Research Article

Stability Analysis for Markovian Jump Neutral Systems with Mixed Delays and Partially Known Transition Rates

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The delay-dependent stability problem is studied for Markovian jump neutral systems with partial information on transition probabilities, and the considered delays are mixed and model dependent. By constructing the new stochastic Lyapunov-Krasovskii functional, which combined the introduced free matrices with the analysis technique of matrix inequalities, a sufficient condition for the systems with fully known transition rates is firstly established. Then, making full use of the transition rate matrix, the results are obtained for the other case, and the uncertain neutral Markovian jump system with incomplete transition rates is also considered. Finally, to show the validity of the obtained results, three numerical examples are provided.

1. Introduction

A switched system is a dynamic system consisted of a number of subsystems and a rule that manages the switching between these subsystems. In the past, a large number of excellent papers and monographs on the stability of switched systems have been published such as [1–7] and the references cited therein. Among the results for switched systems, the stabilization problem of switched neutral systems has also been explored by some researchers [8–22], and mainly two kinds of switching rule are designed in these articles. Some state-dependent switching rules are obtained assuming the convex combination of the systems matrix, see, for example, [8, 10, 20]. To reduce the conservative, the authors in [11] have investigated the stabilization for switched neutral systems without the assumption that the restraint of
the convex combination on systems matrices, the dwell time, and state-dependent rules were
designed. Similarly, the authors in [12] have also studied the problem of the BIBO stability
for switched neutral systems. Using the Razumikhin-like approach [13] and the Leibniz-
Newton formula, the global exponential stability for a class of switched neutral systems with
interval-time-varying state delay and two classes of perturbations is investigated in [14], with
arbitrary switching signal. Moreover, with the constructed state-dependent switching rule,
the authors in [15] have investigated the global exponential stability of switched neutral
systems. With the dwell time approach, the improved stability conditions for a class of
switched neutral systems with mixed time-varying delays have also been obtained in [16],
and the upper bound of derivative of the discrete time-varying delay is not restricted to
one. In [17], the robust reliable stabilization of uncertain switched neutral systems with
delayed switching has been considered. The $H_\infty$ fault estimator for switched nonlinear
systems of neutral type has been designed in [9]. In [18], the authors have studied the
problem of exponential stability for neutral switched systems with interval time-varying
mixed delays and nonlinear perturbations, obtaining the less conservative conditions based
on the introduced free matrices. More recently, the markovian jump parameters have been
considered for the analysis of switched neutral systems in [19].

In the past few decades, as a special switched system, markovian jump systems (MJSs)
have been widely studied due to the fact that many dynamical systems subject to random
abrupt variations can be modeled by MJSs such as manufacturing systems, networked control
systems, and fault-tolerant control systems. There are a lot of useful results that have been
presented in the literature, such as [23–29], and the references therein. For the analysis of
MJSs, the transition probabilities in the jumping process determine the system behavior to
a large extent. However, the likelihood of obtaining such available knowledge is actually
questionable, and the cost is probably expensive. Rather than having a large complexity
to measure or estimate all the transition probabilities, it is significant and necessary, from
control perspectives, to further study more general jump systems with partly unknown
transition probabilities. Recently, many results on the Markovian jump systems with partly
unknown transition probabilities are obtained [30–37]. By introducing the free matrices
based on the property of transition rate matrix, [32] gave less conservative conditions than
that in [30] for Markovian jump systems with partial information on transition probability.
And [33] provided with a new approach to obtain the necessary and sufficient conditions
for markovian jump linear systems with incomplete transition probabilities, which may be
appropriate to discuss the counterpart of delay systems. Most of these improved results
just require some free matrices or the knowledge of the known elements in transition rate
matrix, such as the bounds or structures of uncertainties, and some else of the unknown
elements need not be considered. It is a great progress on the analysis of markovian
jump systems. However, a few of these papers have considered the effect of delay on the
stability or stabilization for the corresponding neutral systems, except for [19]. The global
exponential stability of the markovian jumping neutral systems with interval time-varying
delays has been studied by [19]; however, the transition probabilities are fully known, and
the constructed Lyapunov did not fully consider the effect of the transition probabilities on
the integrand. To the best of the authors’ knowledge, the markovian jump neutral systems
have not been fully investigated, and it is very challenging. All of these motivate this paper.

In this paper, the delay-dependent stability problem of neutral Markovian jump linear
systems with partly unknown transition probabilities is investigated. The obtained results
are presented in the form of linear matrix inequalities, which is easily computed by the
Matlab toolbox. The considered systems are more general than the systems with completely
known or completely unknown transition probabilities, which can be viewed as two special cases of the ones tackled here. Moreover, in contrast with the recent research on uncertain transition probabilities, our proposed concept of the partly unknown transition probabilities does not require any knowledge of the unknown elements, such as the bounds or structures of uncertainties. In addition, the relationship between the stability criteria currently obtained for the usual MJLS and switched linear system under arbitrary switching is exposed by our proposed systems. Furthermore, the number of matrix inequalities conditions obtained in this paper is much more than some existing results due to the introduced free matrices based on the system itself and the information of transition probabilities in this paper, which may increase the complexity of computation. However, it would decrease the conservativeness for the delay-dependent stability conditions. Finally, two numerical examples are provided to illustrate the effectiveness of our results.

2. Problem Statement and Preliminaries

Consider the following neutral system with markovian jump parameters:

$$
\dot{x}(t) - C(r_t)x(t - \tau_2(r_t)) = A(r_t)x(t) + B(r_t)x(t - \tau_1(r_t)),
$$

$$
x(t_0 + \theta) = \varphi(\theta), \quad \forall \theta \in [-\tau, 0],
$$

where \(\{r_t\}, \ t \geq 0\) is a right-continuous Markov process on the probability space taking values in a finite state space, \(\wp = \{1, 2, \ldots, N\}\) with generator \(\Gamma = (\lambda_{ij}), i, j \in \wp\) given by

$$
\Pr\{r_{t+\Delta} = j \mid r_t = i\} = \begin{cases} 
\lambda_{ij}\Delta + o(\Delta), & j \neq i, \\
1 + \lambda_{ii}\Delta + o(\Delta), & j = i,
\end{cases}
$$

where \(\Delta > 0, \lim_{\Delta \to 0} (o(\Delta)/\Delta) = 0, \lambda_{ij} \geq 0, \) for \(j \neq i,\) is the transition rate from mode \(i\) at time \(t\) to mode \(j\) at time \(t + \Delta,\) \(\lambda_{ij} = -\sum_{j=1}^{N} \lambda_{ij}.\) \(A(r_t), B(r_t),\) and \(C(r_t)\) are known matrix functions of the markovian process, \(x(t) \in \mathbb{R}^n\) is the state vector, and \(\varphi(\theta)\) is the initial condition function. \(\tau_1(r_t)\) and \(\tau_2(r_t)\) are mode-dependent delays, when \(r_t = i \in \wp, \tau_1(r_t) = \tau_{i1}, \tau_2(r_t) = \tau_{i2},\) and \(\tau = \max(\tau_{i1}, \tau_{i2}).\)

Since the transition probability depends on the transition rates for the continuous-time MJSSs, the transition rates of the jumping process are considered to be partly accessible in this paper. For instance, the transition rate matrix \(\Gamma\) with \(N\) operation modes may be expressed as

$$
\Gamma = \begin{pmatrix}
\lambda_{11} & ? & \cdots & ? \\
? & \lambda_{13} & \cdots & ? \\
\vdots & \vdots & \ddots & \vdots \\
? & \lambda_{N2} & \cdots & \lambda_{NN}
\end{pmatrix},
$$

where \(\?\) represents the unknown transition rate. For notational clarity, for all \(i \in \wp,\) the set \(U^i\) denotes \(U^i = U^i_k \cup U^i_{uk}\) with \(U^i_k \triangleq \{j : \lambda_{ij} \text{ is known for } j \in \wp\}, U^i_{uk} \triangleq \{j : \lambda_{ij} \text{ is unknown for } j \in \wp\},\) and \(\lambda_{ij}^i = \sum_{j \in U^i_k} \lambda_{ij}.\)
Moreover, if \( U^i_k \neq \emptyset \), it is further described as
\[
U^i_k = \{ k^i_1, k^i_2, \ldots, k^i_m \},
\]
(2.4)
where \( m \) is a nonnegative integer with \( 1 \leq m \leq N \), and \( k^i_j \in \mathbb{Z}^+ \), \( 1 \leq k^i_j \leq N \) \( j = 1, 2, \ldots, m \) represent the \( j \)th known element of the set \( U^i_k \) in the \( i \)th row of the transition rate matrix \( \mathbb{I} \).

**Remark 2.1.** It is worthwhile to note that if \( U^i_k = \emptyset \), \( U^i = U^i_{uk} \) which means that any information between the \( i \)th mode and the other \( N - 1 \) modes is not accessible, then MJJs with \( N \) modes can be regarded as ones with \( N - 1 \) modes. It is clear that when \( U^i_{uk} = \emptyset \), \( U^i = U^i_k \), the system (2.1) becomes the usual assumption case.

For the sake of simplicity, the solution \( x(t, \varphi(\theta), r_0) \) with \( r_0 \in \mathcal{P} \) is denoted by \( x(t) \). It is known from [38] that \( \{x(t), t\} \) is a Markov process with an initial state \( \{\varphi(\theta), r_0\} \), and its weak infinitesimal generator, acting on function \( V \), is defined in [39]:
\[
LV(x(t), t, i) = \lim_{\Delta \to 0^+} \frac{1}{\Delta} \left[ V(x(t + \Delta), t + \Delta, r_{t+\Delta}) - V(x(t), r_t = i) \right].
\]
(2.5)

Throughout this paper, the following definition is necessary. More details refer to [23].

**Definition 2.2 (see, [32]).** The system (2.1) is said to be stochastically stable if the following holds:
\[
\epsilon \left\{ \int_0^\infty \|x(t)\|^2 dt \mid \varphi, r_0 \right\} < \infty,
\]
(2.6)
for every initial condition \( \varphi \in \mathbb{R}^n \) and \( r_0 \in \mathcal{P} \).

### 3. Stability Analysis for Neutral Markovian Jump Systems

The purpose of this section is to state the stability analysis for neutral markovian jump systems with partly unknown transition rates. Throughout the paper, the matrix \( C(r_t) \) is supposed to be \( \rho(C(r_t)) < 1 \). Before giving the stability result of systems (2.1) with a partly unknown transition rate matrix (2.3), the stability of neutral markovian jump systems (2.1) with all transition probabilities known is firstly investigated. With the introduced free matrices and the novel analysis technique of matrix, the stability conditions are presented in this section.
Theorem 3.1. The system (2.1) with a fully known transition rate matrix is stochastically stable if there exist matrices $P_i > 0$, $Q_{1i} > 0$, $Q_{2i} > 0$, $R_1 > 0$, and $R_2 > 0$ and any matrices $N_1$, $N_2$, $N_3$, and $N_4$ with appropriate dimensions satisfying the following linear matrix inequalities:

$$
\varphi_i = \begin{pmatrix}
\varphi_{i11} & \varphi_{i12} & \varphi_{i13} & \varphi_{i14} \\
\ast & \varphi_{i22} & \varphi_{i23} & \varphi_{i24} \\
\ast & \ast & \varphi_{i33} & \varphi_{i34} \\
\ast & \ast & \ast & \varphi_{i44}
\end{pmatrix} < 0,
$$

$$
\sum_{j=1}^N \lambda_{ij} Q_{1j} + \sum_{j=1}^N \lambda_{ij} \tau_{1j} R_1 - R_1 \leq 0,
$$

$$
\sum_{j=1}^N \lambda_{ij} Q_{2j} + \sum_{j=1}^N \lambda_{ij} \tau_{2j} R_2 - R_2 \leq 0,
$$

with

$$
\varphi_{i11} = P_i A_i + A_i^T P_i + \sum_{j=1}^N \lambda_{ij} P_j + Q_{i1} + N_1 A_i + A_i^T N_1^T + \tau_{1i} R_1,
$$

$$
\varphi_{i12} = -N_1 + A_i^T N_2^T,
$$

$$
\varphi_{i13} = N_1 B_i + A_i^T N_3^T + P_i B_i,
$$

$$
\varphi_{i14} = N_1 C_i + A_i^T N_4^T + P_i C_i,
$$

$$
\varphi_{i22} = -N_2 - N_2^T + Q_{2i} + \tau_{2i} R_2,
$$

$$
\varphi_{i23} = N_2 B_i - N_3^T,
$$

$$
\varphi_{i24} = N_2 C_i - N_4^T,
$$

$$
\varphi_{i33} = N_3 B_i + B_i^T N_3^T - Q_{i1} + \sum_{j=1}^N \lambda_{ij} \tau_{1j} Q_{1j},
$$

$$
\varphi_{i34} = N_3 C_i + B_i^T N_4^T,
$$

$$
\varphi_{i44} = N_4 C_i + C_i^T N_4^T - Q_{2i} + \sum_{j=1}^N \lambda_{ij} \tau_{2j} Q_{2j}.
$$

Proof. Construct a stochastic Lyapunov functional candidate as

$$
V(x_i, t, r_i) = \sum_{i=1}^5 V_i(x_i, t, r_i),
$$
where

\[ V_1(x_t, t, r_i) = x^T(t)P(r_i)x(t), \]
\[ V_2(x_t, t, r_i) = \int_{t-r_i}^{t} x^T(s)Q_1(r_i)x(s)ds, \]
\[ V_3(x_t, t, r_i) = \int_{t-r_i}^{t} \hat{x}^T(s)Q_2(r_i)\hat{x}(s)ds, \]
\[ V_4(x_t, t, r_i) = \int_{-r_i}^{0} \int_{t+\theta}^{t} x^T(s)R_1x(s)ds d\theta, \]
\[ V_5(x_t, t, r_i) = \int_{-r_i}^{0} \int_{t+\theta}^{t} \hat{x}^T(s)R_2\hat{x}(s)ds d\theta, \]

(3.6)

where \( P(r_i), Q_1(r_i), Q_2(r_i), R_1, R_2, \) and \( r_i \in \mathbb{R} \) are all positive definite matrices with appropriate dimensions to be determined. Then, for given \( r_i = i \in \mathbb{R} \), \( P(r_i) = P_i, Q_1(r_i) = Q_{1i}, Q_2(r_i) = Q_{2i} \), and the weak infinitesimal operator \( L \) of the stochastic process \( x(t) \) along the evolution of \( V_k(x_t, t, i)(k = 1, \ldots, 7) \) are given as

\[ LV_1(x_t, t, i) = 2x^T(t)P_1\dot{x}(t) + x^T(t)\sum_{j=1}^{N} \lambda_{ij}P_jx(t) \]
\[ = 2x^T(t)P_1(A_1x(t) + B_1x(t - \tau_{1i}) + C_1\dot{x}(t - \tau_{2i})) + x^T(t)\sum_{j=1}^{N} \lambda_{ij}P_jx(t). \]

(3.7)

According to the definition of the weak infinitesimal operator \( L \) and the expression (2.2), it can be shown that

\[ LV_2(x_t, t, i) \]
\[ = \lim_{\Delta \to 0^+} \frac{1}{\Delta} \left[ \varepsilon \left( \int_{t+\Delta-\tau_i}^{t+\Delta} x^T(s)Q_1(r_{i+\Delta})x(s)ds \right) |x(t), r_i = i \right] - V_2(x(t), t, i) \]
\[ = \lim_{\Delta \to 0^+} \frac{1}{\Delta} \left[ \int_{t+\Delta-\tau_i}^{t+\Delta} x^T(s)N \sum_{j=1}^{N} \Pr[r_{i+\Delta} = j | r_i = i]Q_{1j}x(s)ds \right] - V_2(x(t), t, i) \]
\[ = \lim_{\Delta \to 0^+} \frac{1}{\Delta} \left[ \int_{t+\Delta-\tau_i}^{t+\Delta} x^T(s)\left[ Q_{1i} + \sum_{j=1}^{N} (\lambda_{ij}\Delta + O(\Delta))Q_{1j} \right] x(s)ds \right. \]
\[ - \left. \int_{t-\tau_i}^{t} x^T(s)Q_{1i}x(s)ds \right] \]
\[ = \lim_{\Delta \to 0^+} \frac{1}{\Delta} \left[ \int_{t+\Delta-\tau_i}^{t+\Delta} x^T(s)\left[ Q_{1i} + \sum_{j=1}^{N} (\lambda_{ij}\Delta + O(\Delta))Q_{1j} \right] x(s)ds \right. \]
\[ - \left. \int_{t-\tau_i}^{t} x^T(s)Q_{1i}x(s)ds - \int_{t-\tau_i}^{t} x^T(s)Q_{1i}x(s)ds \right] \]
\[
+ \lim_{\Delta \to 0^+} \frac{1}{\Delta} \left[ \int_{t}^{t+\Delta} x^T(s) \left( \sum_{j=1}^{N} (\lambda_{ij} \Delta + O(\Delta)) Q_{ij} \right) x(s) ds \right] \\
= \lim_{\Delta \to 0^+} \frac{1}{\Delta} \int_{t}^{\tau_{l}} x^T(s) Q_{1i} x(s) ds \\
+ \lim_{\Delta \to 0^+} \frac{1}{\Delta} \int_{t+\Delta-\tau_{l}-\sum_{j=1}^{N} (\lambda_{ij} \Delta + O(\Delta)) \tau_{j}}^{t-\tau_{l}} x^T(s) Q_{1i} x(s) ds \\
+ \lim_{\Delta \to 0^+} \frac{1}{\Delta} \int_{t+\Delta-\tau_{l}-\sum_{j=1}^{N} (\lambda_{ij} \Delta + O(\Delta)) \tau_{j}}^{t+\Delta} x^T(s) \sum_{j=1}^{N} (\lambda_{ij} \Delta + O(\Delta)) Q_{1j} x(s) ds \\
= x^T(t) Q_{1i} x(t) - \left( 1 - \sum_{j=1}^{N} \lambda_{ij} \tau_{ij} \right) x^T(t - \tau_{l}) Q_{1i} x(t - \tau_{l}) \\
+ \int_{t}^{t+\Delta} x^T(s) \sum_{j=1}^{N} \lambda_{ij} Q_{1j} x(s) ds, \\
\]

LV_4(x_t, t, i) \\
= \lim_{\Delta \to 0^+} \frac{1}{\Delta} \left[ \varepsilon \left( \int_{t}^{t+\Delta} x^T(s) R_1 x(s) ds \right) x(t), r_l = i \right] - \int_{t-\tau_{l}}^{t} \int_{t+\Delta-\theta}^{t+\Delta} x^T(s) R_1 x(s) ds \\
= \lim_{\Delta \to 0^+} \frac{1}{\Delta} \left[ \varepsilon \left( \int_{t}^{t+\Delta} x^T(s) R_1 x(s) ds \right) x(t), r_l = i \right] \\
- \int_{t-\tau_{l}}^{t} x^T(s) R_1 x(s) ds \\
= \lim_{\Delta \to 0^+} \frac{1}{\Delta} \left[ \varepsilon \left( \int_{t}^{t+\Delta} x^T(s) R_1 x(s) ds \right) x(t), r_l = i \right] \\
+ \int_{t-\tau_{l}}^{t+\Delta} x^T(s) R_1 x(s) ds - \int_{t-\tau_{l}}^{t} \int_{t+\Delta-\theta}^{t+\Delta} x^T(s) R_1 x(s) ds \\
= \tau_{l} x^T(t) R_1 x(t) - \int_{t-\tau_{l}}^{t} x^T(s) R_1 x(s) ds + \int_{t-\tau_{l}}^{t+\Delta} x^T(s) \sum_{j=1}^{N} \lambda_{ij} \tau_{ij} R_1 x(s) ds. \\
(3.8)

Similar to the above, we can obtain

\[
LV_5(x_t, t, i) = x^T(t) Q_{2i} x(t) - \left( 1 - \sum_{j=1}^{N} \lambda_{ij} \tau_{2j} \right) x^T(t - \tau_{2i}) Q_{2i} x(t - \tau_{2i})
\]
Moreover, there exist matrices \( N_k (k = 1, \ldots, 4) \) with appropriate dimensions, such that the following equality holds according to (2.1):

\[
2 \xi^T(t) N (-\dot{x}(t) + A_i x(t) + B_i x(t - \tau_{1i}) + C_i \dot{x}(t - \tau_{2i})) = 0,
\]

where

\[
N^T = (N^T_1 N^T_2 N^T_3 N^T_4),
\]

\[
\xi^T(t) = (x^T(t) \ x^T(t) \ x^T(t - \tau_{1i}) \ x^T(t - \tau_{2i})).
\]

From (3.7)–(3.10) and with (3.2)-(3.3), one can obtain that

\[
LV(x_i, t, i) = \sum_{j=1}^{4} LV_j(x_i, t, i) = x^T(t) \varphi_i x(t) < 0,
\]

where \( \varphi_i \) are defined in this theorem. Therefore,

\[
\varepsilon \left\{ \int_0^\infty \|x(t)\|^2 dt \mid \varphi, r_0 \right\} < \infty,
\]

which means that systems (2.1) are stochastic stability. The proof is completed.

Based on the result of Theorem 3.1, the next theorem will relate to the stability condition of systems (2.1) with partially known transition probabilities.

**Theorem 3.2.** The system (2.1) with a partly unknown transition rate matrix (2.4) is stochastically stable if there exist matrices \( P_i > 0, Q_{1i} > 0, Q_{2i} > 0 (i = 1, 2, \ldots, N), R_1 > 0, \) and \( R_2 > 0 \) and
any matrices $N_k (k = 1, \ldots, 4), W_i = W_i^T$ with appropriate dimensions satisfying the following linear matrix inequalities:

$$
\phi_i = \begin{pmatrix} \phi_{i11} & \phi_{i12} & \phi_{i13} & \phi_{i14} \\ * & \phi_{i22} & \phi_{i23} & \phi_{i24} \\ * & * & \phi_{i33} & \phi_{i34} \\ * & * & * & \phi_{i44} \end{pmatrix} < 0, \quad \forall j \in U_{uk} \text{ if } i \in U_{ik},
$$

(3.14)

$$
\psi_i = \begin{pmatrix} \psi_{i11} & \psi_{i12} & \psi_{i13} & \psi_{i14} \\ * & \psi_{i22} & \psi_{i23} & \psi_{i24} \\ * & * & \psi_{i33} & \psi_{i34} \\ * & * & * & \psi_{i44} \end{pmatrix} < 0, \quad \forall j \in U_{uk} \text{ if } i \in U_{ik},
$$

(3.15)

$$
\sum_{j \in U_k} \lambda_{ij} Q_{ij} + \sum_{j \in U_k} \lambda_{ij} \tau_{ij} R_1 - R_1 - \lambda_k^i Q_{ij} - \lambda_k^i \tau_{ij} R_1 \leq 0, \quad \forall j \in U_{uk} \text{ if } i \in U_{ik},
$$

(3.16)

$$
\sum_{j \in U_k} \lambda_{ij} Q_{ij} + \sum_{j \in U_k} \lambda_{ij} \tau_{ij} R_1 - R_1 + \lambda_k^i Q_{ij} - \lambda_k^i Q_{ij} + \lambda_k^i \tau_{ij} R_1
$$

$$
- \lambda_k^i \tau_{ij} R_1 - \lambda_k^i \tau_{ij} R_1 \leq 0, \quad \forall j \in U_{uk} \text{ if } i \in U_{ik},
$$

(3.17)

$$
\sum_{j \in U_k} \lambda_{ij} Q_{2ij} + \sum_{j \in U_k} \lambda_{ij} \tau_{2ij} R_2 - R_2 - \lambda_k^i Q_{2ij} - \lambda_k^i \tau_{2ij} R_2 \leq 0, \quad \forall j \in U_{uk} \text{ if } i \in U_{ik},
$$

(3.18)

$$
\sum_{j \in U_k} \lambda_{ij} Q_{2ij} + \sum_{j \in U_k} \lambda_{ij} \tau_{2ij} R_2 - R_2 + \lambda_k^i Q_{2ij} - \lambda_k^i Q_{2ij} + \lambda_k^i \tau_{2ij} R_2
$$

$$
- \lambda_k^i \tau_{2ij} R_2 - \lambda_k^i \tau_{2ij} R_2 \leq 0, \quad \forall j \in U_{uk} \text{ if } i \in U_{ik},
$$

(3.19)

$$
P_j - W_i \leq 0, \quad j \in U_{uk}, \ j \neq i,
$$

(3.20)

$$
P_j - W_i \geq 0, \quad j \in U_{uk}, \ j = i,
$$

(3.21)

with

$$
\phi_{i11} = P_i A_i + A_i^T P_i + \sum_{j \in U_k} \lambda_{ij} \left(P_j - W_i\right) + Q_{ij} + N_1 A_i + A_i^T N_1^T + \tau_{ij} R_1,
$$

$$
\phi_{i12} = -N_1 + A_i^T N_2^T,
$$

$$
\phi_{i13} = N_1 B_i + A_i^T N_3^T + P_i B_i,
$$

$$
\phi_{i14} = \cdots
$$
\[ \phi_{i14} = N_1 C_i + A_i^T N_4^T + P_i C_i, \]
\[ \phi_{i22} = -N_2 - N_2^T + Q_{2i} + t_{2i} R_2, \]
\[ \phi_{i23} = N_2 B_i - N_3^T, \]
\[ \phi_{i24} = N_2 C_i - N_4^T, \]
\[ \phi_{i33} = N_3 B_i + B_i^T N_3^T - Q_{ii} + \sum_{j \in U_k} \lambda_{ij} t_{ij} Q_{ii} - \lambda_i^T t_{ij} Q_{ii}, \]
\[ \phi_{i34} = N_3 C_i + B_i^T N_4^T, \]
\[ \phi_{i44} = N_4 C_i + C_i^T N_4^T - Q_{2i} + \sum_{j \in U_k} \lambda_{ij} t_{2j} Q_{2i} - \lambda_i^T t_{2j} Q_{2i}, \]
\[ \psi_{i33} = N_3 B_i + B_i^T N_3^T - Q_{ii} + \sum_{j \in U_k} \lambda_{ij} t_{ij} Q_{ii} + \lambda_i^T t_{ij} Q_{ii} - \lambda_i^T t_{ij} Q_{ii}, \]
\[ \psi_{i44} = N_4 C_i + C_i^T N_4^T - Q_{2i} + \sum_{j \in U_k} \lambda_{ij} t_{2j} Q_{2i} + \lambda_i^T t_{2j} Q_{2i} - \lambda_i^T t_{2j} Q_{2i}, \]
\[ \psi_{i11} = \phi_{i11}, \quad \psi_{i12} = \phi_{i12}, \quad \psi_{i13} = \phi_{i13}, \quad \psi_{i14} = \phi_{i14}, \]
\[ \psi_{i22} = \phi_{i22}, \quad \psi_{i23} = \phi_{i23}, \quad \psi_{i24} = \phi_{i24}, \quad \psi_{i34} = \phi_{i34}, \]

(3.22)

and \( \lambda_i^T \) is a given lower bound for the unknown diagonal element.

**Proof.** For the case of the systems (2.1) with partly unknown transition probabilities, and taking into account the situation that the information of transition probabilities is not accessible completely, due to \( \sum_{j=1}^{N} \lambda_{ij} = 0 \), the following zero equation holds for arbitrary matrices \( W_i = W_i^T \) is satisfied:

\[
-x^T(t) \left( \sum_{j=1}^{N} \lambda_{ij} W_i \right) x(t) = 0, \quad \forall i \in \mathfrak{i},
\]

(3.23)

and the inequality (3.12) can be rewritten as

\[
LV(x_i, t, i) = x^T(t) \sum_{j \in U_k} \lambda_{ij} (P_j - W_i) x(t) + \xi^T(t) \tilde{\phi}_i \xi(t) < 0,
\]

(3.24)

where \( \xi(t) \) has already been defined on the above

\[
\tilde{\phi}_i = \begin{pmatrix} \tilde{\phi}_{i11} & \tilde{\phi}_{i12} & \tilde{\phi}_{i13} & \tilde{\phi}_{i14} \\ * & \tilde{\phi}_{i22} & \tilde{\phi}_{i23} & \tilde{\phi}_{i24} \\ * & * & \tilde{\phi}_{i33} & \tilde{\phi}_{i34} \\ * & * & * & \tilde{\phi}_{i44} \end{pmatrix}, \quad i = 1, \ldots, N.
\]

(3.25)
with the elements are the same as those in \( \varphi_i \), except for

\[
\tilde{\varphi}_{i11} = P_i A_i + A_i^T P_i + \sum_{j \in U_i^k} \lambda_{ij} (P_i - W_i) + Q_{ii} N_i A_i + A_i^T N_i^T,
\]

and note that \( \lambda_{ii} = -\sum_{j \in U_i^k} \lambda_{ij} \) and \( \lambda_{ij} \geq 0 \) for all \( j \neq i \), namely, \( \lambda_{ii} < 0 \) for all \( i \in \mathcal{P} \). Therefore, it follows from easy computation that if \( i \in U_i^k \), inequalities (3.20) and the formula (3.25) less than 0 imply that

\[
LV(x_i, t, i) < 0.
\]

On the other hand, for the same reason, if \( i \in U_i^{ik} \), inequalities (3.20)-(3.21) and the formula (3.25) less than 0 also imply that inequality (3.27) holds. Therefore,

\[
\epsilon \left\{ \int_0^\infty \| x(t) \|^2 dt \mid \varphi, r_0 \right\} < \infty,
\]

which means that systems (2.1) with partly unknown transition probabilities are stochastically stable.

Note that the formula (3.25) less than 0 can be represented as follows:

\[
\tilde{\varphi}_i = \Phi_i + \sum_{j \in U_{ia}} \tilde{\lambda}_{ij} \text{diag}(0, 0, \tau_{ij} Q_{ii}, \tau_{ij} Q_{ii}) < 0,
\]

where \( \tilde{\lambda}_{ij} \) is an unknown element in transition probabilities matrix, and

\[
\Phi_i = \begin{pmatrix}
\Phi_{i11} & \Phi_{i12} & \Phi_{i13} & \Phi_{i14} \\
* & \Phi_{i22} & \Phi_{i23} & \Phi_{i24} \\
* & * & \Phi_{i33} & \Phi_{i34} \\
* & * & * & \Phi_{i44}
\end{pmatrix},
\]

with

\[
\Phi_{i11} = \tilde{\varphi}_{i11}, \quad \Phi_{i12} = \tilde{\varphi}_{i12}, \quad \Phi_{i13} = \tilde{\varphi}_{i13}, \quad \Phi_{i14} = \tilde{\varphi}_{i14},
\]

\[
\Phi_{i22} = \tilde{\varphi}_{i22}, \quad \Phi_{i23} = \tilde{\varphi}_{i23}, \quad \Phi_{i24} = \tilde{\varphi}_{i24}, \quad \Phi_{i34} = \tilde{\varphi}_{i34},
\]

\[
\Phi_{i33} = N_3 B_i + B_i^T N_3^T - Q_{ii} + \sum_{j \in U_i^k} \lambda_{ij} \tau_{ij} Q_{ii},
\]

\[
\Phi_{i44} = N_4 C_i + C_i^T N_4^T - Q_{ii} + \sum_{j \in U_i^k} \lambda_{ij} \tau_{ij} Q_{ii}.
\]

One can note that (3.29) can be separated into two cases, \( i \in U_i^k \) and \( i \in U_{ia}^k \).
Case I \((i \in U^i_{1k})\): it should be first noted that in this case one has \(\lambda^i_k < 0\). In fact, we only need to consider \(\lambda^i_k < 0\) because \(\lambda^i_k = 0\) means all the elements in the \(i\) throw of the transition rate matrix.

Now the formula (3.29) can be rewritten as

\[
\tilde{q}_i = \Phi_i + \sum_{j \in U^i_{1k}} \tilde{\lambda}_{ij} \text{diag}(0, 0, \tau_i Q_{1i}, \tau_i Q_{2i}) = \Phi_i - \lambda^i_k \sum_{j \in U^i_{1k}} \frac{\tilde{\lambda}_{ij}}{-\lambda^i_k} \text{diag}(0, 0, \tau_i Q_{1i}, \tau_i Q_{2i}) < 0.
\]

(3.32)

It follows from \(0 \leq \tilde{\lambda}_{ij} / -\lambda^i_k \leq 1\) and \(\sum_{j \in U^i_{1k}} (\tilde{\lambda}_{ij} / -\lambda^i_k) = 1\) that

\[
\tilde{q}_i = \sum_{j \in U^i_{1k}} \frac{\tilde{\lambda}_{ij}}{-\lambda^i_k} \left( \Phi_i - \lambda^i_k \text{diag}(0, 0, \tau_i Q_{1i}, \tau_i Q_{2i}) \right) < 0.
\]

(3.33)

Similar to the above proof, (3.2) and (3.3) can be rewritten as (3.16) and (3.18), respectively, for this case. Accordingly, for \(0 \leq \tilde{\lambda}_{ij} \leq \lambda^i_k\), \(\tilde{q}_i < 0\) is equivalent to (3.14) which is satisfied for all \(j \in U^i_{1k}\), which also implies that, in the presence of unknown elements \(\tilde{\lambda}_{ij}\), the system stability is ensured if (3.14), (3.16), (3.18), and (3.20) hold.

Case II \((i \in U^i_{2k})\): for the sake of simple expression, let \(\Psi_j = \text{diag}(0, 0, \tau_j Q_{1i}, \tau_j Q_{2i})\).

In this case, \(\tilde{\lambda}_{ii}\) is unknown, \(\lambda^i_k \geq 0\), and \(\tilde{\lambda}_{ij} \leq 0\), and following the same analysis of the above case, we just consider \(\tilde{\lambda}_{ii} < -\lambda^i_k\). And now the formula (3.29) can be rewritten as

\[
\tilde{q}_i = \Phi_i + \tilde{\lambda}_{ii} \text{diag}(0, 0, \tau_i Q_{1i}, \tau_i Q_{2i}) + \sum_{j \in U^i_{2k}, j \neq i} \tilde{\lambda}_{ij} \text{diag}(0, 0, \tau_i Q_{1i}, \tau_i Q_{2i})
\]

\[
= \Phi_i + \tilde{\lambda}_{ii} \text{diag}(0, 0, \tau_i Q_{1i}, \tau_i Q_{2i}) + \left( -\lambda^i_{ii} - \lambda^i_k \right) \sum_{j \in U^i_{2k}, j \neq i} \frac{\tilde{\lambda}_{ij}}{-\lambda^i_{ii} - \lambda^i_k} \Psi_j.
\]

(3.34)

Similarly, since we have

\[
0 \leq \frac{\tilde{\lambda}_{ij}}{-\lambda^i_{ii} - \lambda^i_k} \leq 1, \quad \sum_{j \in U^i_{2k}, j \neq i} \frac{\tilde{\lambda}_{ij}}{-\lambda^i_{ii} - \lambda^i_k} = 1,
\]

(3.35)

one can know that

\[
\tilde{q}_i = \sum_{j \in U^i_{2k}} \frac{\tilde{\lambda}_{ij}}{-\lambda^i_{ii} - \lambda^i_k} \left( \Phi_i + \tilde{\lambda}_{ii} \Psi_i + \left( -\lambda^i_{ii} - \lambda^i_k \right) \Psi_j \right),
\]

(3.36)

which means that \(\tilde{q}_i < 0\) is equivalent to all \(j \in U^i_{2k}, j \neq i\),

\[
\Phi_i + \tilde{\lambda}_{ii} \Psi_i + \left( -\lambda^i_{ii} - \lambda^i_k \right) \Psi_j < 0,
\]

(3.37)
and from the defined $\lambda_i^d$ in Theorem 3.2, we have that $\lambda_i^d \leq \lambda_{ii} < -\lambda_i^k$, which means that $\lambda_{ii}$ may take any value between $[\lambda_i^d, -\lambda_i^k + \varepsilon]$ for some $\varepsilon < 0$ arbitrarily small. Then, $\lambda_{ii}$ can be further written as a convex combination

$$\lambda_{ii} = \alpha (-\lambda_i^k + \varepsilon) + (1 - \alpha) \lambda_i^d,$$

where $\alpha$ takes value arbitrarily in $[0, 1]$. So, (3.37) holds if and only if for all $j \in U_{ik}^i, j \neq i$,

$$\Phi_i - \lambda_i^j \Psi_j + \varepsilon (\Psi_i - \Psi_j) < 0,$$

$$\Phi_i + \lambda_i^j \Psi_j - \lambda_i^j \Psi_j - \lambda_i^k \Psi_i < 0$$

simultaneously hold. Since $\varepsilon$ is arbitrarily small, (3.39) holds if and only if

$$\Phi_i - \lambda_i^j \Psi_j < 0,$$

which is the case in (3.40) when $j = i$ for all $j \in U_{ik}^i$. Hence, (3.37) is equivalent to (3.15). Furthermore, following the same line of this proof, (3.2) and (3.3) can be represented as (3.17) and (3.19), respectively, in this case.

Therefore, from the above discussion, in the presence of unknown elements in the transition probabilities matrix, we can readily draw a conclusion that the system (2.1) with partly known transition rates is stable if the inequalities in Theorem 3.2 are satisfied. It completes this proof.

**Remark 3.3.** In order to obtain the less conservative stability criterion of MJSs with partial information on transition probabilities, similar to [32], the free-connection weighting matrices are introduced by making use of the relationship of the transition rates among various subsystems, that is, $\sum_{j=1}^{N} \lambda_{ij} = 0$ for all $i \in \mathcal{P}$, which overcomes the conservativeness of using the fixed-connection weighting matrices. However, it is difficult to decrease the conservative using free-connection matrices only based on the above equalities, but not on the systems and themselves Newton-Leibniz formula. Moreover, this paper is inspired by [30], and the delay-dependent stability results in this theorem are the extension of [30] to delay systems to some extent. Although the large number of introduced free weighting matrices may increase the complexity of computation, using the technique of free weighting matrices would reduce the conservativeness, which would be reflected in the fifth section.

**Remark 3.4.** It should be noted that the more known elements are there in (2.3), the lower the conservative of the condition will be. In other word, the more unknown elements are there in (2.3), the lower the maximum of time delay will be in Theorem 3.2. Actually, if all transition probabilities are unknown, the corresponding system can be viewed as a switched linear system under arbitrary switching. Thus, the conditions obtained in Theorem 3.2 will thereby cover the results for arbitrary switched linear system with mixed delays. In that case, one can see that the stability condition in Theorem 3.2 becomes seriously conservative, for many constraints. Fortunately, we can use the common Lyapunov functional method to analyze the stability for the system under the assumption that all transition probabilities are not known.
For the stability analysis of the neutral markovian jump systems with all transition probabilities is not known, one can construct the following common Lyapunov functional:

\[
V(x_t, t, r_t) = x^T(t)Px(t) + \int_{t-\tau_i(n)}^{t} x^T(s)Q_1 x(s)\,ds + \int_{t-\tau_i(n)}^{t} \dot{x}^T(s)Q_2 \dot{x}(s)\,ds
+ \int_{-\tau_i(n)}^{0} \int_{t+\theta}^{t} x^T(s)R_1 x(s)\,ds\,d\theta + \int_{-\tau_i(n)}^{0} \int_{t+\theta}^{t} \dot{x}^T(s)R_2 \dot{x}(s)\,ds\,d\theta,
\]

and following a similar line as in the proof of Theorem 3.1, we can obtain the following corollary.

**Corollary 3.5.** The system (2.1) with all elements unknown in transition rate matrix (2.3) is stochastically stable if there exist positive definite matrices \(P, Q_1, Q_2, R_1 > 0\) and \(R_2 > 0\) and any matrices \(N_k (k = 1, \ldots, 4)\) with appropriate dimensions satisfying the following linear matrix inequalities:

\[
\chi_i = \begin{pmatrix}
\chi_{i11} & \chi_{i12} & \chi_{i13} & \chi_{i14} \\
\ast & \chi_{i22} & \chi_{i23} & \chi_{i24} \\
\ast & \ast & \chi_{i33} & \chi_{i34} \\
\ast & \ast & \ast & \chi_{i44}
\end{pmatrix} < 0,
\]

with

\[
\begin{align*}
\chi_{i11} &= PA_i + A_i^T P + Q_1 + N_1 A_i + A_i^T N_1^T + \tau_i R_1, \\
\chi_{i12} &= -N_1 + A_i^T N_2^T, \\
\chi_{i13} &= N_1 B_i + A_i^T N_3^T + P B_i, \\
\chi_{i14} &= N_1 C_i + A_i^T N_4^T + P C_i, \\
\chi_{i22} &= -N_2 - N_2^T + Q_2 + \tau_i R_2, \\
\chi_{i23} &= N_2 B_i - N_3^T, \\
\chi_{i24} &= N_2 C_i - N_4^T, \\
\chi_{i33} &= N_3 B_i + B_i^T N_3^T - Q_1, \\
\chi_{i34} &= N_3 C_i + B_i^T N_4^T, \\
\chi_{i44} &= N_4 C_i + C_i^T N_4^T - Q_2.
\end{align*}
\]
4. Extension to Uncertain Neutral Markov Jump Systems

In this section, we will consider the uncertain neutral Markov jump systems with partially unknown transition probabilities as follows:

\[
\dot{x}(t) - C(r_t)x(t - \tau_2(r_t)) = (A(r_t) + \Delta A(r_t))x(t) + (B(r_t) + \Delta B(r_t))x(t - \tau_1(r_t)).
\] (4.1)

\(A(r_t), B(r_t)\) are known mode-dependent constant matrices with appropriate dimensions, while \(\Delta A(r_t), \Delta B(r_t)\) are the time-varying but norm-bounded uncertainties satisfying

\[
(\Delta A(r_t) \ \Delta B(r_t)) = L_{rt}F_{rt}(t)(H_1(r_t) \ H_2(r_t)),
\] (4.2)

where \(L_{rt}, H_1(r_t), H_2(r_t)\) are known mode-dependent matrices with appropriate dimensions, and \(F_{rt}(t)\) is the time-varying unknown matrix function with Lebesgue norm measurable elements satisfying \(F_{rt}^T(t)F_{rt}(t) \leq I\).

**Theorem 4.1.** The uncertain neutral Markovian jump system (4.1) with a partly unknown transition rate matrix (2.3) is stochastically stable if there exist matrices \(\bar{P}_1 > 0, \bar{Q}_{1i} > 0, \bar{Q}_{2i} > 0, \bar{R}_i > 0, \) and \(\bar{R}_2 > 0\) and any matrices \(\bar{W}_i = \bar{W}_i^T, \bar{M}_i = \bar{M}_i^T, \bar{S}_i = \bar{S}_i^T, \bar{F}_k, \bar{G}_k,\) and \(\bar{N}_k(k = 1, \ldots, 6)\) with appropriate dimensions satisfying the following linear matrix inequalities:

\[
\begin{bmatrix}
\bar{\phi}_{i11} & \bar{\phi}_{i12} & \bar{\phi}_{i13} & \bar{\phi}_{i14} & H_{11}^T & \bar{P}_1L_i + \bar{N}_1L_i \\
* & \bar{\phi}_{i22} & \bar{\phi}_{i23} & \bar{\phi}_{i24} & 0 & \bar{N}_2L_i \\
* & * & \bar{\phi}_{i33} & \bar{\phi}_{i34} & H_{21}^T & \bar{N}_3L_i \\
* & * & * & \bar{\phi}_{i44} & 0 & \bar{N}_4L_i \\
* & * & * & * & * & -I
\end{bmatrix} < 0, \quad \forall j \in \mathcal{U}_{uk}^i \text{ if } i \in \mathcal{U}_{uk}^i, \quad (4.3)
\]

\[
\begin{bmatrix}
\bar{\psi}_{i11} & \bar{\psi}_{i12} & \bar{\psi}_{i13} & \bar{\psi}_{i14} & H_{11}^T & \bar{P}_1L_i + \bar{N}_1L_i \\
* & \bar{\psi}_{i22} & \bar{\psi}_{i23} & \bar{\psi}_{i24} & 0 & \bar{N}_2L_i \\
* & * & \bar{\psi}_{i33} & \bar{\psi}_{i34} & H_{21}^T & \bar{N}_3L_i \\
* & * & * & \bar{\psi}_{i44} & 0 & \bar{N}_4L_i \\
* & * & * & * & * & -I
\end{bmatrix} < 0, \quad \forall j \in \mathcal{U}_{uk}^i \text{ if } i \in \mathcal{U}_{uk}^i, \quad (4.4)
\]
\[ \sum_{j \in U'_k} \lambda_{ij} \tilde{Q}_{1j} + \sum_{j \in U'_k} \lambda_{ij} \tau_{1j} \tilde{R}_1 - \tilde{Q}_{1j} - \lambda_k' \tilde{Q}_{1j} - \lambda_k' \tau_{1j} \tilde{R}_1 \leq 0, \forall j \in U'_k \text{ if } i \in U'_k, \]  
\[ (4.5) \]
\[ \sum_{j \in U'_k} \lambda_{ij} \tilde{Q}_{1j} + \sum_{j \in U'_k} \lambda_{ij} \tau_{1j} \tilde{R}_1 - \tilde{Q}_{1j} + \lambda_k' \tilde{Q}_{1j} - \lambda_k' \tau_{1j} \tilde{R}_j \leq 0, \quad \forall j \in U'_k \text{ if } i \in U'_k, \]  
\[ \sum_{j \in U'_k} \lambda_{ij} \tilde{Q}_{2j} + \sum_{j \in U'_k} \lambda_{ij} \tau_{2j} \tilde{R}_2 - \tilde{Q}_{2j} - \lambda_k' \tilde{Q}_{2j} - \lambda_k' \tau_{2j} \tilde{R}_2 \leq 0, \quad \forall j \in U'_k \text{ if } i \in U'_k, \]  
\[ \sum_{j \in U'_k} \lambda_{ij} \tilde{Q}_{2j} + \sum_{j \in U'_k} \lambda_{ij} \tau_{2j} \tilde{R}_2 - \tilde{Q}_{2j} + \lambda_k' \tilde{Q}_{2j} - \lambda_k' \tau_{2j} \tilde{R}_2 \leq 0, \quad \forall j \in U'_k \text{ if } i \in U'_k, \]  
\[ \tilde{P}_j - \tilde{W}_i \leq 0, \quad j \in U'_k, j \neq i, \]  
\[ \tilde{P}_j - \tilde{W}_i \geq 0, \quad j \in U'_k, j = i, \]  
\[ (4.9) \]
\[ (4.10) \]

with

\[ \tilde{\phi}_{i11} = \tilde{P}_i A_i + A_i^T \tilde{P}_i + \sum_{j \in U'_k} \lambda_{ij} (\tilde{P}_j - \tilde{W}_i) + \tilde{Q}_{1i} + \tilde{N}_1 A_i + A_i^T \tilde{N}_1^T + \tau_{1i} \tilde{R}_1, \]
\[ \tilde{\phi}_{i12} = -\tilde{N}_1 + A_i^T \tilde{N}_2^T, \]
\[ \tilde{\phi}_{i13} = \tilde{N}_1 B_i + A_i^T \tilde{N}_3^T + \tilde{P}_i B_i, \]
\[ \tilde{\phi}_{i14} = \tilde{N}_1 C_i + A_i^T \tilde{N}_4^T + \tilde{P}_i C_i, \]
\[ \tilde{\phi}_{i22} = -\tilde{N}_2 - \tilde{N}_2^T + \tilde{Q}_{2i} + \tau_{2i} \tilde{R}_2, \]
\[ \tilde{\phi}_{i23} = \tilde{N}_2 B_i - \tilde{N}_3^T, \]
\[ \tilde{\phi}_{i24} = \tilde{N}_2 C_i - \tilde{N}_4^T, \]
\[ \tilde{\phi}_{i33} = \tilde{N}_3 B_i + B_i^T \tilde{N}_3^T - \tilde{Q}_{1i} + \sum_{j \in U'_k} \lambda_{ij} \tau_{1j} \tilde{Q}_{1j} - \lambda_k' \tau_{1j} \tilde{Q}_{1j}, \]
\[ \tilde{\phi}_{i34} = \tilde{N}_3 C_i + B_i^T \tilde{N}_4^T, \]
\[ \tilde{\phi}_{i44} = \tilde{N}_4 C_i + C_i^T \tilde{N}_4^T - \tilde{Q}_{2i} + \sum_{j \in U'_k} \lambda_{ij} \tau_{2j} \tilde{Q}_{2j} - \lambda_k' \tau_{2j} \tilde{Q}_{2j}, \]
\[ \tilde{\phi}_{i33} = \tilde{N}_3 B_i + B_i^T \tilde{N}_3^T - \tilde{Q}_{1i} + \sum_{j \in U'_k} \lambda_{ij} \tau_{1j} \tilde{Q}_{1j} + \lambda_k' \tau_{1j} \tilde{Q}_{1j} - \lambda_k' \tau_{1j} \tilde{Q}_{1j}, \]
\[ \tilde{\phi}_{i44} = \tilde{N}_4 C_i + C_i^T \tilde{N}_4^T - \tilde{Q}_{2i} + \sum_{j \in U'_k} \lambda_{ij} \tau_{2j} \tilde{Q}_{2j} + \lambda_k' \tau_{2j} \tilde{Q}_{2j} - \lambda_k' \tau_{2j} \tilde{Q}_{2j}, \]
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\[ \tilde{\psi}_{i11} = \tilde{\phi}_{i11}, \quad \tilde{\psi}_{i12} = \tilde{\phi}_{i12}, \quad \tilde{\psi}_{i13} = \tilde{\phi}_{i13}, \quad \tilde{\psi}_{i14} = \tilde{\phi}_{i14}, \]
\[ \tilde{\psi}_{i22} = \tilde{\phi}_{i22}, \quad \tilde{\psi}_{i23} = \tilde{\phi}_{i23}, \quad \tilde{\psi}_{i24} = \tilde{\phi}_{i24}, \quad \tilde{\psi}_{i34} = \tilde{\phi}_{i34}, \]
(4.11)

and \( \lambda_d^i \) is a given lower bound for the unknown diagonal element.

**Proof.** \( \tilde{\phi}_i < 0 \) can be written as

\[ \tilde{\phi}_i = \phi_i + D_i^T F_i(t) G_i^T + G_i F_i^T(t) D_i < 0, \]
(4.12)

where

\[ G_i^T = (L_i^T P_i + L_i^T N_1^T L_i^T N_2^T L_i^T N_3^T L_i^T N_4^T), \]
\[ D_i = (H_{1i} \quad 0 \quad H_{2i} \quad 0), \]
(4.13)

and \( \phi_i \) are defined in Theorem 3.2. According to the approach in [40] with Lemma 2.4 in [41], there exists a scalar \( \varepsilon \) such that (4.12) are equivalents to

\[ \tilde{\phi}_i = \varepsilon \phi_i + \varepsilon^2 D_i^T D_i + G_i G_i^T < 0. \]
(4.14)

Introducing new variables \( \tilde{P}_i = \varepsilon P_i, \quad \tilde{Q}_{1i} = \varepsilon Q_{1i}, \quad \tilde{Q}_{2i} = \varepsilon Q_{2i}, \quad \tilde{R}_1 = \varepsilon R_1, \quad \tilde{R}_2 = \varepsilon R_2, \quad \tilde{W}_i = \varepsilon W_i, \) and \( \tilde{N}_k = \varepsilon N_k(k = 1, \ldots, 4) \), and with Schur’s complement [42], yields inequalities (4.3). Similarly, it concludes (4.4) with the same proof. On the other hand, with the same variables substitution, we note that pre- and postmultiplying, respectively, (3.16)–(3.21) by a scalar \( \varepsilon \) yield (4.5)–(4.10), which completes this proof. \( \square \)

**Remark 4.2.** It should be mentioned that Theorem 4.1 is an extension of (2.1) to uncertain neutral Markovian jump systems (4.1) with incomplete transition descriptions. In fact, this technology is frequently adopted in dealing with the robust stability analysis of uncertain systems.

### 5. Examples

In order to show the effectiveness of the approaches presented in the above sections, two numerical examples are provided.

**Example 5.1.** Consider the MJLS (2.1) with four operation modes whose state matrices are listed as follows:

\[ A_1 = \begin{pmatrix} -1.15 & 0 \\ 0 & -1.5 \end{pmatrix}, \quad A_2 = \begin{pmatrix} -3.15 & 0 \\ 0 & -7.1 \end{pmatrix}, \]
The partly transition rate matrix $\Pi$ is considered as

\[
Case I: \Pi = \begin{bmatrix}
-1.3 & 0.2 & ? & ? \\
? & 0.3 & 0.3 & ? \\
0.6 & ? & -1.5 & ? \\
0.4 & ? & ? & ? \\
\end{bmatrix}, \quad Case II: \Pi = \begin{bmatrix}
-1.3 & 0.2 & 0.6 & 0.5 \\
0.2 & -0.8 & 0.3 & 0.3 \\
0.6 & 0.7 & -1.5 & 0.2 \\
0.4 & 0.1 & 0.4 & -0.9 \\
\end{bmatrix}. \quad (5.3)
\]

Our purpose here is to check the stability of the above system for the two different cases of transition probabilities. For Case I, it is clear to see that $\lambda_{22}$ and $\lambda_{44}$ are not valued, one can set $\lambda_{22}^1 = -0.8, \lambda_{44}^1 = -0.9$, and let $\tau_{12} = 1.1000, \tau_{13} = 0.8000, \tau_{14} = 0.9900, \tau_{21} = \tau_{11}, \tau_{22} = \tau_{12}, \tau_{23} = \tau_{13},$ and $\tau_{24} = \tau_{14}$. Solving the inequalities in Theorem 3.2 using LMI toolbox, the maximum of the time delay $\tau_{11}$ can be computed as $\tau_{11} = 1.1210$. However, in Case II, the maximum of the time delay $\tau_{11}$ can be computed as $\tau_{11} = 2.0530$ by Theorem 3.1. It is easily seen that the more transition probabilities knowledge we have, the larger the maximum of delay can be obtained for ensuring stability. This shows the trade-off between the cost of obtaining transition probabilities and the system performance.

Furthermore, when the transition probabilities are not fully known, as the delay for one of the subsystems decreases, the maximum of other delays may increase. However, when all transition probabilities are fully known, the conclusion may be on the opposite in some interval. In fact, the above observation is in accordance with the actual. Then, we assume that $\lambda_{22}^1 = -0.8, \lambda_{44}^1 = -0.9$, and let $\tau_{12} = 1.1000, \tau_{14} = 0.9900, \tau_{21} = \tau_{11}, \tau_{22} = \tau_{12}, \tau_{23} = \tau_{13},$ and $\tau_{24} = \tau_{14}, \tau_{13}$ be different with $\tau_{11}$, and with the same computation in Theorem 3.1, as shown in Table 1. However, just according to the approach of Theorem 3 in [32], not introducing some other free matrices and some other skills, we cannot find the feasible solutions which contain time delay to verify the stability of the system. Therefore, this example shows that the stability criterion in this paper gives much less conservative delay-dependent stability conditions. This example also shows that the approach presented in this paper is effectiveness.
Example 5.2. Consider the above MJLS in Example 5.1 with partially unknown transition probabilities of Case I, and the uncertain structure matrices are described by (4.2) where

\[
L_1 = \begin{pmatrix} 0.1 \\0.2 \end{pmatrix}, \quad L_2 = \begin{pmatrix} 0.2 \\-0.4 \end{pmatrix}, \quad L_3 = \begin{pmatrix} 0.4 \\0.3 \end{pmatrix}, \quad L_4 = \begin{pmatrix} 0.25 \\0.27 \end{pmatrix},
\]

\[
H_{11} = (0.72 \hspace{1em} 1.14), \quad H_{12} = (1.22 \hspace{1em} 1.66), \quad H_{13} = (1.41 \hspace{1em} -1.13), \quad H_{14} = (-1.39 \hspace{1em} 1.63),
\]

\[
H_{21} = (-1.11 \hspace{1em} 1.32), \quad H_{22} = (1.31 \hspace{1em} 1.14), \quad H_{23} = (1.27 \hspace{1em} 1.15), \quad H_{24} = (-1.26 \hspace{1em} 1.37).
\]

In this case, we check the robust stability result provided by Theorem 4.1. One can also set \(\lambda_2 = -0.8, \lambda_3 = -0.9\), and let \(\tau_{12} = 1.1000, \tau_{13} = 0.7500, \tau_{14} = 0.9900, \tau_{21} = \tau_{11}, \tau_{22} = \tau_{12}, \tau_{23} = \tau_{13}\), and \(\tau_{24} = \tau_{14}\). Solving the inequalities in Theorem 4.1 by using LMI toolbox, the maximum of the time delay \(\tau_{11}\) can be computed as \(\tau_{11} = 1.0892\). Some of the feasible solutions can be obtained as follows:

\[
P_1 = \begin{pmatrix} 6.6762 \hspace{1em} -1.8338 \\-1.8338 \hspace{1em} 9.5418 \end{pmatrix}, \quad P_2 = \begin{pmatrix} 6.6930 \hspace{1em} -1.8283 \\-1.8283 \hspace{1em} 9.5446 \end{pmatrix},
\]

\[
P_3 = \begin{pmatrix} 10.7916 \hspace{1em} -5.0166 \\-5.0166 \hspace{1em} 12.0252 \end{pmatrix}, \quad P_4 = \begin{pmatrix} 10.7923 \hspace{1em} -5.0178 \\-5.0178 \hspace{1em} 12.0274 \end{pmatrix},
\]

\[
W_1 = \begin{pmatrix} 10.7928 \hspace{1em} -270.5249 \\260.4880 \hspace{1em} 12.0285 \end{pmatrix}, \quad W_2 = \begin{pmatrix} 6.6915 \hspace{1em} 371.7539 \\375.4115 \hspace{1em} 9.5439 \end{pmatrix},
\]

\[
W_3 = \begin{pmatrix} 10.7943 \hspace{1em} 82.9434 \\-92.9861 \hspace{1em} 12.0338 \end{pmatrix}, \quad W_4 = \begin{pmatrix} 10.7920 \hspace{1em} -212.1720 \\202.1374 \hspace{1em} 12.0265 \end{pmatrix},
\]

\[
N_4 = \begin{pmatrix} -0.3069 \hspace{1em} 0.1342 \\0.0548 \hspace{1em} 0.1402 \end{pmatrix}.
\]

In a word, this example shows that the robust stability condition of Theorem 4.1 is feasible. It is also approved that the approach provided in this paper is effectiveness.

Example 5.3. Consider the MJLS (2.1) with two operation modes whose state matrices are listed as follows:

\[
A_1 = \begin{pmatrix} -2.16\hspace{1em}0.05 \\-0.15 \hspace{1em} -1.35 \end{pmatrix}, \quad A_2 = \begin{pmatrix} -3.13 \hspace{1em} -0.19 \\0.21 \hspace{1em} -2.21 \end{pmatrix}, \quad B_1 = \begin{pmatrix} 0.22 \hspace{1em} -0.02 \\0.54 \hspace{1em} -0.05 \end{pmatrix},
\]

\[
B_2 = \begin{pmatrix} 0.43 \hspace{1em} -0.16 \\0.27 \hspace{1em} 1.07 \end{pmatrix}, \quad C_1 = \begin{pmatrix} -0.08 \hspace{1em} -0.06 \\0.03 \hspace{1em} 0.04 \end{pmatrix}, \quad C_2 = \begin{pmatrix} 0.05 \hspace{1em} 0.31 \\0.23 \hspace{1em} -0.06 \end{pmatrix}.
\]
The partly transition rate matrix $\Pi$ is considered as

$$\Pi = \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix}. \quad (5.8)$$

The above matrix (5.8) implies that systems (2.1) with all transition probabilities are not known, which viewed the systems as switched systems with arbitrary switching. Given that $\tau_{11} = 2.4279$, $\tau_{12} = 1.1000$, $\tau_{13} = 0.95$, $\tau_{14} = 0.9900$, $\tau_{21} = \tau_{11}$, $\tau_{22} = \tau_{12}$, $\tau_{23} = \tau_{13}$, and $\tau_{24} = \tau_{14}$, according to the approach of Theorem 3.2, we cannot find the feasible solutions. However, using the Matlab LMI toolbox, we solve the LMI in Corollary 3.5, and the feasible solutions can be obtained as follows:

$$P = \begin{pmatrix} 15.9364 & 0.7491 \\ 0.7491 & 11.2785 \end{pmatrix}, \quad R_1 = \begin{pmatrix} 2.7619 & 0.2382 \\ 0.2382 & 1.5829 \end{pmatrix},$$

$$R_2 = \begin{pmatrix} 0.9940 & 0.0974 \\ 0.0974 & 0.5010 \end{pmatrix}, \quad Q_1 = \begin{pmatrix} 6.8645 & 0.3173 \\ 0.3173 & 7.3064 \end{pmatrix}, \quad Q_2 = \begin{pmatrix} 4.3109 & 0.1752 \\ 0.1752 & 3.8836 \end{pmatrix}. \quad (5.9)$$

For the case of all transition probabilities that are unknown, this example also shows that Corollary 3.5 is less conservative than Theorem 3.2 on the stability analysis for the neutral markovian jump system.

6. Conclusion

The delay-dependent stability for neutral markovian jump systems with partly known transition probabilities has been investigated. Based on a new class of stochastic Lyapunov-Krasovskii functionals constructed, and combined with the technique of analysis for matrix inequalities, some new stability criteria are obtained. The main contribution of this paper contains the following two-fold: one is the extension of delay-dependent stability conditions for markovian jump delay systems to markovian neutral jump systems; the other is the new method presented to decrease the conservative brought by the markovian jump with partly known transition probabilities. The future work is to investigate the systems with mode-dependent interval mixed time delays and the systems with unsynchronised control input. Three examples have shown the effectiveness of the conditions presented in this paper.

Nomenclature

$\mathbb{R}^n$: $n$-dimensional real space

$\mathbb{R}^{m\times n}$: Set of all real $m$ by $n$ matrices

$x^T$ or $A^T$: Transpose of vector $x$ (or matrix $A$)

$P > 0$: ($P < 0$, resp.) Matrix $P$ is symmetric positive (negative, resp.) definite

$P \geq 0$: ($P \leq 0$, resp.) Matrix $P$ is symmetric positive (negative, resp.) semidefinite

$: The elements below the main diagonal of a symmetric block matrix

$x_t(\theta)$: $x(t + \theta), \theta \in [-\tau, 0]$. 
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