Hindawi Publishing Corporation
Abstract and Applied Analysis
Volume 2012, Article ID 383592, 10 pages
doi:10.1155/2012/383592

Research Article

n-Bazilevic Functions

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Received 31 December 2011; Revised 2 March 2012; Accepted 3 March 2012

Academic Editor: Khalida Inayat Noor

The aim of this paper is to define and study a class of Bazilevic functions using the generalized Salagean operator. Some properties of this class are investigated: inclusion relation, some convolution properties, coefficient bounds, and other interesting results.

1. Introduction

Let H be the set of analytic functions in the open unit disc $E = \{z : |z| < 1\}$. Let A be the set of functions $f \in H$, with $f(z) = z + \sum_{k=2}^{\infty} a_k z^k$, and let A_0 be the set of functions $f \in H$, with $f(0) = 1$. Let S be the class of functions $f \in A$, which are univalent in E. Denote by $ST(\gamma)(CV(\gamma))(K(\gamma))$, $\gamma < 1$, the class of starlike (convex)(close-to-convex) functions of order γ. Note that when $0 \le \gamma < 1$, then $ST(\gamma)(CV(\gamma))(K(\gamma)) \subset S$, let $ST(0)(CV(0))(K(0)) \equiv ST(CV)(K)$. A function $F \in A_0$, where $F \neq 0$ belongs to the Kaplan class $K(\alpha,\beta)$, $\alpha \ge 0$, $\beta \ge 0$, [1] if

$$-\alpha\pi + \frac{1}{2}(\alpha-\beta)(\theta_2-\theta_1) \le \arg F\left(re^{i\theta_2}\right) - \arg F\left(re^{i\theta_1}\right) \le \frac{1}{2}(\alpha-\beta)(\theta_2-\theta_1) + \beta\pi, \tag{1.1}$$

for $\theta_1 < \theta_2 < \theta_1 + 2\pi$ and $0 < r < 1$.

The Dual of $\nu \subset A_0$ is defined as

$$\nu^{\star} = \left\{g \in A_0 : f * g \neq 0 \text{ in } \Delta,\ f \in \nu\right\}, \tag{1.2}$$

where $*$ denotes Hadamard product (convolution).

A set $v \subset A_0$ is called a test set for $u \subset A$ (denoted by $v \rightsquigarrow u$) if $v \subset u \subset v^{\star\star}$. Note that if $v \rightsquigarrow u$, then $v^{\star} \subset u^{\star}$.

Denote by P the class of functions $p \in A_0$, such that $\operatorname{Re} p > 0$, in E, and let $P^{\alpha} = \{f^{\alpha} \in A_0,\ f \in P\}$. Note that for $0 \le \alpha \le \beta$,

$$K(\alpha,\beta) = P^{\alpha} \cdot K(0,\beta-\alpha), \tag{1.3}$$

and that $f \in ST(\alpha)$, $\alpha < 1$, if, and only if, $f/z \in K(0, 2-2\alpha)$.

For $\alpha \ge 0$ and $\beta \ge 0$, define the class $T(\alpha,\beta)$ as

$$T(\alpha,\beta) = \left\{ \frac{(1+xz)^{[\alpha]}(1+yz)^{\alpha-[\alpha]}}{(1+uz)^{\beta}} : |x| = |y| = |u| = 1 \right\}. \tag{1.4}$$

Note that $T(\alpha,\beta) \rightsquigarrow K(\alpha,\beta)$, $\alpha \ge 1$, $\beta \ge 1$.

A function $f \in A_0$, is called prestarlike of order α, $\alpha \le 1$, (denoted by $R(\alpha)$) if and only if $f/z \in T(0, 3-2\alpha)^{\star}$, or $f \in R(\alpha)$ if and only if

$$\begin{gathered} f * \frac{z}{(1-z)^{2-2\alpha}} \in ST(\alpha) \quad \alpha < 1, \\ \operatorname{Re} \frac{f}{z} > \frac{1}{2}, \quad z \in E\ \alpha = 1. \end{gathered} \tag{1.5}$$

Let $B(\alpha,\beta)$, $\alpha > 0$, $\beta \in \mathbb{R}$, denote the class of Bazilevic functions in E, introduced by Bazilevic [2], $f \in B(\alpha,\beta)$, $\alpha > 0$, $\beta \in \mathbb{R}$, if and only if there exists a $g \in ST(1-\alpha)$, such that for $z \in E$

$$\frac{zf'}{g}\left(\frac{f}{z}\right)^{\alpha+i\beta-1} \in P, \tag{1.6}$$

where $(f/z)^{\alpha+i\beta} = 1$ at $z = 0$. We denote $B(\alpha,0)$ by $B(\alpha)$. Bazilevic shows that $B(\alpha,\beta) \subset S$, for $\alpha > 0$, $\beta \in \mathbb{R}$. Note that

$$CV \subset ST \subset K \subset B(\alpha,\beta). \tag{1.7}$$

For further information, see [3–7].

The generalized Salagean operator $D_{\lambda}^{n} f : A \longrightarrow A$, $n \in \mathbb{N}_0 = \mathbb{N} \cup \{0\}$, $\lambda \ge 0$, is defined [8] as

$$D_{\lambda}^{n} f(z) = h_{n,\lambda}(z) * f(z), \tag{1.8}$$

where

$$h_{n,\lambda}(z) = \underbrace{h_\lambda(z) * \cdots * h_\lambda(z)}_{(n\text{-times})},$$
$$h_\lambda(z) = \frac{z-(1-\lambda)z^2}{(1-z)^2} = z + \sum_{k=2}^{\infty}[1+\lambda(k-1)]z^k. \tag{1.9}$$

The Operator $D_\lambda^n f$ satisfies the following identity:

$$D_\lambda^{n+1} f(z) = (1-\lambda)D_\lambda^n f(z) + \lambda z\left(D_\lambda^n f(z)\right)'. \tag{1.10}$$

Not that for $\lambda = 1$, $D_1^n f(z) \equiv D^n f(z)$, Salagean differential operator [9].

Let

$$k_{n,\lambda}(z) = h_{n,\lambda}^{(-1)}(z) = z + \sum_{k=2}^{\infty}\frac{z^k}{[1+\lambda(k-1)]^n}, \quad \lambda > 0, \tag{1.11}$$

we mean by $f^{(-1)}$, the solution of $f * f^{(-1)} = z/(1-z)$. Hence

$$k_{n,\lambda}(z) = \underbrace{k_\lambda(z) * \cdots * k_\lambda(z)}_{(n\text{-times})}, \tag{1.12}$$

where $k_\lambda(z) = h_\lambda^{(-1)}(z)$. It is known [10] that $k_\lambda/z \in T(1, 1+1/\lambda)^*$, hence $k_\lambda \in R(1-1/\lambda)$, and that

$$k_\lambda(z) * \frac{z}{(1-z)^{(1/\lambda)+1}} = \frac{z}{(1-z)^{1/\lambda}}. \tag{1.13}$$

The class $ST^n(\gamma)$, $\gamma \leq 1$ is defined as $f \in ST^n(\gamma)$ if and only if $D_\lambda^n f \in ST(\gamma)$. For $\lambda = 1$, we get Salagean-type n-starlike functions [9].

The operator $D_\lambda^n f$ is now called "Al-Oboudi Operator" and has been extensively studied latly, [5, 11, 12].

In this paper we define and study a class of Bazilevic functions using the operator $D_\lambda^n f$ and study some of its basic properties, inclusion relation, convolution properties coefficient bounds, and other interesting results.

2. Definition and Preliminaries

In this section, the class of n-Bazilevic functions B_λ^n, $\lambda > 0$, where $B_\lambda^0 \equiv B(1/\lambda, 0)$, is defined and some preliminary lemmas are given.

2.1. Definition

Let $f \in A$. Then $f \in B_\lambda^n$, $n \in N_0$, $\lambda > 0$, if and only if there exists a $g \in ST^n(1-1/\lambda)$, such that

$$\frac{D_\lambda^{n+1} z(f/z)^{1/\lambda}}{D_\lambda^n g(z)} \in P, \quad z \in E, \tag{2.1}$$

where the power $(f/z)^{1/\lambda}$ is chosen as a principal one.

Denote by $B_{1\lambda}^n$ the class of functions $f \in B_\lambda^n$, where $g \equiv z$.

Using (1.10), we see that $D_\lambda^{n+1} z(f/z)^{1/\lambda} = f'(z) D_\lambda^n z(f/z)^{1/\lambda - 1}$, from which the following special cases are clear.

2.1.1. Special Cases

(1) For $n = 0$, $B_{1/\alpha}^0 \equiv B(\alpha)$, $\alpha > 0$, Bazilevic [2].

(2) For $\lambda = 1$, $B_1^n \equiv K_n(0)$, Salagean type close to convex functions, Blezu [13].

(3) For $n = 0$, $\lambda = 1$, $B_1^0 \equiv K$, Kaplan [14].

(4) For $\lambda = 1$, $B_{11}^n \equiv B_{n+1}(1)$, Abdul Halim [15] and $B_{11}^n \equiv T_{n+1}^1(0)$, Opoola [16].

2.2. Lemmas

The following lemmas are needed to prove our results.

Lemma 2.1 (see [10]). *Let $\alpha, \beta \geq 1$ and $f \in T(\alpha,\beta)^*$, $g \in K(\alpha-1,\beta-1)$. Then for $F \in A$*

$$\frac{(\varphi * g\ F)}{(\varphi * g)}(E) \subset \overline{CO}\ (F(E)). \tag{2.2}$$

Lemma 2.2. *If $D_\lambda^{n+1} f \in ST(1-1/\lambda)$, $\lambda > 0$, then $D_\lambda^n f \in ST(1-1/\lambda)$.*

Proof. Since $D_\lambda^n f = k_\lambda * D_\lambda^{n+1} f$, we will show that $(k_\lambda * D_\lambda^{n+1} f) \in ST(1-1/\lambda)$. Now $D_\lambda^{n+1} f \in ST(1-1/\lambda) \subset ST(1/2 - 1/2\lambda)$, implies

$$\left(\frac{z}{(1-z)^{1/\lambda+1}}\right)^{(-1)} * D_\lambda^{n+1} f(z) \in R\left(\frac{1}{2} - \frac{1}{2\lambda}\right) \subset R\left(1 - \frac{1}{2\lambda}\right). \tag{2.3}$$

Hence

$$\left(\frac{z}{(1-z)^{1/\lambda}}\right)\left(\frac{z}{(1-z)^{1/\lambda+1}}\right)^{(-1)} * D_\lambda^{n+1} f(z) \in ST\left(1 - \frac{1}{2\lambda}\right) \subset ST\left(1 - \frac{1}{\lambda}\right). \tag{2.4}$$

From (1.13), we get the required result. □

Lemma 2.3 (see [1]). *Let $\alpha, \beta \geq 1$ and $f \in T(\alpha, \beta)^*$, $g \in K(\alpha, \beta)$. Then $f * g \in K(\alpha, \beta)$.*

For $X \subset A$, let $r_c(X)$ denote the largest positive number so that every $f \in X$ is convex in $|z| < r_c(X)$. The following result is due to Al-Amiri [17].

Lemma 2.4. *One has*

$$r_c(h_\lambda) = r_c = \frac{1}{1 + |c| + \sqrt{1 + |c| + |c|^2}}, \quad c = 2\lambda - 1, \ 0 < \lambda \leq 1. \tag{2.5}$$

Lemma 2.5 (see [18]). *Let $f, g \in H$, with $f(0) = g(0) = 0$ and $f'(0)g'(0) \neq 0$. Let $\varphi \in \nu^\star$ in $|z| < r < 1$, where*

$$\nu = \left\{ \frac{1 + xz}{1 + yz} g(z) : |x| = |y| = 1 \right\}. \tag{2.6}$$

Then for each $F \in H$,

$$\frac{(\varphi * Fg)}{(\varphi * g)}(|z| < r) \subset \overline{CO}\ (F(E)), \tag{2.7}$$

where $\overline{CO}$ stands for closed convex hull.

Remark 2.6. In [1], it was shown that condition (2.6) is satisfied for all z in E whenever φ is in CV and g is in ST.

Lemma 2.7 (see [10]). *Let $\alpha, \beta, \gamma, \delta, \mu, \nu \in \mathbb{R}$ be such that*

$$0 \leq \gamma \leq \alpha - 1, \quad 0 \leq \delta \leq \beta - 1, \quad 0 \leq \mu \leq \alpha - \gamma, \quad 0 \leq \gamma \leq \beta - \delta, \tag{2.8}$$

and let $f \in T(\alpha, \beta)^*$, $g \in K(\gamma, \delta)$, $F \in K(\mu, \nu)$. Then

$$\frac{(f * gF)}{(f * g)} \in P^{\max\{\mu, \nu\}}. \tag{2.9}$$

From (1.12) and (1.13), we immediately have;

Lemma 2.8. *One has*

$$k_{n+1,\lambda} = \frac{z}{(1 - z)^{(1/\lambda) - n}} * \left(\frac{z}{(1 - z)^{(1/\lambda) + 1}} \right)^{(-1)}, \tag{2.10}$$

Lemma 2.9 (see [10]). *Let $f \in K(\alpha,\beta)$, $\alpha,\ \beta \geq 1$. Then*

$$f \ll \frac{(1+z)^{\alpha}}{(1-z)^{\beta}}, \tag{2.11}$$

where $\ll$ stands for coefficient majorization.

3. Main Results

Theorem 3.1. *One has*

$$B_{\lambda}^{n+1} \subset B_{\lambda}^{n}, \quad \lambda > 0. \tag{3.1}$$

Proof. Let $f \in B_{\lambda}^{n+1}$. Then there exists $g \in ST^{n+1}(1-1/\lambda)$, such that

$$D_{\lambda}^{n+2} z\left(\frac{f}{z}\right)^{1/\lambda} = D_{\lambda}^{n+1} g(z) \cdot p(z), \quad p \in K(1,1). \tag{3.2}$$

Hence

$$\left(\frac{f}{z}\right)^{1/\lambda} = \frac{k_{n+1,\lambda}}{z} * \frac{k_{\lambda}}{z} * \frac{D_{\lambda}^{n+1} g}{z} \cdot p, \quad p \in K(1,1). \tag{3.3}$$

Since $D_{\lambda}^{n+1} g/z \in K(0,2/\lambda)$, and $k_{\lambda}/z \in T(1,1+1/\lambda)^{*}$, application of Lemma 2.3 gives

$$\frac{k_{\lambda}}{z} * \frac{D_{\lambda}^{n+1} g}{z} \cdot p = \left(\frac{k_{\lambda}}{z} * \frac{D_{\lambda}^{n+1} g}{z}\right) p_0, \quad p_0 \in K(1,1) \tag{3.4}$$

hence

$$\left(\frac{f}{z}\right)^{1/\lambda} = \frac{k_{n+1,\lambda}}{z} * \frac{D_{\lambda}^{n} g}{z} p_0, \quad p_0 \in K(1,1) \tag{3.5}$$

Using Lemma 2.2 we deduce that $f \in B_{\lambda}^{n}$.

As a consequence of (3.1) we immediately have the following.

Corollary 3.2. *One has*

$$B_\lambda^n \subset S. \tag{3.6}$$

□

Corollary 3.3. *If $f \in B_\lambda^n$, $n \in N_0$, $\lambda > 0$, then, for $z \in E$*

$$\left(\frac{f}{z}\right)^{1/\lambda} \in K\left(1, 1 + \frac{2}{\lambda}\right). \tag{3.7}$$

Proof. Since $f \in B_\lambda^n$, there exists a $g \in ST^n(1 - 1/\lambda)$ or $D_\lambda^n g/z \in K(0, 2/\lambda)$ such that

$$\left(\frac{f}{z}\right)^{1/\lambda} = \frac{k_{n+1,\lambda}}{z} * \frac{D_\lambda^n g}{z} \cdot p, \quad p \in K(1,1), \tag{3.8}$$

$$= \frac{\mathrm{k}_{n+1,\lambda}}{z} * F, \quad F \in K\left(1, 1 + \frac{2}{\lambda}\right), \tag{3.9}$$

using (1.3). From (3.6), we conclude that

$$\left(\frac{f}{z}\right)^{1/\lambda} = \frac{k_{n+1,\lambda}}{z} * F \neq 0, \quad 0 < |z| < 1, \tag{3.10}$$

which implies that

$$\frac{k_{n+1,\lambda}}{z} \in K\left(1, 1 + \frac{2}{\lambda}\right)^* \equiv T\left(1, 1 + \frac{2}{\lambda}\right)^*, \tag{3.11}$$

Applying Lemma 2.3 to (3.9), we get the result. □

Theorem 3.4. *Let $f \in B_\lambda^n$. Then*

$$\left(\frac{D_\lambda^{n+1} z (f/z)^{1/\lambda}}{z}\right)^\lambda \in K(\lambda, \boldsymbol{\lambda + 2}). \tag{3.12}$$

Proof. From (2.1), we see that

$$D_\lambda^{n+1} z \left(\frac{f}{z}\right)^{1/\lambda} = D_\lambda^n g(z) \cdot p(z), \quad p \in K(1,1). \tag{3.13}$$

Since $(D_\lambda^n g/z)^\lambda \in K(0,2)$, and $p(z)^\lambda \in K(\lambda,\lambda)$, then

$$\left(\frac{D_\lambda^{n+1} z(f/z)^{1/\lambda}}{z}\right)^\lambda = \left(\frac{D_\lambda^n g}{z}\right)^\lambda p(z)^\lambda \in K(\lambda, \boldsymbol{\lambda} + \mathbf{2}), \tag{3.14}$$

which is the required result. □

In the following we prove the converse of Theorem 3.1, for $0 < \lambda \le 1$.

Theorem 3.5. *Let $f \in B_\lambda^n$, $0 < \lambda \le 1$. Then $f \in B_\lambda^{n+1}$ in $|z| < r_c$, where r_c is given by* (2.5)

Proof. $f \in B_\lambda^n$ implies (2.1), where $D_\lambda^n g \in ST(1 - 1/\lambda) \subset ST$.
Now

$$\frac{D_\lambda^{n+2} z(f/z)^{1/\lambda}}{D_\lambda^{n+1} g} = \frac{h_\lambda * D_\lambda^{n+1} z(f/z)^{1/\lambda}}{h_\lambda * D_\lambda^n g}. \tag{3.15}$$

Using Lemma 2.4, we see that $h_\lambda \in CV$ in $|z| < r_c$, for $0 < \lambda \le 1$, where r_c is given by (2.5).
From Remark 2.6, we conclude

$$h_\lambda * \left\{\frac{1 + xz}{1 + yz} D_\lambda^n g : |x| = |y| = 1\right\} \neq 0. \tag{3.16}$$

Applying Lemma 2.5, we deduce

$$\left(\frac{h_\lambda * \left(D_\lambda^{n+1} z(f/z)^{1/\lambda} / D_\lambda^n g\right) D_\lambda^n g}{h_\lambda * D_\lambda^n g}\right)(|z| < r_c) \subset \overline{CO}\left(\frac{D_\lambda^{n+1} z(f/z)^{1/\lambda}}{D_\lambda^n g}\right)(E), \tag{3.17}$$

hence $D_\lambda^{n+2} z(f/z)^{1/\lambda} / D_\lambda^{n+1} g \in P$ in $|z| < r_c$, as required. □

Corollary 3.3 can be improved for $0 < \lambda \le 1$, as follows.

Theorem 3.6. *Let $f \in B_\lambda^n$, $0 < \lambda \le 1$. Then*

$$\frac{f}{z} \in K(1,2). \tag{3.18}$$

Proof. We will use Ruscheweyh's.method of proof [10]. $f \in B_\lambda^n$ implies (3.8), where $D_\lambda^{n+1} g/z \in K(0, 2/\lambda)$, $k_{n+1,\lambda}/z \in T(1, 1 + 2/\lambda)^*$.
Let $D_\lambda^n g/z = l \cdot m$, where $l = (D_\lambda^n g/z)^{(1+\lambda)/2}$ and $m = (D_\lambda^n g/z)^{(1-\lambda)/2}$.
Then $l \in K(0, 1/\lambda + 1) \cdot m \in K(0, 1/\lambda - 1)$ and $m \cdot p = F \in K(1, 1/\lambda)$. This implies

$$\left(\frac{f}{z}\right)^{1/\lambda} = \frac{k_{n+1,\lambda}}{z} * lF. \tag{3.19}$$

Using Lemma 2.7, we get

$$\frac{(k_{n+1,\lambda}/z) * lF}{(k_{n+1,\lambda}/z) * l} = F_0 \in K\left(\frac{1}{\lambda}, \frac{1}{\lambda}\right), \quad 0 < \lambda \leq 1. \tag{3.20}$$

Hence

$$\left(\frac{f}{z}\right)^{1/\lambda} = \left(\frac{k_{n+1,\lambda}}{z} * l\right) F_0, \quad F_0 \in K\left(\frac{1}{\lambda}, \frac{1}{\lambda}\right). \tag{3.21}$$

To prove that $(f/z)^{1/\lambda} \in K(1/\lambda, 2/\lambda)$, we have to show that $((k_{n+1,\lambda}/z) * l) \in K(0, 1/\lambda)$, or equivalently $k_{n+1,\lambda} * z\, l \in ST(1 - 1/2\lambda)$.

Since $z\, l \in ST((\lambda - 1)/2\lambda)$, then from (1.5)

$$\begin{gathered} \left(\frac{z}{(1-z)^{1/\lambda+1}}\right)^{(-1)} * z\, l \in R\left(\frac{\lambda - 1}{2\lambda}\right) \subset R\left(\frac{(n+2)\lambda - 1}{2\lambda}\right), \\ \frac{z}{(1-z)^{1/\lambda - n}} * \left(\frac{z}{(1-z)^{1/\lambda+1}}\right)^{(-1)} * z\, l \in ST\left(\frac{(n+2)\lambda - 1}{2\lambda}\right) \subset ST\left(1 - \frac{1}{2\lambda}\right). \end{gathered} \tag{3.22}$$

From Lemma 2.8, (1.13), and (3.22), we see that $((k_{n+1,\lambda}/z) * l) \in K(0, 1/\lambda)$. Using (3.21), we obtain $(f/z)^{1/\lambda} \in K(1/\lambda, 2/\lambda)$. From (1.1) we get the required result. □

Remark 3.7. For $n = 0$, $\lambda = 1/\alpha$ Theorem 3.6 and other stronger results depending on α, are proved by Sheil-Small [7].

For the coefficient bounds of $f \in B_\lambda^n$, Theorem 3.6 is not strong enough to settle this problem for $0 < \lambda < 1$, In 1962, Zamorski [19] proved the Bieberbach conjecture for $f \in B(\alpha)$, when $\alpha = 1, 1/2, 1/3, \ldots$, in the following we prove this result for $f \in B_\lambda^n$, using the extreme points of Kaplan class $K(\alpha, \beta)$.

Theorem 3.8. *For $f \in B_\lambda^n$, $\lambda = m \in \mathbb{N}$,*

$$f \ll \frac{z}{(1-z)^{2-mn}}. \tag{3.23}$$

Proof. From (3.9) and Lemma 2.9, we get

$$\begin{aligned} \left(\frac{f}{z}\right)^{1/\lambda} &\ll \frac{k_{n+1,\lambda}}{z} * \frac{1+z}{(1-z)^{1+2/\lambda}}, \\ &= \frac{1}{(1-z)^{2/\lambda - n}}, \end{aligned} \tag{3.24}$$

using (2.10). Raising both sides of (3.24) to the mth power, where $\lambda = m \in \mathbb{N}$, we get the required result. □

Remark 3.9. For $n = 0$, we get the result of Zamorski [19], and the result of Sheil-Small [7], from which we get the idea of proof.

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