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Research Article

An Optimal Approach to Study the Nonlinear Behaviour of a Rotating Electrical Machine

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We introduce a novel analytical approximate technique, called Optimal Variational Method (OVM), to investigate the nonlinear behaviour of a rotating electrical machine modelled as an oscillator with cubic elastic restoring force and time variable coefficients. The proposed procedure involves the presence of some initially unknown convergence-control parameters whose values are later optimally determined. Comparisons between the obtained results and exact ones reveal that OVM is very effective and convenient providing highly accurate results.

1. Introduction

Mathematical modelling of many physical and engineering systems often leads to nonlinear ordinary or partial differential equations. In order to analyze such mathematical models, an effective method is required to provide solutions conforming to physical reality. In some cases, inherent difficulties are overcome by replacing a nonlinear differential equation with a corresponding linear differential equation that approximates the original one close enough to give useful results, but this is only a harsh approximation.

There are some known approaches intended to obtain approximate solutions to nonlinear dynamical systems. The most common and widely studied methods for solving nonlinear differential equations are the perturbation methods [1–3], but almost all perturbation methods are based on such an assumption that a small parameter must exist in the equation under investigation. The approximate solutions obtained through perturbation methods are valid, in most cases, only for small values of parameters, and moreover, there is no criterion on how small the parameters should be.

In order to overcome these limitations related to small parameters, several effective methods were developed, such as the method of Lie group [4], the Adomian decomposition method [5], the weighted-linearization method [6], and the method of harmonic balance [7].

The aim of this paper is to propose a novel variational approach to study the nonlinear behaviour of a rotating electrical machine modelled as an oscillator with cubic elastic force and time variable coefficients.

Rotating electrical machines are complex dynamical systems exhibiting typical problems from rotor dynamics, especially for high speeds or heavy duty rotor shafts with high inertial loads [8, 9]. An important engineering challenge in this field is to develop such mathematical models and methods, which allows quite accurate predictions of the dynamical phenomena at the stage of design, when compared against experimental measurements obtained after the stage of machine building. These models and methods should be made available in order to provide an easy-to-use tool to operate design modifications when these may be required to suppress undesired dynamic phenomena.

The most common sources of vibration in rotating machines are related to the unbalanced forces of the rotor, shaft misalignment and nonlinearity of the bearing stiffness, variable elasticity, bad bearings and mechanical looseness, and other electrical and mechanical faults which generate nonlinear vibration in the system.

These phenomena should be controlled in order to make the machine run smoothly and reliably. It is well known that rotating parts cannot be perfectly balanced. From engineering point of view, it is impossible to make any rotor perfectly mass balanced. Hence, residual unbalance is always present to some extent, even though the rotating structure is well constructed, but if it deteriorates, then damaging vibration occurs. Supplementary problems could arise in case of some horizontal rotating machines, when the gravity effect is not negligible for certain stiffness conditions. In case of gravity deflection, the shaft center leaves the bearing centerline, which leads to vibration occurrence. Misalignment could also occur in the electrical rotating machine after some amount of running. All these could be highly detrimental, affecting the integrity of the system [8, 9].

In this paper, a new analytical procedure, namely, the Optimal Variational Method (OVM) is employed to study the problem of nonlinear vibrations of an electric machine supported by nonlinear bearings characterized by nonlinear stiffness of Duffing type while the entire system is subject to a parametric excitation caused by an axial thrust and a forcing excitation caused by an unbalanced force of the rotor. In these conditions, the dynamical behaviour of the investigated electrical machine will be governed by the following second-order strongly nonlinear differential equation [8]:

$$m\ddot{u} + k_1(1 - q\sin\omega_2 t)u + k_2u^3 = f\sin\omega_1 t, \tag{1.1}$$

with the initial conditions

$$u(0) = A, \qquad \dot{u}(0) = 0, \tag{1.2}$$

which can be written in the more convenient form

$$\ddot{u} + \omega^2 u - \alpha u\sin\omega_2 t + \beta u^3 - \gamma\sin\omega_1 t = 0, \tag{1.3}$$

where

$$\omega^2 = \frac{k_1}{m}, \qquad \alpha = \frac{k_1 q}{m}, \qquad \beta = \frac{k_2}{m}, \qquad \gamma = \frac{f}{m}, \tag{1.4}$$

and the dot denotes derivative with respect to time and A is the initial amplitude of the oscillations. Note that it is unnecessary to assume the existence of any small or large parameters in (1.3).

Equation (1.3) describes a system oscillating with an unknown period T. We switch to a scalar time $\tau = \Omega t$. Under the transformation

$$\tau = \Omega t; \qquad u(\mathrm{t}) = Ax(\tau), \tag{1.5}$$

the original Equation (1.3) becomes

$$\Omega^2 x'' + \omega^2 x - \alpha x \sin\frac{\omega_2}{\Omega}\tau + \beta A^2 x^3 - \frac{\gamma}{A}\sin\frac{\omega_1}{\Omega}\tau = 0, \tag{1.6}$$

with the initial conditions

$$x(0) = 1, \qquad x'(0) = 0, \tag{1.7}$$

where the prime denotes derivative with respect to τ.

2. A Novel Variational Method and Solutions

In order to show the basics of OVM, we consider the following differential equation:

$$F(\tau, x, x', x'') = 0. \tag{2.1}$$

The variational principle for (2.1) can be easily established if there exists a functional

$$J = \int_{\tau_1}^{\tau_2} L(\tau, x, x')\, d\tau, \tag{2.2}$$

which admits as extremals the solutions of (2.1), where L is the Lagrangian of the system (2.1) and $[\tau_1, \tau_2]$ is the domain of interest.

This problem is based on the study of the conditions under which there exists a function $L(\tau, x, x')$ such that Euler's equation of the functional (2.2) coincide with the system (2.1), that is,

$$\frac{\partial L}{\partial x} - \frac{d}{d\tau}\left(\frac{\partial L}{\partial x'}\right) = F(\tau, x, x', x''). \tag{2.3}$$

On physical grounds, the primary significance of this problem, called "the inverse problem in Newtonian mechanics" [10] rests on the fact that the acting forces in Newtonian system (2.1) need not necessarily be derivable from a potential. Equation (2.2) is called action functional or action for short.

In our procedure, we assume that the approximate solution $\overline{x}$ of (2.1) depends on several parameters $C_1, C_2, \ldots, C_s$:

$$\overline{x} = \overline{x}(\tau, C_1, C_2, \ldots, C_s), \tag{2.4}$$

such that the action functional (2.2) becomes

$$J(C_1, C_2, \ldots, C_s) = \int_{\tau_1}^{\tau_2} L(\tau, \overline{x}(\tau, C_i), \overline{x}'(\tau, C_i)\, d\tau), \quad i = 1, 2, \ldots, s. \tag{2.5}$$

The parameters C_i from (2.5), called convergence-control parameters, can be determined optimally applying the Ritz method [8]:

$$\frac{\partial J}{\partial C_1} = \frac{\partial J}{\partial C_2} = \cdots = \frac{\partial J}{\partial C_s}. \tag{2.6}$$

From (2.6) and from the initial condition $(1.7)_1$ which becomes

$$\overline{x}(0, C_1, C_2, \ldots, C_s) = 1, \tag{2.7}$$

we can obtain optimally the parameters C_i, $i = 1, 2, \ldots, s$ and the frequency Ω of the system (2.1).

We remark that the condition $(1.7)_2$ is identically verified by the solution (2.4). On the other hand, the expression of the solution (2.4) is not unique.

The validity of the proposed approach is illustrated on (1.6). In this case, the Lagrangian of (1.6) can be written as

$$L(\tau, x, x') = -\frac{1}{2}\Omega^2 x'^2 + \frac{1}{2}\omega^2 x^2 - \frac{1}{2}\alpha x^2 \sin\frac{\omega_2}{\Omega}\tau + \frac{1}{4}\beta A^2 x^4 - \frac{\gamma}{A} x \sin\frac{\omega_1}{\Omega}\tau. \tag{2.8}$$

If we consider $s = 3$ in (2.4), then the approximate solution of (1.6) can be written as

$$\overline{x}(\tau) = C_1 \cos\tau + C_2 \cos\frac{\omega_1}{\Omega}\tau + C_3 \cos\frac{\omega_2}{\Omega}\tau. \tag{2.9}$$

Also, we can choose this approximate solution in the form

$$\overline{x}(\tau) = C_1 \cos\tau + C_2 \cos\frac{\omega_1 + \omega_2}{\Omega}\tau + C_3 \cos\frac{\omega_1 - \omega_2}{\Omega}\tau + C_4 \cos\frac{\Omega + \omega_1}{\Omega}\tau, \tag{2.10}$$

or

$$\overline{x}(\tau) = C_1 \cos \tau + C_2 \cos \frac{\Omega - \omega_1}{\Omega} + C_3 \cos \frac{\Omega + \omega_1}{\Omega}\tau + C_4 \cos \frac{\Omega - \omega_2}{\Omega}\tau + C_5 \cos \frac{\Omega + \omega_2}{\Omega}\tau, \tag{2.11}$$

and so on.

Substituting (2.9) into (2.8) and this into (2.10) we have the following results for $\tau_1 = 0$ and $\tau_2 = 2\pi$:

$$\begin{aligned} J(C_1, C_2, C_3) = {} & K_0 + K_1 \cos \frac{2\pi\omega_1}{\Omega} + K_2 \cos \frac{2\pi\omega_2}{\Omega} \\ & + K_3 \cos \frac{4\pi\omega_1}{\Omega} + K_4 \cos \frac{4\pi\omega_2}{\Omega} + K_5 \cos \frac{6\pi\omega_2}{\Omega} \\ & + K_6 \cos \frac{2\pi(\omega_1 - \omega_2)}{\Omega} + K_7 \cos \frac{2\pi(\omega_1 + \omega_2)}{\Omega} + K_8 \cos \frac{2\pi(2\omega_1 + \omega_2)}{\Omega} \\ & + K_9 \cos \frac{2\pi(2\omega_1 - \omega_2)}{\Omega} + K_{10} \cos \frac{2\pi(\omega_1 + 2\omega_2)}{\Omega} + K_{11} \cos \frac{2\pi(\omega_1 - 2\omega_2)}{\Omega} \\ & + K_{12} \sin \frac{2\pi\omega_1}{\Omega} + K_{13} \sin \frac{2\pi\omega_2}{\Omega} + K_{14} \sin \frac{4\pi\omega_1}{\Omega} + K_{15} \sin \frac{4\pi\omega_2}{\Omega} \\ & + K_{16} \sin \frac{6\pi\omega_1}{\Omega} + K_{17} \sin \frac{6\pi\omega_2}{\Omega} + K_{18} \sin \frac{8\pi\omega_1}{\Omega} + K_{19} \sin \frac{8\pi\omega_2}{\Omega} \\ & + K_{20} \sin \frac{2\pi(\omega_1 + \omega_2)}{\Omega} + K_{21} \sin \frac{2\pi(\omega_1 - \omega_2)}{\Omega} + K_{22} \sin \frac{4\pi(\omega_1 + \omega_2)}{\Omega} \\ & + K_{23} \sin \frac{4\pi(\omega_1 - \omega_2)}{\Omega} + K_{24} \sin \frac{2\pi(2\omega_1 + \omega_2)}{\Omega} + K_{25} \sin \frac{2\pi(2\omega_1 - \omega_2)}{\Omega} \\ & + K_{26} \sin \frac{2\pi(3\omega_1 + \omega_2)}{\Omega} + K_{27} \sin \frac{2\pi(3\omega_1 - \omega_2)}{\Omega} + K_{28} \sin \frac{2\pi(\omega_1 + 2\omega_2)}{\Omega} \\ & + K_{29} \sin \frac{2\pi(\omega_1 - 2\omega_2)}{\Omega} + K_{30} \sin \frac{2\pi(\omega_1 + 3\omega_2)}{\Omega} + K_{31} \sin \frac{2\pi(\omega_1 - 3\omega_2)}{\Omega}, \end{aligned} \tag{2.12}$$

where

$$\begin{aligned} K_0 = {} & \frac{\pi}{2}\left(\omega^2 - \Omega^2\right)\left(C_1^2 + C_2^2 + C_3^2\right) + \frac{\alpha\Omega}{12\omega_2}\left(3C_1^2 + 3C_2^2 + 2C_3^2\right) - \frac{\alpha\Omega\omega_2 C_1^2}{4\left(4\Omega^2 - \omega_2^2\right)} \\ & + \frac{\alpha\Omega\omega_2 C_2^2}{8\left(4\omega_1^2 - \omega_2^2\right)} + \frac{\alpha\Omega(\omega_1 - \omega_2)C_1C_2}{2\left[\Omega^2 - (\omega_1 - \omega_2)^2\right]} + \frac{\alpha\Omega(\omega_1 + \omega_2)C_1C_2}{2\left[\Omega^2 - (\omega_1 + \omega_2)^2\right]} - \frac{\alpha\Omega\omega_2 C_1C_3}{\Omega^2 - 4\omega_2^2} \\ & + \frac{\alpha\Omega C_2C_3}{4(2\omega_2 - \omega_1)} + \frac{\alpha\Omega C_2C_3}{4(2\omega_2 + \omega_1)} - \frac{\gamma\Omega\omega_1 C_1}{A\left(\Omega^2 - \omega_1^2\right)} + \frac{\gamma\Omega C_2}{4A\omega_1} + \frac{\gamma\Omega C_3}{2A(\omega_1 - \omega_2)} \\ & + \frac{\gamma\Omega C_3}{2A(\omega_1 + \omega_2)} + \frac{3\pi\beta A^2}{16}\left(C_1^4 + C_2^4 + C_3^4 + 4C_1^2C_2^2 + 4C_1^2C_3^2 + 4C_2^2C_3^2\right), \end{aligned}$$

$$K_1 = \frac{\gamma\Omega\omega_1 C_1}{A(\Omega^2-\omega_1^2)}, \qquad K_2 = \frac{\alpha\Omega\omega_2 C_1^2}{4(4\Omega^2-\omega_2^2)} - \frac{\alpha\Omega(C_1^2+C_2^2)}{4\omega_2}, \qquad K_3 = -\frac{\lambda\Omega C_2}{4A\omega_1},$$

$$K_4 = \frac{\alpha\Omega\omega_2 C_1 C_3}{\Omega^2-4\omega_2^2}, \qquad K_5 = -\frac{\alpha\Omega C_3^2}{24\omega_2}, \qquad K_6 = \frac{\alpha\Omega(\omega_2-\omega_1)C_1C_2}{2\left[\Omega^2-(\omega_2-\omega_1)^2\right]} - \frac{\gamma\Omega C_3}{2A(\omega_1-\omega_2)},$$

$$K_7 = -\frac{\alpha\Omega(\omega_1+\omega_2)C_1C_2}{2\left[\Omega^2-(\omega_1+\omega_2)^2\right]} - \frac{\gamma\Omega C_3}{2A(\omega_1+\omega_2)}, \qquad K_8 = -\frac{\alpha\Omega C_2^2}{8(2\omega_1+\omega_2)}, \qquad K_9 = \frac{\alpha\Omega C_2^2}{8(2\omega_1-\omega_2)},$$

$$K_{10} = -\frac{\alpha\Omega C_2 C_3}{4(\omega_1+2\omega_2)}, \qquad K_{11} = -\frac{\alpha\Omega C_2 C_3}{4(2\omega_2-\omega_1)},$$

$$K_{12} = \frac{\Omega\omega_1(\Omega\omega_1+\omega^2)C_1C_2}{\Omega^2-\omega_1^2} + \frac{\beta A^2}{8}\left[\frac{\Omega^2 C_1^3 C_2 + \Omega C_1 C_2\omega_1(2C_1^2+3C_2^2+10C_3^2)}{\Omega^2-\omega_1^2} + \frac{3\Omega^2 C_1^3 C_2}{9\Omega^2-\omega_1^2}\right],$$

$$K_{13} = \frac{\Omega\omega_2(\Omega\omega_2+\omega^2)C_1C_3}{\Omega^2-\omega_2^2} + \frac{\beta A^2}{8}\left[\frac{\Omega^2 C_1^3 C_3 + \Omega C_1 C_3\omega_2(2C_1^2+10C_2^2+3C_3^2)}{\Omega^2-\omega_2^2} + \frac{3\Omega^2 C_1^3 C_3}{9\Omega^2-\omega_2^2}\right],$$

$$K_{14} = -\frac{(\omega^2+\omega_1^2)\Omega C_2^2}{8\omega_1} + \frac{\beta A^2}{32}\left[\frac{5\Omega\omega_1 C_1^2 C_2^2}{\Omega^2-\omega_1^2} - \frac{2\Omega C_2^2(3C_1^2+C_2^2+3C_3^2)}{\omega_1}\right],$$

$$K_{15} = -\frac{(\omega^2+\omega_2^2)\Omega C_3^2}{8\omega_2} + \frac{\beta A^2}{32}\left[\frac{5\Omega\omega_2 C_1^2 C_3^2}{\Omega^2-\omega_2^2} - \frac{2\Omega C_3^2(3C_1^2+3C_2^2+C_3^2)}{\omega_2}\right],$$

$$K_{16} = \frac{3\beta A^2\Omega\omega_1 C_1 C_2^3}{8(\Omega^2-9\omega_1^2)}, \qquad K_{17} = \frac{3\beta A^2\Omega\omega_2 C_1 C_3^3}{8(\Omega^2-9\omega_2^2)}, \qquad K_{18} = \frac{\beta A^2\Omega C_2^4}{128\omega_1}, \qquad K_{19} = \frac{\beta A^2\Omega C_3^4}{128\omega_2},$$

$$K_{20} = -\frac{(\omega^2+\omega_1\omega_2)\Omega C_2C_3}{2(\omega_1+\omega_2)} + \frac{\beta A^2}{4}\left[\frac{3\Omega(\omega_1+\omega_2)C_1^2C_2C_3}{4\Omega^2-(\omega_1+\omega_2)^2} - \frac{3\Omega C_2C_3(2C_1^2+C_2^2+C_3^2)}{2(\omega_1+\omega_2)}\right],$$

$$K_{21} = -\frac{(\omega^2+\omega_1\omega_2)\Omega C_2C_3}{2(\omega_1-\omega_2)} + \frac{\beta A^2}{4}\left[\frac{3\Omega(\omega_1-\omega_2)C_1^2C_2C_3}{4\Omega^2-(\omega_1-\omega_2)^2} - \frac{3\Omega C_2C_3(2C_1^2+C_2^2+C_3^2)}{2(\omega_1-\omega_2)}\right],$$

$$K_{22} = -\frac{3\beta A^2\Omega C_2^2 C_3^2}{32(\omega_1+\omega_2)}, \qquad K_{23} = -\frac{3\beta A^2\Omega C_2^2 C_3^2}{32(\omega_1-\omega_2)}, \qquad K_{24} = \frac{3\beta A^2\Omega(2\omega_1+\omega_2)C_1C_2^2C_3}{8\left[\Omega^2-(2\omega_1+\omega_2)^2\right]},$$

$$K_{25} = \frac{3\beta A^2\Omega(2\omega_1-\omega_2)C_1C_2^2C_3}{8\left[\Omega^2-(2\omega_1-\omega_2)^2\right]}, \qquad K_{26} = -\frac{\beta A^2\Omega C_2^3 C_3}{8(3\omega_1+\omega_2)}, \qquad K_{27} = -\frac{\beta A^2\Omega C_2^3 C_3}{8(3\omega_1-\omega_2)},$$

$$K_{28} = \frac{3\beta A^2\Omega(\omega_1+2\omega_2)C_1C_2C_3^2}{8\left(\Omega^2-(\omega_1+2\omega_2)^2\right]}, \qquad K_{29} = \frac{3\beta A^2\Omega(\omega_1-2\omega_2)C_1C_2C_3^2}{8\left(\Omega^2-(\omega_1-2\omega_2)^2\right]},$$

$$K_{30} = -\frac{\beta A^2\Omega C_2 C_3^3}{8(\omega_1+3\omega_2)}, \qquad K_{31} = -\frac{\beta A^2\Omega C_2 C_3^3}{8(3\omega_2-\omega_1)}. \tag{2.13}$$

The values of the parameters C_1, C_2, C_3 and the frequency Ω are obtained from (2.6) which become:

$$\frac{\partial J}{\partial C_1} = \frac{\partial J}{\partial C_2} = \frac{\partial J}{\partial C_3} = 0, \tag{2.14}$$

and from the initial condition (2.7) which can be written as

$$C_1 + C_2 + C_3 = 1. \tag{2.15}$$

3. Test Examples

The validity of the proposed procedure for solving the investigated problem is illustrated on three examples, considering different parameters and initial amplitudes.

Case a. As a first example, we consider the following set of parameters:

$$\begin{aligned} &A = 1, \quad m = 2, \quad k_1 = 200, \quad k_2 = 1000, \\ &q = 0.1, \quad \omega_1 = 2.1, \quad \omega_2 = 1.1, \quad f = 6, \end{aligned} \tag{3.1}$$

or for the corresponding coefficients of (1.6):

$$\omega = 10, \quad \alpha = 10, \quad \beta = 500, \quad \gamma = 3. \tag{3.2}$$

The parameters C_1, C_2, C_3 and the frequency Ω are obtained from (2.14) and (2.15):

$$C_1 = 0.981045, \quad C_2 = 0.00376699, \quad C_3 = 0.0151884, \quad \Omega = 21.5167. \tag{3.3}$$

The approximate solution of (1.1) in this case becomes

$$u(t) = 0.981045 \cos \Omega t + 0.00376699 \cos \omega_1 t + 0.0151884 \cos \omega_2 t. \tag{3.4}$$

Figure 1 shows the comparison between the present solution (3.4) and numerical integration results obtained using a fourth-order Runge-Kutta scheme.

Case b. As a second example, we consider the following parameters:

$$A = 0.8, \quad \omega_1 = 2.7, \quad \omega_2 = 1.0. \tag{3.5}$$

Keeping the rest of the coefficients unchanged and following the same procedure, we obtain the optimal values of the convergence-control parameters C_1, C_2, C_3 and the frequency Ω:

$$C_1 = 0.966202, \quad C_2 = 0.00106843, \quad C_3 = 0.0327299, \quad \Omega = 18.0654, \tag{3.6}$$

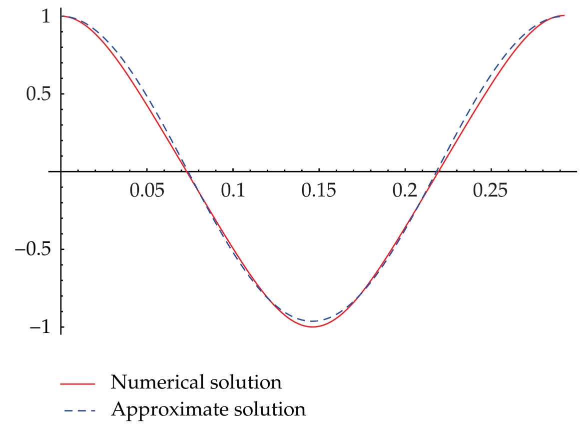

Figure 1: Comparison between the numerical solution of (1.1) and the approximate solution (3.4) for $A = 1$, $m = 2$, $k_1 = 200$, $k_2 = 1000$, $q = 0.1$, $\omega_1 = 2.1$, $\omega_2 = 1.1$, $f = 6$.

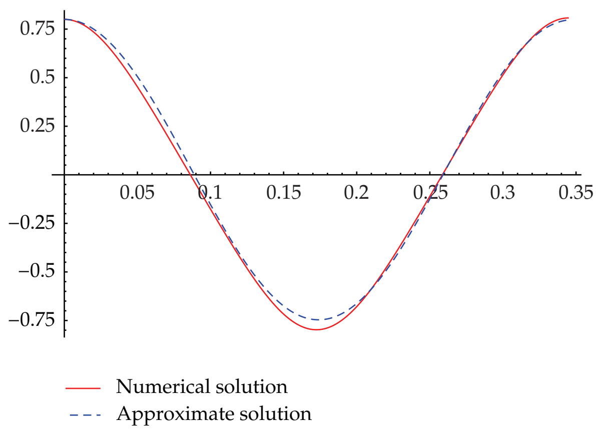

Figure 2: Comparison between the numerical solution of (1.1) and the approximate solution (3.7) for $A = 0.8$, $m = 2$, $k_1 = 200$, $k_2 = 1000$, $q = 0.1$, $\omega_1 = 2.7$, $\omega_2 = 1.0$, $f = 6$.

and consequently, the approximate solution of (1.1) becomes in this case:

$$u(t) = 0.772961 \cos \Omega t + 0.000854743 \cos \omega_1 t + 0.0261839 \cos \omega_2 t. \tag{3.7}$$

The comparison between the present solution (3.7) and numerical integration results in the second case is presented in Figure 2.

Case c. Finally, for the last example we consider the following set of parameters:

$$A = 2, \qquad \omega_1 = 2.5, \qquad \omega_2 = 1.5. \tag{3.8}$$

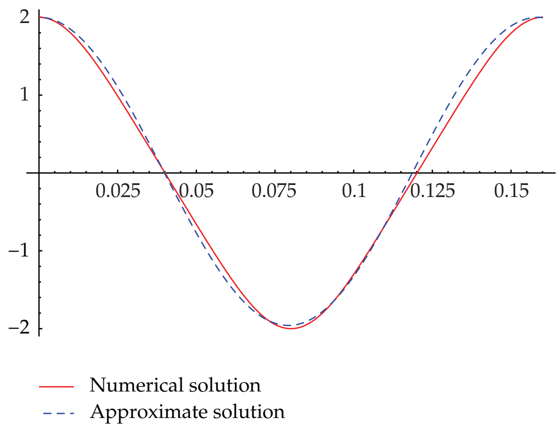

Figure 3: Comparison between the numerical solution of (1.1) and the approximate solution (3.10) for $A = 2$, $m = 2$, $k_1 = 200$, $k_2 = 1000$, $q = 0.1$, $\omega_1 = 2.5$, $\omega_2 = 1.5$, $f = 6$.

In this case, following the procedure described above, we obtain

$$C_1 = 0.989719, \qquad C_2 = 0.00281954, \qquad C_3 = 0.00746148, \qquad \Omega = 39.6388, \tag{3.9}$$

and the approximate solution of (1.1) becomes

$$u(t) = 1.97944 \cos \Omega t + 0.00563909 \cos \omega_1 t + 0.014923 \cos \omega_2 t. \tag{3.10}$$

The comparison between the obtained approximate analytical solution (3.10) and numerical integration results in the third case is presented in Figure 3.

It can be observed from Figures 1–3 that the approximate analytical results obtained using the proposed procedures are in good agreement with the numerical ones.

4. Conclusions

In this work, an optimal variational approach is employed to propose a new analytic approximate solution for nonlinear conservative oscillations of an electrical machine. The construction of our variational approach is different from traditional approach especially concerning the involvement of some initially unknown parameters C_i called convergence-control parameters, whose optimal values ensure a fast convergence of the approximate analytical solutions. This is in fact the main advantage of the OVM, which provides us with a simple and rigorous way to control and adjust the convergence of the solutions. The proposed procedure is valid even if the nonlinear equation does not contain any small or large parameters. Three test examples illustrate that the proposed analytical approach is very effective and yields accurate results comparing to those obtained via numerical integration using a fourth-order Runge-Kutta method.

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