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Research Article

A Generalized Alternative Theorem of Partial and Generalized Cone Subconvexlike Set-Valued Maps and Its Applications in Linear Spaces

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We first introduce a new notion of the partial and generalized cone subconvexlike set-valued map and give an equivalent characterization of the partial and generalized cone subconvexlike set-valued map in linear spaces. Secondly, a generalized alternative theorem of the partial and generalized cone subconvexlike set-valued map was presented. Finally, Kuhn-Tucker conditions of set-valued optimization problems were established in the sense of globally proper efficiency.

1. Introduction

Generalized convexity plays an important role in set-valued optimization. The generalization of convexity from vector-valued maps to set-valued maps happened in the 1970s. Borwein [1] and Giannessi [2] introduced and studied the cone convexity of set-valued maps. Based on Borwein and Giannessi's work, some authors [3–7] established a series of optimality conditions of set-valued optimization problems under different types of generalized convexity of set-valued maps in topological spaces. Since linear spaces are wider than topological spaces, generalizing some results of the above mentioned references from topological spaces to linear spaces is an interesting topic. Li [8] introduced a cone subconvexlike set-valued map involving the algebraic interior and established Kuhn-Tucker conditions. Huang and Li [9] studied Lagrangian multiplier rules of set-valued optimization problems with generalized cone subconvexlike set-valued maps in linear spaces. When the algebraic interior of the convex cone is empty, Hernández et al. [10] used the relative algebraic interior of the convex cone to introduce cone subconvexlikeness of set-valued maps and investigated Benson proper efficiency of set-valued optimization problems in linear spaces.

The aim of this paper is to study globally proper efficiency of set-valued optimization problems in linear spaces. This paper is organized as follows. In Section 2, we recalled some basic notions and gave some lemmas. In Section 3, we presented a generalized alternative theorem of the partial and generalized cone subconvexlike set-valued map and established Kuhn-Tucker conditions of set-valued optimization problems in the sense of globally proper efficiency.

2. Preliminaries

In this paper, let Y and Z be two real-ordered linear spaces, and let 0 denote the zero element of every space. Let K be a nonempty subset in Y. The cone hull of K is defined as cone $K := \{\lambda k \mid k \in K, \lambda \geq 0\}$. K is called a convex cone if and only if

$$\lambda_1 k_1 + \lambda_2 k_2 \in K, \quad \forall \lambda_1, \lambda_2 \geq 0, \ \forall k_1, k_2 \in K. \tag{2.1}$$

A cone K is said to be pointed if and only if $K \cap (-K) = \{0\}$. A cone K is said to be nontrivial if and only if $K \neq \{0\}$ and $K \neq Y$.

Let Y^* and Z^* stand for the algebraic dual spaces of Y and Z, respectively. Let C and D be nontrivial, pointed, and convex cones in Y and Z, respectively. The algebraic dual cone C^+ of C is defined as $C^+ := \{y^* \in Y^* \mid \langle y, y^* \rangle \geqslant 0, \forall y \in C\}$, and the strictly algebraic dual cone C^{+i} of C is defined as $C^{+i} := \{y^* \in Y^* \mid \langle y, y^* \rangle > 0, \forall y \in C \setminus \{0\}\}$, where $\langle y, y^* \rangle$ denotes the value of the linear functional y^* at the point y. The meaning of D^+ is similar to that of C^+.

Let K be a nonempty subset of Y. The linear hull span K of K is defined as span $K := \{k \mid k = \sum_{i=1}^{n} \lambda_i k_i, \lambda_i \in \mathbb{R}, k_i \in K, \ i = 1, \ldots, n\}$, and the affine hull aff K of K is defined as aff $K := \{k \mid k = \sum_{i=1}^{n} \lambda_i k_i, \sum_{i=1}^{n} \lambda_i = 1, \lambda_i \in \mathbb{R}, k_i \in K, \ i = 1, \ldots, n\}$. The generated linear subspace $L(K)$ of K is defined as $L(K) := \operatorname{span}(K - K)$.

Definition 2.1 (see [11]). Let K be a nonempty subset of Y. The algebraic interior of K is the set

$$\operatorname{cor} K := \left\{k \in K \mid \forall k' \in Y, \exists \lambda' > 0, \forall \lambda \in \left[0, \lambda'\right], k + \lambda k' \in K\right\} . \tag{2.2}$$

Definition 2.2 (see [12]). Let K be a nonempty subset of Y. The relative algebraic interior of K is the set

$$\operatorname{icr} K = \{k \in K \mid \forall v \in \operatorname{aff} K - k, \exists \lambda_0 > 0, \forall \lambda \in [0, \lambda_0], k + \lambda v \in K\}. \tag{2.3}$$

Clearly, aff $K - k = L(K)$, for all $k \in K$. Therefore, Definition 2.2 is consistent with the definition of the relative algebraic interior of K in [13, 14]. However, Definition 2.2 seems to be more convenient than the ones in [13, 14].

It is worth noting that if K is a nontrivial and pointed cone in Y, then $0 \notin \operatorname{icr} K$, and if K is a convex cone, then $\operatorname{icr} K$ is a convex set, and $\operatorname{icr} K \cup \{0\}$ is a convex cone.

Lemma 2.3 (see [13]). *If K is a convex cone in Y, then $K + \operatorname{icr} K = \operatorname{icr} K$.*

Lemma 2.4 (see [10, 12, 14]). *If K is a nonempty subset in Y, then*

(a) $\operatorname{aff} K - k = \operatorname{aff} K - K$, *for all* $k \in K$;

if K is convex in Y and $\operatorname{icr} K \neq \emptyset$, *then*

(b) $\operatorname{icr}(\operatorname{icr} K) = \operatorname{icr} K$;

(c) $\operatorname{aff}(\operatorname{icr} K) = \operatorname{aff} K$.

Lemma 2.5 (see [12]). *Let K be a convex set with* $\operatorname{icr}(K) \neq \emptyset$ *in Y. If* $0 \notin \operatorname{icr} K$, *then there exists* $y^* \in Y^* \setminus \{0\}$ *such that*

$$\langle k, y^* \rangle \geq 0, \quad \forall k \in K. \tag{2.4}$$

3. Main Results

Let A be a nonempty set, and let $F : A \rightrightarrows Y$ and $G : A \rightrightarrows Z$ be two set-valued maps on A. Write $F(A) := \bigcup_{x \in A} F(x)$ and $\langle F(x), y^* \rangle := \{\langle y, y^* \rangle \mid y \in F(x)\}$. The meanings of $G(A)$ and $\langle G(x), z^* \rangle$ are similar to those of $F(A)$ and $\langle F(x), y^* \rangle$.

Now, we introduce a new notion of the partial and generalized cone subconvexlike set-valued map.

Definition 3.1. A set-valued map $J = (F, G) : A \rightrightarrows Y \times Z$ is called partial and generalized $C \times D$-subconvexlike on A if and only if $\operatorname{cone}(J(A)) + \operatorname{icr} C \times D$ is a convex set in $Y \times Z$.

The following theorem will give some equivalent characterizations of the partial and generalized $C \times D$-subconvexlike set-valued map in linear spaces.

Theorem 3.2. *Let* $\operatorname{icr} C \neq \emptyset$. *Then the following statements are equivalent:*

(a) *the set-valued map $J : A \rightrightarrows Y \times Z$ is partial and generalized $C \times D$-subconvexlike on A,*

(b) *For all* $(c, d) \in \operatorname{icr} C \times D, \forall x_1, x_2 \in A, \forall \lambda \in]0, 1[$,

$$(c, d) + \lambda J(x_1) + (1 - \lambda) J(x_2) \subseteq \operatorname{cone}(J(A)) + \operatorname{icr} C \times D, \tag{3.1}$$

(c) $\exists c' \in \operatorname{icr} C, \forall x_1, x_2 \in A, \forall \lambda \in]0, 1[, \forall \varepsilon > 0$,

$$\varepsilon(c', 0) + \lambda J(x_1) + (1 - \lambda) J(x_2) \subseteq \operatorname{cone}(J(A)) + C \times D, \tag{3.2}$$

(d) $\exists c'' \in C, \forall x_1, x_2 \in A, \forall \lambda \in]0, 1[, \forall \varepsilon > 0$,

$$\varepsilon(c'', 0) + \lambda J(x_1) + (1 - \lambda) J(x_2) \subseteq \operatorname{cone}(J(A)) + C \times D. \tag{3.3}$$

Proof. (a) $\Rightarrow$ (b). Let $(c,d) \in \operatorname{icr} C \times D, x_1, x_2 \in A, \lambda \in]0,1[, (y_1,z_1) \in J(x_1)$, and $(y_2,z_2) \in J(x_2)$. Clearly,

$$\begin{aligned} (y_1,z_1) + (c,d) \in \operatorname{cone}(J(A)) + \operatorname{icr} C \times D, \\ (y_2,z_2) + (c,d) \in \operatorname{cone}(J(A)) + \operatorname{icr} C \times D. \end{aligned} \tag{3.4}$$

Since J is partial and generalized $C \times D$-subconvexlike on A, it follows from (3.4) that

$$\begin{aligned} &(c,d) + \lambda(y_1,z_1) + (1-\lambda)(y_2,z_2) \\ &\quad = \lambda((y_1,z_1) + (c,d)) + (1-\lambda)((y_2,z_2) + (c,d)) \in \operatorname{cone}(J(A)) + \operatorname{icr} C \times D, \end{aligned} \tag{3.5}$$

which implies that (3.1) holds.

The implications (b) $\Rightarrow$ (c) $\Rightarrow$ (d) are clear.

(d) $\Rightarrow$ (a). Let $(m_i,n_i) \in \operatorname{cone}(J(A)) + \operatorname{icr} C \times D\ (i=1,2), \lambda \in]0,1[$. Then there exist $\rho_i \geq 0, x_i \in A, (y_i,z_i) \in J(x_i)$, and $(c_i,d_i) \in \operatorname{icr} C \times D\ (i=1,2)$ such that $(m_i,n_i) = \rho_i(y_i,z_i) + (c_i,d_i)$.

Case one: if $\rho_1 = 0$ or $\rho_2 = 0$, we have $\lambda(m_1,n_1) + (1-\lambda)(m_2,n_2) \in \operatorname{cone}(J(A)) + \operatorname{icr} C \times D$.

Case two: if $\rho_1 > 0$ and $\rho_2 > 0$, we have

$$\begin{aligned} &\lambda(m_1,n_1) + (1-\lambda)(m_2,n_2) \\ &\quad = \lambda(\rho_1(y_1,z_1) + (c_1,d_1)) + (1-\lambda)(\rho_2(y_2,z_2) + (c_2,d_2)) \\ &\quad = [\lambda(c_1,d_1) + (1-\lambda)(c_2,d_2)] + [\lambda\rho_1(y_1,z_1) + (1-\lambda)\rho_2(y_2,z_2)] \\ &\quad = \beta\left\{\frac{1}{\beta}[\lambda(c_1,d_1) + (1-\lambda)(c_2,d_2)] + \left[\frac{\lambda\rho_1}{\beta}(y_1,z_1) + \frac{(1-\lambda)\rho_2}{\beta}(y_2,z_2)\right]\right\}, \end{aligned} \tag{3.6}$$

where $\beta = \lambda\rho_1 + (1-\lambda)\rho_2$.

By Lemma 2.4, we obtain

$$\begin{aligned} -c'' \in C - C \subseteq \operatorname{aff} C - C = \operatorname{aff} C - \frac{1}{\beta}[\lambda c_1 + (1-\lambda)c_2] \\ = \operatorname{aff}(\operatorname{icr} C) - \frac{1}{\beta}[\lambda c_1 + (1-\lambda)c_2]. \end{aligned} \tag{3.7}$$

Since $(1/\beta)[\lambda c_1 + (1-\lambda)c_2] \in \operatorname{icr} C = \operatorname{icr}(\operatorname{icr} C)$, there exists $\lambda_0 > 0$ such that

$$\frac{1}{\beta}[\lambda c_1 + (1-\lambda)c_2] + \lambda_0(-c'') \in \operatorname{icr} C. \tag{3.8}$$

By (3.3), (3.6), (3.8), and Lemma 2.3, we have

$$\begin{aligned}
\lambda(m_1,n_1)+(1-\lambda)(m_2,n_2) &= \beta\left\{\frac{1}{\beta}[\lambda(c_1,d_1)+(1-\lambda)(c_2,d_2)]\right. \\
&\quad \left. +\lambda_0(-c'',0)+\left[\lambda_0(c'',0)+\frac{\lambda\rho_1}{\beta}(y_1,z_1)+\frac{(1-\lambda)\rho_2}{\beta}(y_2,z_2)\right]\right\} \\
&= \beta\left\{\left(\frac{1}{\beta}[\lambda c_1+(1-\lambda)c_2]+\lambda_0(-c''),\frac{1}{\beta}[\lambda d_1+(1-\lambda)d_2]\right)\right. \\
&\quad \left. +\left[\lambda_0(c'',0)+\frac{\lambda\rho_1}{\beta}(y_1,z_1)+\frac{(1-\lambda)\rho_2}{\beta}(y_2,z_2)\right]\right\} \\
&\in \beta(\operatorname{icr} C\times D)+\operatorname{cone}(J(A))+C\times D\subseteq \operatorname{cone}(J(A))+\operatorname{icr} C\times D.
\end{aligned} \tag{3.9}$$

Cases one and two imply that $\operatorname{cone}(J(A))+\operatorname{icr} C\times D$ is a convex set in $Y\times Z$. Therefore, (a) holds. □

Remark 3.3. Theorem 3.2 generalizes the sixth item of Proposition 2.4 in [14], Lemma 2.1 in [15], and Lemma 2 in [16].

Now, we will give a generalized alternative theorem of the partial and generalized $C\times D$-subconvexlike map. We consider the following two systems.

System 1. There exists $x_0\in A$ such that $-J(x_0)\cap(\operatorname{icr} C\times D)\neq\emptyset$.
System 2. There exists $(y^*,z^*)\in(C^+\times D^+)\setminus\{(0,0)\}$ such that

$$\langle y,y^*\rangle+\langle z,z^*\rangle\geq 0,\quad \forall(y,z)\in J(A). \tag{3.10}$$

Theorem 3.4 (generalized alternative theorem). *Let* $\operatorname{icr}(\operatorname{cone}(J(A))+\operatorname{icr} C\times D)\neq\emptyset$, *and let the set-valued map* $J: A\rightrightarrows Y\times Z$ *be partial and generalized* $C\times D$*-subconvexlike on* A*. Then,*

(i) *if System 1 has no solutions, then System 2 has a solution;*

(ii) *if* $(y^*,z^*)\in C^{+i}\times D^+$ *is a solution of System 2, then System 1 has no solutions.*

Proof. (i) Firstly, we assert that $(0,0)\notin\operatorname{cone}(J(A))+\operatorname{icr} C\times D$. Otherwise, there exist $x_0\in A$ and $\alpha\geq 0$ such that $(0,0)\in\alpha J(x_0)+\operatorname{icr} C\times D$.

Case one: if $\alpha=0$, then $0\in\operatorname{icr} C$. Since C is a nontrivial, pointed, and convex cone, $0\notin\operatorname{icr} C$. Thus, we obtain a contradiction.

Case two: if $\alpha>0$, then there exists $(y_0,z_0)\in J(x_0)$ such that

$$-(y_0,z_0)\in\frac{1}{\alpha}(\operatorname{icr} C\times D)\subseteq\operatorname{icr} C\times D, \tag{3.11}$$

which contradicts that System 1 has no solutions.

Cases one and two show that our assertion is true. Since the set-valued map J is partial and generalized $C\times D$-subconvexlike on A, $\operatorname{cone}(J(A))+\operatorname{icr} C\times D$ is a convex set in $Y\times Z$. Note that $\operatorname{icr}(\operatorname{cone}(J(A)) + \operatorname{icr} C \times D) \neq \emptyset$. Thus, all conditions of Lemma 2.5 are satisfied. Therefore, there exists $(y^*, z^*) \in (Y^* \times Z^*) \setminus \{(0,0)\}$ such that

$$\langle ry + c, y^* \rangle + \langle rz + d, z^* \rangle \geq 0, \quad \forall r \geq 0, x \in A, y \in F(x), z \in G(x), c \in \operatorname{icr} C, d \in D. \tag{3.12}$$

Letting $r = 1$ in (3.12), we have

$$\langle y + c, y^* \rangle + \langle z + d, z^* \rangle \geq 0, \quad \forall x \in A, y \in F(x), z \in G(x), c \in \operatorname{icr} C, d \in D. \tag{3.13}$$

We again assert that $y^* \in C^+$. Otherwise, there exists $y' \in C$ such that $\langle y', y^* \rangle < 0$. Let $\overline{x} \in A, \overline{y} \in F(\overline{x}), \overline{z} \in G(\overline{x}), \overline{c} \in \operatorname{icr} C$, and $\overline{d} \in D$ be fixed. Then there exists sufficiently large positive number λ such that $\lambda \langle y', y^* \rangle + \langle \overline{y} + \overline{c}, y^* \rangle + \langle \overline{z} + \overline{d}, z^* \rangle < 0$, that is,

$$\langle \overline{y} + (\overline{c} + \lambda y'), y^* \rangle + \left\langle \overline{z} + \overline{d}, z^* \right\rangle < 0. \tag{3.14}$$

By Lemma 2.3, $\overline{c} + \lambda y' \in \operatorname{icr} C$. Thus, (3.14) contradicts (3.13). Therefore, $y^* \in C^+$. Similarly, we can prove that $z^* \in D^+$.

Let $c \in \operatorname{icr} C$ be fixed in (3.13). Then, $\beta c \in \operatorname{icr} C, \forall \beta > 0$. Letting $d = 0$ in (3.13), we have

$$\langle y, y^* \rangle + \beta \langle c, y^* \rangle + \langle z, z^* \rangle \geq 0, \quad \forall x \in A,\ y \in F(x),\ z \in G(x). \tag{3.15}$$

Letting $\beta \to 0$ in (3.15), we obtain

$$\langle y, y^* \rangle + \langle z, z^* \rangle \geq 0, \quad \forall x \in A, y \in F(x), z \in G(x), \tag{3.16}$$

which implies that System 2 has a solution.

(ii) If $(y^*, z^*) \in C^{+i} \times D^+$ is a solution of System 2, then

$$\langle y, y^* \rangle + \langle z, z^* \rangle \geq 0, \quad \forall x \in A,\ y \in F(x),\ z \in G(x). \tag{3.17}$$

We assert that System 1 has no solutions. Otherwise, there exist $p \in F(x_0)$ and $q \in G(x_0)$ such that $-p \in \operatorname{icr} C \subseteq C \setminus \{0\}$ and $-q \in D$. Therefore, we have $\langle p, y^* \rangle + \langle q, z^* \rangle < 0$, which contradicts (3.17). Therefore, our assertion is true. □

Remark 3.5. If $Y \times Z$ is a finite-dimensional space, then the partial and generalized $C \times D$-subconvexlikeness of $J : A \rightrightarrows Y \times Z$ implies that $\operatorname{cone}(J(A)) + \operatorname{icr} C \times D$ is a nonempty convex in $Y \times Z$, which in turn implies that the condition $\operatorname{icr}(\operatorname{cone}(J(A)) + \operatorname{icr} C \times D) \neq \emptyset$ holds trivially.

Remark 3.6. Theorem 3.4 generalizes Theorem 3.7 in [14], Theorem 2.1 in [15], and Theorem 1 in [16].

From now on, we suppose that $\operatorname{icr} C \neq \emptyset$.

Definition 3.7 (see [17]). Let $B \subseteq Y$. $\overline{y} \in B$ be called a global properly efficient point with respect to C (denoted by $\overline{y} \in \operatorname{GPE}(B, C)$) if and only if there exists a nontrivial, pointed, and convex cone C' with $C \setminus \{0\} \subseteq \operatorname{icr} C'$ such that $(B - \overline{y}) \cap (-C' \setminus \{0\}) = \emptyset$.

Now, we consider the following set-valued optimization problem:

$$\begin{aligned} &\text{Min} && F(x) \\ &\text{subject to} && -G(x) \cap D \neq \emptyset. \end{aligned} \tag{3.18}$$

The feasible set of (3.18) is defined by $S := \{x \in A \mid -G(x) \cap D \neq \emptyset\}$.

Definition 3.8. Let $\overline{x} \in S$ be called a global properly efficient solution of (3.18) if and only if there exists $\overline{y} \in F(\overline{x})$ such that $\overline{y} \in \mathrm{GPE}(F(S), C)$. The pair $(\overline{x}, \overline{y})$ is called a global properly efficient element of (3.18).

Now, we will establish Kuhn-Tucker conditions of set-valued optimization problem (3.18) in the sense of globally proper efficiency.

Theorem 3.9. *Suppose that the following conditions hold:*

(i) (x_0, y_0) *is a global properly efficient element of* (3.18)*;*

(ii) *the set-valued map* $I : A \rightrightarrows Y \times Z$ *is partial and generalized* $C \times D$*-subconvexlike on* A*, where* $I(x) = (F(x) - y_0, G(x))$*, for all* $x \in A$*.*

Then, there exists $(y^*, z^*) \in (C^+ \times D^+) \setminus \{(0,0)\}$ such that

$$\inf_{x \in A} \big(\langle F(x), y^* \rangle + \langle G(x), z^* \rangle \big) = \langle y_0, y^* \rangle, \quad \inf \langle G(x_0), z^* \rangle = 0. \tag{3.19}$$

Proof. Since (x_0, y_0) is a global properly efficient element of (3.18), there exists a nontrivial, pointed, and convex cone C' with $C \setminus \{0\} \subseteq \mathrm{icr}\, C'$ such that

$$-\big(F(x) - y_0\big) \cap \big(C' \setminus \{0\}\big) = \emptyset, \quad \forall x \in A. \tag{3.20}$$

It follows from (3.20) that

$$-\big(F(x) - y_0\big) \cap \mathrm{icr}\, C = \emptyset, \quad \forall x \in A. \tag{3.21}$$

By (3.21), we obtain

$$-I(x) \cap (\mathrm{icr}\, C \times D) = \emptyset, \quad \forall x \in A. \tag{3.22}$$

Since I is partial and generalized $C \times D$-subconvexlike on A, it follows from (3.22) and Theorem 3.4 that there exists $(y^*, z^*) \in (C^+ \times D^+) \setminus \{(0,0)\}$ such that

$$\langle F(x) - y_0, y^* \rangle + \langle G(x), z^* \rangle \geq 0, \quad \forall x \in A, \tag{3.23}$$

that is

$$\langle F(x), y^* \rangle + \langle G(x), z^* \rangle \geq \langle y_0, y^* \rangle, \quad \forall x \in A. \tag{3.24}$$

Because $x_0 \in S$, there exists $p \in G(x_0)$ such that $-p \in D$. Since $z^* \in D^+$, we have

$$\langle p, z^* \rangle \leq 0. \tag{3.25}$$

Letting $x = x_0$ in (3.24), we obtain

$$\langle p, z^* \rangle \geq 0. \tag{3.26}$$

It follows from (3.25) and (3.26) that

$$\langle p, z^* \rangle = 0. \tag{3.27}$$

Therefore, we have

$$\langle y_0, y^* \rangle \in \langle F(x_0), y^* \rangle + \langle G(x_0), z^* \rangle. \tag{3.28}$$

By (3.24) and (3.28), we have $\inf_{x \in A}(\langle F(x), y^* \rangle + \langle G(x), z^* \rangle) = \langle y_0, y^* \rangle$. Letting $x = x_0$ in (3.24), we have

$$\langle G(x_0), z^* \rangle \geq 0. \tag{3.29}$$

It follows from (3.27) and (3.29) that $\inf \langle G(x_0), z^* \rangle = 0$. □

The following theorem, which can be found in [17], is a sufficient condition of global properly efficient elements of (3.18).

Theorem 3.10. *Suppose that the following conditions hold:*

(i) $x_0 \in S$,

(ii) *there exist* $y_0 \in F(x_0)$ *and* $(y^*, z^*) \in C^{+i} \times D^+$ *such that*

$$\inf_{x \in A}(\langle F(x), y^* \rangle + \langle G(x), z^* \rangle) \geq \langle y_0, y^* \rangle. \tag{3.30}$$

Then, (x_0, y_0) *is a global properly efficient element of* (3.18).

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References

[1] J. Borwein, "Multivalued convexity and optimization: a unified approach to inequality and equality constraints," *Mathematical Programming*, vol. 13, no. 1, pp. 183–199, 1977.

[2] F. Giannessi, "Theorems of the alternative and optimality conditions," *Journal of Optimization Theory and Applications*, vol. 42, no. 3, pp. 331–365, 1984.

[3] W. D. Rong and Y. N. Wu, "Characterizations of super efficiency in cone-convexlike vector optimization with set-valued maps," *Mathematical Methods of Operations Research*, vol. 48, no. 2, pp. 247–258, 1998.

[4] Z. F. Li, "Benson proper efficiency in the vector optimization of set-valued maps," *Journal of Optimization Theory and Applications*, vol. 98, no. 3, pp. 623–649, 1998.

[5] X. M. Yang, X. Q. Yang, and G. Y. Chen, "Theorems of the alternative and optimization with set-valued maps," *Journal of Optimization Theory and Applications*, vol. 107, no. 3, pp. 627–640, 2000.

[6] X. M. Yang, D. Li, and S. Y. Wang, "Near-subconvexlikeness in vector optimization with set-valued functions," *Journal of Optimization Theory and Applications*, vol. 110, no. 2, pp. 413–427, 2001.

[7] P. H. Sach, "New generalized convexity notion for set-valued maps and application to vector optimization," *Journal of Optimization Theory and Applications*, vol. 125, no. 1, pp. 157–179, 2005.

[8] Z. Li, "The optimality conditions for vector optimization of set-valued maps," *Journal of Mathematical Analysis and Applications*, vol. 237, no. 2, pp. 413–424, 1999.

[9] Y. W. Huang and Z. M. Li, "Optimality condition and Lagrangian multipliers of vector optimization with set-valued maps in linear spaces," *Operation Research Transactions*, vol. 5, no. 1, pp. 63–69, 2001.

[10] E. Hernández, B. Jiménez, and V. Novo, "Weak and proper efficiency in set-valued optimization on real linear spaces," *Journal of Convex Analysis*, vol. 14, no. 2, pp. 275–296, 2007.

[11] J. van Tiel, *Convex Analysis*, John Wiley and Sons, New York, NY, USA, 1984.

[12] S. Z. Shi, *Convex Analysis*, Shanghai Science and Technology Press, Shanghai, China, 1990.

[13] M. Adán and V. Novo, "Efficient and weak efficient points in vector optimization with generalized cone convexity," *Applied Mathematics Letters*, vol. 16, no. 2, pp. 221–225, 2003.

[14] M. Adán and V. Novo, "Partial and generalized subconvexity in vector optimization problems," *Journal of Convex Analysis*, vol. 8, no. 2, pp. 583–594, 2001.

[15] Q. L. Wang and Z. M. Li, "Alternative theorem for (u, O_2)-generalized subconvexlike maps and its application to vector optimization," *Applied Mathematics A*, vol. 20, no. 4, pp. 459–464, 2005.

[16] Z. A. Zhou, "Weak efficient solutions of generalized convex set-valued vector optimization problems," *Journal of Southwest China Normal University*, vol. 30, no. 2, pp. 221–225, 2005.

[17] Z. A. Zhou, X. M. Yang, and J. W. Peng, "Optimality conditions of set-valued optimization problem involving relative algebraic interior in ordered linear spaces," *Optimization*. In press.