) is not true, then there exist a positive solution $(x_1(t), x_2(t), u(t))$ of (1.3) and a constant $T_2 \ge T_1$ such that

$$x_1(t) < \alpha_0, \tag{3.28}$$

for all $t \ge T_2$.

From the second equation of (1.3), we have

$$\frac{dx_2(t)}{dt} \le \mu(t)\alpha_0 - d(t)x_2(t) \tag{3.29}$$

for all $t \ge T_2$. Let $\hat{z}(t)$ be the solution of (3.24) with initial condition $\hat{z}(T_2) = x_2(T_2)$, by the comparison theorem, we have $x_2(t) \le \hat{z}(t)$ for all $t \ge T_2$. In (3.25), we choose $t_0 = T_2$ and $\hat{z}_0 = x_2(T_2)$, since $\mu(t)\alpha_0 < \delta_1$, we obtain $\hat{z}(t, t_0, \hat{z}_0) < \alpha_0$ for all $t \ge T_2 + T^{**}$. Hence, we further obtain

$$x_2(t, T_2, x_2(T_2)) < \alpha_0 \quad \forall t \ge T_2 + T^{**}. \tag{3.30}$$

By applying (3.28) and (3.30) to the third equation of (1.3), it follows that

$$\frac{du(t)}{dt} \le -e(t)u(t) + f_1(t)\alpha_0 + f_2(t)\alpha_0, \tag{3.31}$$

for all $t \geq T_2 + T^{**} + \tau$. Let $\widehat{u}(t)$ be the solution of (3.22) with initial condition $\widehat{u}(T_2 + T^{**} + \tau) = u(T_2 + T^{**} + \tau)$, by the comparison theorem, we have

$$u(t) \leq \widehat{u}(t) \quad \forall t \geq T_2 + T^{**} + \tau. \tag{3.32}$$

In (3.23), we choose $t_0 = T_2 + T^{**} + \tau$ and $\widehat{u}_0 = u(T_2 + T^{**} + \tau)$, since $\alpha_0 f_1(t) + \alpha_0 f_2(t) < \delta_0$ for all $t \geq T_2 + T^{**} + \tau$, we obtain

$$\widehat{u}(t, T_2 + T^{**} + \tau, u(T_2 + T^{**} + \tau)) < \varepsilon_0 \quad \forall t \geq T_2 + T^{**} + \tau + T^*. \tag{3.33}$$

Hence, from (3.32), we further obtain

$$u(t, T_2 + T^{**} + \tau, u(T_2 + T^{**} + \tau)) < \varepsilon_0 \quad \forall t \geq T_2 + T^{**} + \tau + T^*. \tag{3.34}$$

Hence, by (3.30) and (3.34) it follows that

$$\frac{dx_1(t)}{dt} \geq x_1(t)\left(b(t) - \mu(t) - 2k(t)\alpha_0 - c_1(t)\varepsilon_0\right), \tag{3.35}$$

for any $t > T_2 + T^{**} + T^* + 2\tau$. Integrating (3.35) from $T_3 = T_2 + T^{**} + T^* + 2\tau$ to t we have

$$x_1(t) \geq x_1(T_3) \exp \int_{T_3}^{t} \left(b(s) - \mu(s) - 2k(s)\alpha_0 - c_1(s)\varepsilon_0\right)ds. \tag{3.36}$$

Thus, from (3.26), we have $x_1(t) \to \infty$ as $t \to \infty$, which leads to a contradiction. So, (3.27) holds.

Now, we prove the conclusion of Theorem 3.2. In fact, if it is not true, then there exists an initial functions sequence $\{Z^{(m)}\} = \{(\phi_1^{(m)}(\theta), \phi_2^{(m)}(\theta), \psi^{(m)}(\theta))\}, \theta \in [-\tau, 0]$ such that

$$\liminf_{t \to \infty} x_1\left(t, Z^{(m)}\right) < \frac{\alpha_0}{(m+1)^2} \quad \forall\, m = 1, 2, \ldots, \tag{3.37}$$

for the solution $(x_1(t, Z^{(m)}), x_2(t, Z^{(m)}), u(t, Z^{(m)}))$ of (1.3). From (3.27) and (3.37), for every m, there are two time sequences $\{s_q^{(m)}\}$ and $\{t_q^{(m)}\}$, satisfying $0 < s_1^{(m)} < t_1^{(m)} < s_2^{(m)} < t_2^{(m)} < \cdots < s_q^{(m)} < t_q^{(m)} < \cdots$ and $\lim_{q \to \infty} s_q^{(m)} = \infty$, such that

$$x_1\left(s_q^{(m)}, Z^{(m)}\right) = \frac{\alpha_0}{m+1}, \qquad x_1\left(t_q^{(m)}, Z^{(m)}\right) = \frac{\alpha_0}{(m+1)^2}, \tag{3.38}$$

$$\frac{\alpha_0}{(m+1)^2} \leq x_1\left(t, Z^{(m)}\right) \leq \frac{\alpha_0}{m+1} \quad \forall t \in \left(s_q^{(m)}, t_q^{(m)}\right). \tag{3.39}$$

From Theorem 3.1, we can choose a positive constant $T^{(m)}$ such that $x_1(t, Z^{(m)}) < M$ and $x_2(t, Z^{(m)}) < M$ and $u(t, Z^{(m)}) < M$, for all $t > T^{(m)}$. Further, there is an integer $K_1^{(m)} > 0$ such that $s_q^{(m)} > T^{(m)} + \tau$ for all $q \geq K_1^{(m)}$. Let $q \geq K_1^{(m)}$, for any $t \in (s_q^{(m)}, t_q^{(m)})$, we have

$$
\begin{aligned}
\frac{dx_1(t, Z^{(m)})}{dt} &\geq x_1(t, Z^{(m)})\left[b(t) - \mu(t) - k(t)M - k(t)M - c_1(t)M\right] \\
&\geq -\gamma_1 x_1(t, Z^{(m)}),
\end{aligned}
\tag{3.40}
$$

where $\gamma_1 = \sup_{t \in R_+}\{|b(t) - \mu(t) - 2k(t)M - c_1(t)M|\}$. Integrating the above inequality from $s_q^{(m)}$ to $t_q^{(m)}$, we further have

$$
\begin{aligned}
\frac{\alpha_0}{(m+1)^2} = x_1(t_q^{(m)}, Z^{(m)}) &\geq x_1(s_q^{(m)}, Z^{(m)})\exp\left[-\gamma_1(t_q^{(m)} - s_q^{(m)})\right] \\
&= \frac{\alpha_0}{m+1}\exp\left[-\gamma_1(t_q^{(m)} - s_q^{(m)})\right].
\end{aligned}
\tag{3.41}
$$

Consequently,

$$
t_q^{(m)} - s_q^{(m)} \geq \frac{\ln(m+1)}{\gamma_1} \quad \forall q \geq K_1^{(m)}, \ m = 1, 2, \ldots,
\tag{3.42}
$$

we can choose a large enough N_0 such that

$$
t_q^{(m)} - s_q^{(m)} \geq T^{**} + T^* + 2\tau + \omega_1 \quad \forall m \geq N_0, \ q \geq K_1^{(m)}.
\tag{3.43}
$$

For any $m \geq N_0, q \geq K_1^{(m)}$, and $t \in [s_q^{(m)}, t_q^{(m)}]$, from (3.39), we can obtain

$$
\begin{aligned}
\frac{dx_2(t, Z^{(m)})}{dt} &\leq \mu(t)\frac{\alpha_0}{m+1} - d(t)x_2(t, Z^{(m)}) \\
&\leq \mu^M\alpha_0 - d(t)x_2(t, Z^{(m)}).
\end{aligned}
\tag{3.44}
$$

Let $\hat{z}(t)$ be solution of (3.24) with initial condition $\hat{z}(s_q^{(m)}) = x_2(s_q^{(m)})$, by the comparison theorem, we have $x_2(t) \leq \hat{z}(t)$ for all $t \geq s_q^{(m)}$. In (3.25), we choose $t_0 = s_q^{(m)}$ and $\hat{z}_0 = z(s_q^{(m)})$, since $\mu^M\alpha_0 < \delta_1$ and $x_2(s_q^{(m)} < M$, we obtain $\hat{z}(t, s_q^{(m)}, z(s_q^{(m)})) < \alpha_0$ for all $t \geq s_q^{(m)} + T^{**}$. By Lemma 2.3, we can obtain that $T^{**} = T^{**}(\alpha_0, M)$ is independent of m. Hence, we further obtain

$$
x_2(t, s_q^{(m)}, x_2(s_q^{(m)})) < \alpha_0,
\tag{3.45}
$$

for all $t > s_q^{(m)} + T^{**}$. From the third equation of (1.3), we obtain

$$
\begin{aligned}
\frac{du(t, Z^{(m)})}{dt} &= -e(t)u\left(t, Z^{(m)}\right) + f_1(t)x_1\left(t - \tau_1(t), Z^{(m)}\right) + f_2(t)x_2\left(t - \tau_2(t), Z^{(m)}\right) \\
&\leq -e(t)u\left(t, Z^{(m)}\right) + f_1(t)\alpha_0 + f_2(t)\alpha_0,
\end{aligned}
\tag{3.46}
$$

for all $t \geq s_q^{(m)} + T^{**} + \tau$. Assume that $\tilde{u}(t)$ is the solution of (3.22) with initial condition $\tilde{u}(s_q^{(m)} + T^{**} + \tau) = u(s_q^{(m)} + T^{**} + \tau)$, then we have

$$
u\left(t, Z^{(m)}\right) \leq \tilde{u}(t) \quad \forall t \in \left[s_q^{(m)} + T^{**} + \tau, t_q^{(m)}\right], \ m \geq N_0, \ q \geq K_1^{(m)}.
\tag{3.47}
$$

In (3.23), we choose $t_0 = s_q^{(m)} + T^{**} + \tau$ and $u_0 = u(s_q^{(m)} + T^{**} + \tau)$. Obviously, $\alpha_0(f_1(t) + f_2(t)) < \delta_0$ for all $t \geq s_q^{(m)} + T^{**} + \tau$. So, we have

$$
\tilde{u}(t) = \tilde{u}\left(t, s_q^{(m)} + T^{**} + \tau, u\left(s_q^{(m)} + T^{**} + \tau\right)\right) < \varepsilon_0
\tag{3.48}
$$

for all $t \in [s_q^{(m)} + T^{**} + \tau + T^*, t_q^{(m)}]$. Using the comparison theorem, it follows that

$$
u\left(t, Z^{(m)}\right) < \varepsilon_0
\tag{3.49}
$$

for all $t \in [s_q^{(m)} + T^{**} + \tau + T^*, t_q^{(m)}]$, $q \geq K_1^{(m)}$, and $m \geq N_0$.

So, for any $m \geq N_0$, $q \geq K_1^{(m)}$, and $t \in [s_q^{(m)} + T^{**} + T^* + 2\tau, t_q^{(m)}]$, from (3.26), (3.45), and (3.49), it follows

$$
\begin{aligned}
\frac{dx_1(t, Z^{(m)})}{dt} &= x_1\left(t, Z^{(m)}\right)\left(b(t) - \mu(t) - k(t)\left(x_1\left(t, Z^{(m)}\right) + x_2\left(t, Z^{(m)}\right)\right) - c_1(t)u\left(t - \sigma_1(t), Z^{(m)}\right)\right) \\
&\geq x_1\left(t, Z^{(m)}\right)\left(b(t) - \mu(t) - 2k(t)\alpha_0 - c_1(t)\varepsilon_0\right).
\end{aligned}
\tag{3.50}
$$

Integrating the above inequality from $t_q^{(m)} - \omega_1$ to $t_q^{(m)}$, then from (3.26), we obtain

$$
\begin{aligned}
\frac{\alpha_0}{(m+1)^2} &= x_1\left(t_q^{(m)}, Z^{(m)}\right) \\
&\geq x_1\left(t_q^{(m)} - \omega_1, Z^{(m)}\right) \exp \int_{t_q^{(m)} - \omega_1}^{t_q^{(m)}} \left(b(t) - \mu(t) - 2k(t)\alpha_0 - c_1(t)\varepsilon_0\right) dt \\
&\geq \frac{\alpha_0}{(m+1)^2} \exp(\alpha_0) > \frac{\alpha_0}{(m+1)^2},
\end{aligned}
\tag{3.51}
$$

which leads to a contradiction. Therefore, this contradiction shows that there exists constant $\eta_1 > 0$ such that

$$\liminf_{t \to \infty} x_1(t) > \eta_1, \tag{3.52}$$

for any positive solution $(x_1(t), x_2(t), u(t))$ of (1.3). Therefore, there exists constant $\eta_2 > 0$ such that

$$\liminf_{t \to \infty} x_2(t) > \eta_2, \tag{3.53}$$

for any positive solution $(x_1(t), x_2(t), u(t))$ of (1.3). This completes the proof. $\qquad\square$

Remark 3.3. In Theorem 3.2, we note that (H_1)–(H_3) are decided by (1.3), which is independent of the feedback controls. So, the feedback controls have no influence on the permanence of (1.3).

4. Extinction

In this section, we discuss the extinction of the component $x_1(t), x_2(t)$ of (1.3).

Theorem 4.1. *Suppose that there exist constants σ_1, σ_2, such that*

$$\limsup_{n \to \infty} \int_t^{t+\sigma_1} (b(s) - \mu(s)) ds < 0; \qquad \liminf_{t \to \infty} \int_t^{t+\sigma_2} k(s) ds > 0 \tag{4.1}$$

hold. Then,

$$\lim_{t \to \infty} x_1(t) = 0, \qquad \lim_{t \to \infty} x_2(t) = 0, \tag{4.2}$$

for any positive solution $(x_1(t), x_2(t), u(t))$ of (1.3).

Proof. By the condition, for every given positive constant ε, there exist constants ε_1 and T_1 such that

$$\int_t^{t+\sigma_1} (b(s) - \mu(s) - k(s)\varepsilon) ds < -\varepsilon_1, \tag{4.3}$$

for all $t > T_1$. First, we show that there exists a $T_2 > T_1$, such that $x_1(T_2) < \varepsilon$. Otherwise, we have

$$x_1(t) \geq \varepsilon, \quad \forall t > T_1. \tag{4.4}$$

Hence, for all $t \geq T_1$, one has

$$x_1'(t) < x_1(t)(b(t) - k(t)\varepsilon - \mu(t)). \tag{4.5}$$

Thus, as $t \rightarrow +\infty$, we have

$$\varepsilon \leq x_1(t) < x_1(T_1) \exp\left\{\int_{T_1}^{t} (b(s) - \mu(s) - k(s)\varepsilon)ds\right\}$$

$$< x_1(T_1) \exp\{-\varepsilon_1(t - T_1)\} \longrightarrow 0. \tag{4.6}$$

So, $\varepsilon < 0$, which leads to a contradiction. Therefore, there exists a $T_2 > T_1$, such that $x_1(T_2) < \varepsilon$.
Second, we show that

$$x_1(t) < \varepsilon \exp\{\mu_1\sigma_1\} \quad \forall t > T_2, \tag{4.7}$$

where $\mu_1 = \max_{t\in[0,+\infty]}\{b(t) + \mu(t) + k(t)\varepsilon\}$. Otherwise, there exists a $T_3 > T_2$, such that $x_1(T_3) > \varepsilon \exp\{\mu_1\sigma_1\}$. By the continuity of $x_1(t)$, there must exist $T_4 \in (T_2, T_3)$ such that $x_1(T_4) = \varepsilon$ and $x_1(t) > \varepsilon$ for $t \in (T_4, T_3)$. Let P_1 be the nonnegative integer such that $T_3 \in (T_4 + P_1\sigma_1, T_4 + (P_1 + 1)\sigma_1)$. Further, we obtain that

$$\varepsilon \exp\{\mu_1\sigma_1\} < x_1(T_3) < x_1(T_4) \exp\left\{\int_{T_4}^{T_3} (b(s) - \mu(s) - k(s)\varepsilon)ds\right\}$$

$$= x_1(T_4) \exp\left\{\int_{T_4}^{T_4+P_1\sigma_1} (b(s) - \mu(s) - k(s)\varepsilon)ds\right.$$

$$\left. + \int_{T_4+P_1\sigma_1}^{T_3} (b(s) - \mu(s) - k(s)\varepsilon)ds\right\} \tag{4.8}$$

$$< \varepsilon \exp\{\mu_1\sigma_1\},$$

which leads to contradiction. This shows that (4.7) holds. By the arbitrariness of ε, it immediately follows that $x_1(t) \rightarrow 0$ as $t \rightarrow +\infty$. Further, we can obtain that $x_2(t) \rightarrow 0$ as $t \rightarrow +\infty$. This completes the proof of Theorem 4.1. □

Acknowledgment

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