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Research Article

Modelling and Regularity of Nonlinear Impulsive Switching Dynamical System in Fed-Batch Culture

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A hybrid system with state-based switchings is proposed to describe the fed-batch production of 1,3-propandiol from glycerol in our previous work. However, the on-off switching of alkali is too frequent, which greatly increases the computational cost of the numerical solution to the system so as to locate the state-based switchings in strict time order and implement the correct mode changes. To deal with this problem, we consider the switching of alkali pump as an impulsive event and present a nonlinear impulsive switching system to describe the fed-batch culture. It is proved that the impulsive switching system is non-Zeno. Some basic properties of solutions to the impulsive switching system are also explored. In order to overcome the discontinuities of the system, the Skorohod topology is induced and a specific form of λ is constructed to prove the main theorem. Additionally, a numerical simulation is carried out to show that the proposed system can describe the fed-batch culture properly and the essential difference with the previous work.

1. Introduction

Glycerol can be converted to 1,3-PD by several microorganisms [1–3]. This bioconversion process is of technical interest since the product 1,3-PD has numerous applications in polymers, cosmetics, food, lubricants, and medicines [4]. The fermentation of glycerol by *K. pneumoniae* under anaerobic conditions is a complex bioprocess, since microbial growth is subjected to multiple inhibitions of substrate and products [5]. Almost all of the existing culture techniques, including batch culture, fed-batch culture, and continuous culture, have been practiced. By the fed-batch fermentation of glycerol, due to greatly eliminating substrate inhibition, more glycerol is used and more biomass is obtained, resulting in high 1,3-PD concentration and productivity [6]. Therefore, a lot of experimental researches have been done on this culture [7–10].

Many biological phenomena and their optimal controls cannot be formulated due to continuous processes using ordinary or partial differential equations merely. The applications of the impulsive differential equations emerge in pharmacokinetics, population dynamics, mathematical in epidemiology, optimal management of renewable resources, and so forth. [11]. Impulsive dynamical systems, which can be viewed as a subclass of hybrid systems, consist of three components: a continuous-time differential equation, which governs the motion of the dynamical system between impulsive and resetting events; a difference equation, which characterizes the jump behavior of the system states when a resetting event occurs; and a condition for determining when the states of the system are to be reset [12].

During the fed-batch fermentation, glycerol and alkali are discontinuously added to the reactor at constant rates, so as to keep the substrate concentration and the pH in the desirable levels. In order to describe the open loop glycerol input and pH logic control, the hybrid system based on an output equation of the pH was firstly established [13]. However, in the real process of fermentation, the effect of alkali feeding to the pH value is much bigger than the effect of glycerol feeding on the order of magnitude. Additionally, in the numerical simulation, the dwell time of alkali feeding is shortest among the four possible modes. So the stepsize has to be very small. The cost of calculation will rise and it is disadvantage for further study on the parameter identification and optimal control of the hybrid system in which the hybrid system should be repeatedly computed. To overcome this difficulty, the process of alkali feeding can be described by an impulsive process. It can greatly reduce the cost of numerical calculation and describe the fed-batch culture properly. When the impulsive switching system is applied to describe the fed-batch culture, it will be found that the continuous dependence of the solution on the initial state or the kinetic parameters fails to be satisfied in the common L_2-norm [13], which is inconsistent with the actual physical system of the fed-batch fermentation. The reason for this discontinuity is that the time scale of the impulsive switching solutions with different initial state or parameters is inconsistent [13] and L_∞-norm is invalid for exploring the continuity property in this sense. In this work, we overcome the structural weaknesses by inducing the Skorohod topology. Skorohod topology was presented in 1960s [14], it grew matured, and had some important properties in theory. However, in the aspect of applied science, it is not trivial to construct the specific form of the λ. In this paper, we construct a specific form of λ to prove the main theorem.

This paper is organized as follows. In Section 2, the background, impulsive switching system of microbial fed-batch fermentation and its properties are introduced. In Section 3, we discuss the main theorem about continuous dependence on parameters and initial state by inducing the Skorohod topology, and a specific form of λ is constructed to prove it. Numerical simulation of a fed-batch experiment is carried out in Section 4. Discussions and conclusions are presented at the end of this paper.

2. Background, Dynamics, and Property

2.1. Model Formulation

The fed-batch culture begins with batch fermentation, then batch-fed glycerol and alkali are poured into the reactor in order that the concentration of glycerol keeps in a proper range and the pH of the solution in the desirable level.

According to the factual experiments, we make the following assumptions.

(H1) The concentrations of reactants are uniform in reactor, while time delay and nonuniform space distribution are ignored.

(H2) The feeding media includes only fixed concentrations of glycerol and alkali.

In the laboratory, the effect of alkali feeding on the pH value is much bigger than the effect of glycerol feeding on the order of magnitude, and the time of alkali feeding is relatively shorter than the whole experiment time. The impulsive switching system is fitter for describing the actual process than the continuous system.

According to the above description, in this paper, we let $F_N = 0$ in [13], that is, the flow rate of alkali is zero. Glycerol feeding strategy is determined by a preassigned times sequence. Let $t_0 = \bar{t}_0 < \bar{t}_1 < \bar{t}_2 < \cdots < \bar{t}_{2N} = T$ be a partition of $\mathcal{T} := [t_0, T]$. $[\bar{t}_0, \bar{t}_1]$ is a period of time of batch fermentation, $\bar{t}_i$, $i \in \{1, 3, \ldots, 2N-1\}$ is the moment of starting the inlet flow of glycerol, and $\bar{t}_{i+1}$ is the moment of ending this input. So, for convenience, we let

$$j(t) = \begin{cases} 0, t \in \left[\bar{t}_i, \bar{t}_{i+1}\right) \Longleftrightarrow F_G = v, \\ 1, t \in \left[\bar{t}_{i+1}, \bar{t}_{i+2}\right) \Longleftrightarrow F_G = 0, \end{cases} \quad i \in \{1, 3, \ldots, 2N-1\}. \tag{2.1}$$

Under the assumptions (H1) and (H2), the fed-batch process can be formulated by

$$\begin{aligned} \frac{dx_1(t)}{dt} &= (\mu - d)x_1(t) - \frac{F_G}{x_7(t)}x_1(t) := f_1^{j(t)}(t, x, p), \\ \frac{dx_2(t)}{dt} &= -q_2 x_1(t) + \frac{F_G}{x_7(t)}(C_{s_0} - x_2(t)) := f_2^{j(t)}(t, x, p), \\ \frac{dx_3(t)}{dt} &= q_3 x_1(t) - \frac{F_G}{x_7(t)}x_3(t) := f_3^{j(t)}(t, x, p), \\ \frac{dx_4(t)}{dt} &= q_4 x_1(t) - \frac{F_G}{x_7(t)}x_4(t) := f_4^{j(t)}(t, x, p), \\ \frac{dx_5(t)}{dt} &= q_5 x_1(t) - \frac{F_G}{x_7(t)}x_5(t) := f_5^{j(t)}(t, x, p), \\ \frac{dx_6(t)}{dt} &= -\frac{F_G}{x_7(t)}x_6(t) := f_6^{j(t)}(t, x, p), \\ \frac{dx_7(t)}{dt} &= F_G := f_7^{j(t)}(t, x, p), \end{aligned} \tag{2.2}$$

where $x := (x_1, x_2, x_3, x_4, x_5, x_6, x_7)$ is the continuous state vector; $p := (d, \mu_m, k_s, m_2, m_3, m_4, m_5, Y_2, Y_3, Y_4, Y_5, \Delta_2, \Delta_3, \Delta_4, \Delta_5, K_2^*, K_3^*, K_4^*, K_5^*, \gamma)$ is the parameter vector. The specific growth rate of cells μ, specific consumption rate of substrate q_2, and specific formation rate

of products q_i, $i = 3,4,5$, are expressed by the following equations based on previous works [5, 13, 15]:

$$\mu = \begin{cases} \mu_m \dfrac{x_2}{x_2 + k_s}\left(1 - \dfrac{x_1}{x_1^*}\right)\left(1 - \dfrac{x_2}{x_2^*}\right)\left(1 - \dfrac{x_3}{x_3^*}\right)\left(1 - \dfrac{x_4}{x_4^*}\right)\left(1 - \dfrac{x_5}{x_5^*}\right), \\ \qquad \text{if } 0 \le x_i \le x_i^*, \quad i = 1,2,3,4,5, \\ 0, \quad \text{otherwise} \end{cases}$$

$$\begin{aligned} q_2 &= m_2 + \frac{\mu}{Y_2} + \Delta_2 \frac{x_2}{x_2 + K_2^*}, \\ q_3 &= m_3 + \mu Y_3 + \Delta_3 \frac{x_2}{x_2 + K_3^*}, \\ q_4 &= m_4 + \mu Y_4 + \Delta_4 \frac{x_2}{x_2 + K_4^*}, \\ q_5 &= m_5 + \mu Y_5 + \Delta_5 \frac{x_2}{x_2 + K_5^*}. \end{aligned} \tag{2.3}$$

According to the factual experiments, we only discuss the fermentation under acidic environment in this paper. Since the added NaOH is the only basic source, we can make the following assumption.

(H3) During the whole process of fed-batch culture, there exists a constant $M > 0$ such that $x_4 - \gamma x_6 \ge M$.

For mathematical convenience, we let $\mathcal{D}_{ad} := \prod_{i=1}^{20} [p_i^l, p_i^u] \subset \mathbb{R}^{20}$ is the admissible set of parameter vector; $W_{ad} := [0, x_1^*] \times [0, x_2^*] \times [0, x_3^*] \times [0, x_4^*] \times [0, x_5^*] \times [0, x_6^*] \times [0, x_7^*] \subset \mathbb{R}_+^7$ is the admissible set of the continuous state vector x.

Under the assumptions (H1)–(H3), the pH at time $t (\in \mathcal{J})$ can be formulated by the following output equation according to Dang [16]:

$$\begin{aligned} \mathrm{pH}(t) &= y_{\mathrm{pH}}(x(t)) \\ &= \begin{cases} pK_a - \lg \dfrac{x_4 - \gamma x_6}{\gamma x_6} & \text{if } x_6 \ge \epsilon_0, \\ -\lg\left(\dfrac{-K_a + \sqrt{K_a^2 + 4K_a x_4 / (1000\gamma)}}{2} + \sqrt{K_w^-} \right) & \text{otherwise.} \end{cases} \end{aligned} \tag{2.4}$$

Here $pK_a = -\lg(K_a)$; $K_w^- = 1 \times 10^{-14}$; ϵ_0 is a sufficient small constant, below which the concentration of NaOH can be ignored while computing the pH.

To ensure that the pH is restricted in its admissible range, the following two inequalities must hold during the entire fermentation time:

$$\begin{aligned} h_0(x(t)) &:= \mathrm{pH}^* - y_{\mathrm{pH}}(x(t)) \ge 0, \\ h_1(x(t)) &:= y_{\mathrm{pH}}(x(t)) - \mathrm{pH}_* \ge 0. \end{aligned} \tag{2.5}$$

2.2. Impulsive Switching System and Properties

Now, we also need to identify the rule which is the volume of alkali at each impulsive instant. In this paper, the volume of alkali can exactly make the value of pH from pH^* to pH_*. Thus, we can deduce the specific form of impulsive volume. Then, we can describe the process of fed-batch fermentation with open loop glycerol input and pH logic control by the following impulsive switching system.

Defining surface $S_k := \{(\tau_k, x(\tau_k)) \mid h_1(x(\tau_k)) = 0\}$, on the basis of (H3), we have $\tau_k < \tau_{k+1}$ and $\tau_k \to \infty$, while $k \to \infty$,

$$\begin{aligned} \dot{x}(t) = f^{j(t)}(t,x,p) = \left(f_1^{j(t)}(t,x),\ldots,f_7^{j(t)}(t,x)\right)^T, \quad (t,x(t)) \in S_k, \\ \Delta x = I(t,x,p), \quad (t,x(t)) \overline{\in} S_k, \\ x\left(t_0^+\right) = x_0, \end{aligned} \tag{2.6}$$

where $I(t,x,p) = (x_1(t)\Delta V/(V+\Delta V),\ldots,x_1(t)\Delta V/(V+\Delta V),(\rho - x_6(t))\Delta V/(V+\Delta V),\Delta V)^T$. The solution of system (2.6) corresponding to a parameter vector is denoted by $x(t;p)$.

Given any $j \in \{0,1\}$, it is easy to verify that the function f^j in (2.6) satisfies the following property referring [17, 18].

Property 1. The functions f^j, $j \in \{0,1\}$, and I defined in (2.6) satisfy

(i) f^j is Lipschitz in x on X and continuous in p on $\mathcal{P}_{ad}$;

(ii) there exists positive constant α such that the linear growth condition holds, that is,

$$\begin{aligned} \left\| f^j(t,x,p) \right\| \le \alpha(\|x\|+1) \quad \forall x \in X,\ p \in \mathcal{P}_{ad}, \\ \left\| I(t,x,p) \right\| \le \alpha(\|x\|+1) \quad \forall x \in X, \end{aligned} \tag{2.7}$$

where $\|\cdot\|$ is the Euclidean norm.

Property 2. Under the assumptions (H1)–(H3), given $p \in \mathcal{P}_{ad}$, the system (2.6) needs at most finitely many times of switchings over the time interval $\mathcal{T}$. That is, the system is non-Zeno.

Proof. On the basis of the proof of Property 2 in [13], we only need to prove that the number of impulsive instants is finite.

Without loss of generality, consider a period of time with the moment to start alkali pump as initial time and the next moment to start alkali pump as end time, and assume that glycerol pump is running during this phase. The fermentation process in this phase can be described by the following subsystem:

$$\dot{x} = f^{j(t)}(t,x,p), \quad x(\tau_k) = x\left(\tau_k^-\right) + I(\tau_k,x,p), \quad t \in [\tau_k, \tau_{k+1}). \tag{2.8}$$

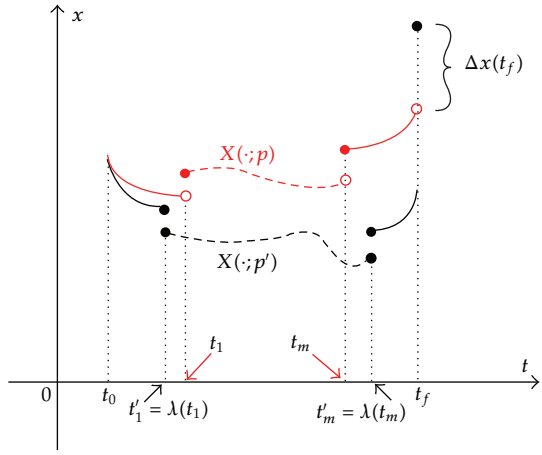

Figure 1: Comparison of two solutions with same initial data and different parameters.

According to Property 1 and the classical theory of differential equations, there exists a unique solution to the subsystem (2.8) on the time interval $[\tau_k, \tau_{k+1})$, denoted by $x^t(\cdot; x^{\tau_k})$, which is given by

$$x^t(\cdot; x^{\tau_k}) = x(\tau_k^-) + \int_{\tau_k}^{t} f^{j(s)}(s, x, p)ds, \quad \forall t \in [\tau_k, \tau_{k+1}). \tag{2.9}$$

According to the impulsive rule of the alkali, τ_k and τ_{k+1} are, respectively, the instants that the pH reaches its allowable upper and lower bounds. Thus, the total variation of $pH(\cdot)$ over $[\tau_k, \tau_{k+1})$, $V_{\tau_k}^{\tau_{k+1}}(pH)$, is given by

$$V_{\tau_k}^{\tau_{k+1}}(\mathrm{pH}) \geq \mathrm{pH}^* - \mathrm{pH}_* \triangleq \Delta\mathrm{pH}. \tag{2.10}$$

Consequently, we can follow the method in [13] step by step to prove the desired results. □

3. Skorohod Topology and Regularity Properties

It should be mentioned that impulsive dynamical systems can be viewed as a subclass of hybrid systems [12]. According to our similar analysis about Property 2 and the proofs of Theorems 1 and 2 in [13], we can verify that the time viable is of continuous dependence on parameters and initial state. However, the property of continuous dependence about the solution of the impulsive switching system is not trivial. For example, we can see from Figure 1, intuitively, it is a common property that the trajectory of the parameter p' should continuously approach the trajectory of the parameter p. But, the property is obviously failed in the L_∞-norm. The nature of these discontinuities is that the time scale of the impulsive switching solutions with different parameters is inconsistent. So, we overcome the structural weaknesses by inducing the following Skorohod topology.

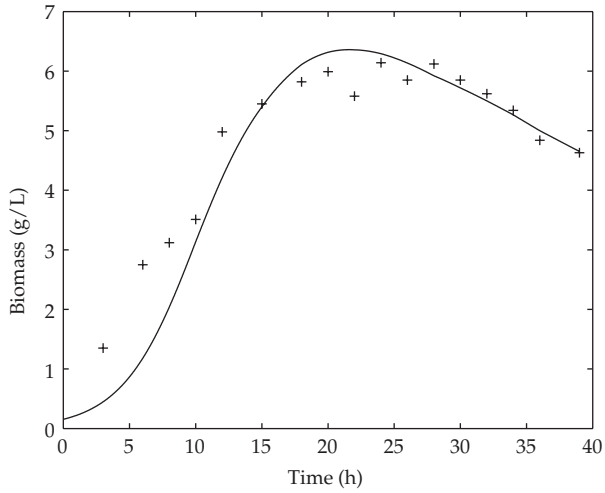

Figure 2: Comparison of biomass concentration between experimental data and computational results.

Since we can never expect to have the continuity of $x(t;p)$ with respect to p at $t^* = \tau_k(x(t^*;p))$ for some $k \geq 1$, we have the definition of continuous dependence on parameters of impulsive dynamical system as follows, which is similar definition of continuous dependence on initial value of impulsive dynamical system in [11].

Definition 3.1. The solutions $x(t;p)$ of system (2.6) are said to have continuous dependence relative to p if and only if

(a)

$$\lim_{p' \to p} x(t;p') = x(t;p) \text{ if } t \neq \tau_k(x(t;p)) \quad \forall k \geq 1, \tag{3.1}$$

(b) for any $\epsilon > 0$ there is a closed set $J_\epsilon \subset J$ and a $\delta > 0$ such that $m(J/J_\epsilon) < \epsilon$ and

$$\|x(t,p') - x(t,p)\| < \epsilon, \quad t \in J_\epsilon \tag{3.2}$$

provided

$$\|p' - p\| < \delta, \tag{3.3}$$

where m denotes the Lebesgue measure.

In (3.2), t and t' may be different in response to different parameters. The impulsive instant is based on the feedback of the state, which is not preassigned. So, in order to prove the continuous dependence on parameters, the general Euclidean norm cannot give us the desired result, we also need the Skorohod topology defined in [14].

Let $D = D_{[t_0,t_f]}$ be the space of functions x on $[t_0, t_f]$ that are right-continuous and have left limits. Λ denotes the class of strictly increasing, continuous mappings of $[t_0, t_f]$ onto itself.

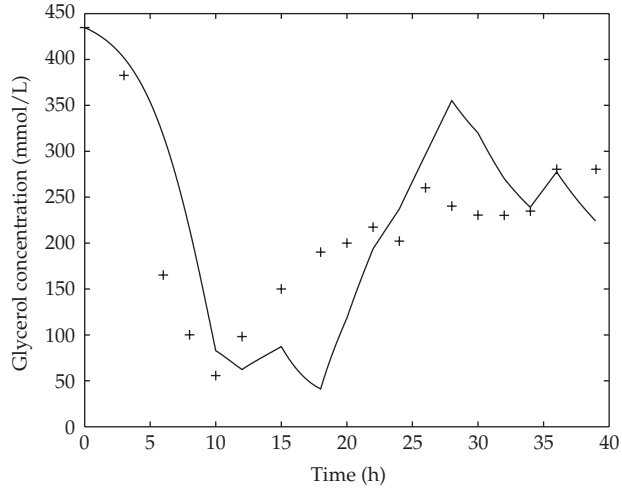

Figure 3: Comparison of glycerol concentration between experimental data and computational results.

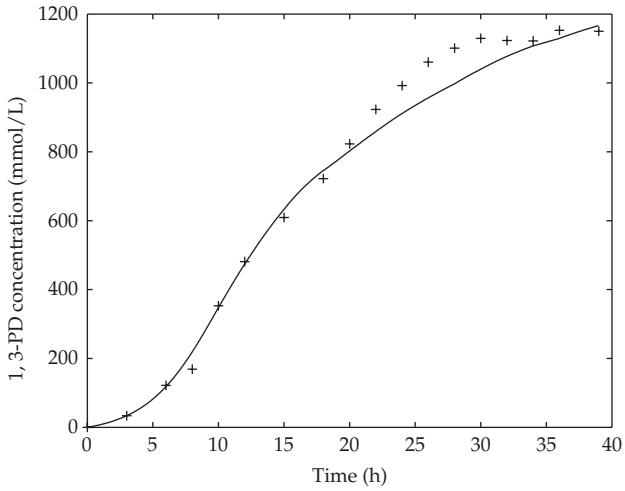

Figure 4: Comparison of 1,3-PD concentration between experimental data and computational results.

If $\lambda \in \Lambda$, then $\lambda(0) = 0$ and $\lambda(t_f) = t_f$. For x and y in D, define the Skorohod topology to be the infimum of those positive ϵ for which there exists in Λ a λ such that

$$\begin{aligned} \sup_t |\lambda(t) - t| &\leq \epsilon, \\ \sup_t \left| x(t) - y(\lambda(t)) \right| &\leq \epsilon. \end{aligned} \tag{3.4}$$

Here, λ is kind of time scaling, we can use it to compare the different solution with respect to the different parameters. With the above preparation, we can prove the next theorem.

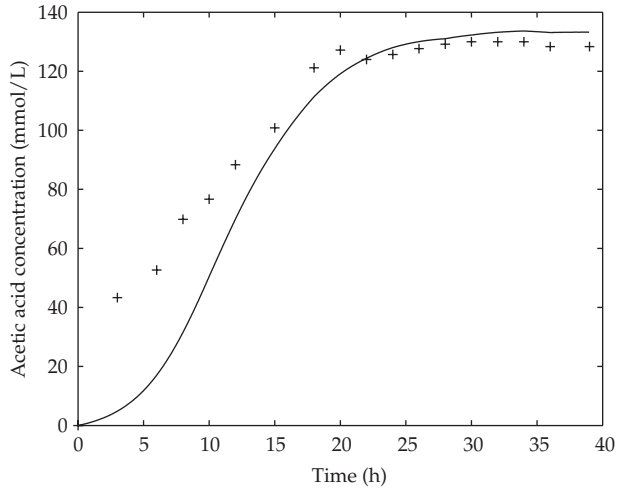

Figure 5: Comparison of acetic acid concentration between experimental data and computational results.

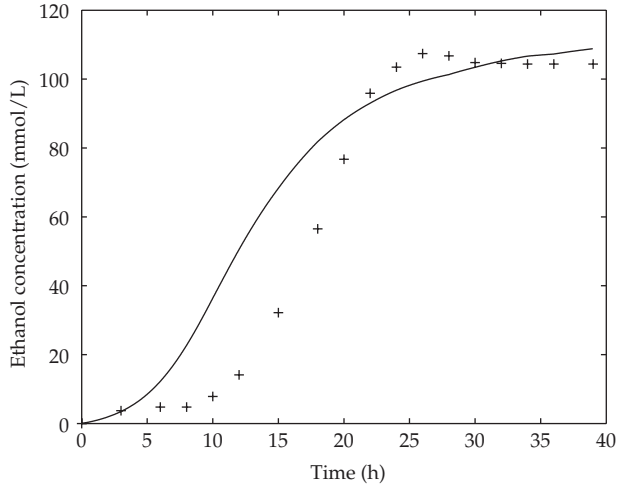

Figure 6: Comparison of ethanol concentration between experimental data and computational results.

Remark 3.2. Skorohod topology was presented in 1960s [14]; it grew matured and had some important properties in theory. However, in the aspect of applied science, it is not trivial to construct the specific form of the λ.

Theorem 3.3. *Under the assumptions (H1)–(H3), given $p \in \Omega_{ad}$ and glycerol switching signal vector σ, the solution of system* (2.6)*is of continuous dependence on parameters. That is,* (3.2) *is satisfied.*

Proof. On the basis of Property 2, without loss of generality, we only need to prove the theorem for a special situation showed in Figure 1, that is, the number of impulsive instant of the solutions $x(t,p)$ is one more time than the solution $x(t,p')$; furthermore, $x(t,p)$ has a

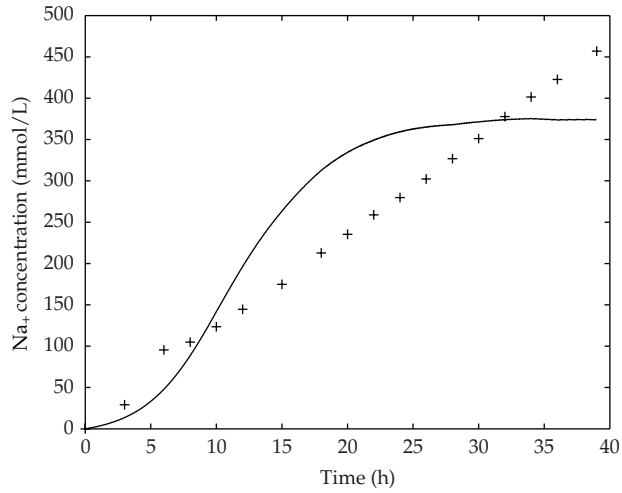

Figure 7: Comparison of Na^+ ions concentration between experimental data and computational results.

impulsive instant at the final time t_f. Consequently, a formulation of λ is presented as follows:

$$\lambda(t) = \begin{cases} t_0(p') + \dfrac{t_1(p') - t_0(p')}{t_1(p) - t_0(p)}(t - t_0(p)), & t \in [t_0(p), t_1(p)], \\ \vdots & \\ t_i(p') + \dfrac{t_{i+1}(p') - t_i(p')}{t_{i+1}(p) - t_i(p)}(t - t_i(p)), & t \in [t_i(p), t_{i+1}(p)], \\ \vdots & \\ t_m(p) + \dfrac{t_m(p') - t_{m-1}(p')}{t_m(p) - t_{m-1}(p)}(t - t_{m-1}(p)), & t \in [t_{m-1}(p), t_m], \\ t_f, & t \in [t_m, t_f]. \end{cases} \tag{3.5}$$

In view of the definition of $\lambda(t)$, the time scale of different impulsive instant is transformed to be consistent, so we only need to prove the desired result on the time interval $[t_m, t_f]$. For different parameters p and p', we have

$$\begin{aligned} x\left(t_f^+, p\right) &= x\left(t_f^-, p\right) + \Delta x(t_f), \\ x(\lambda(t), p') &= x\left(t_M^-, p'\right) + \Delta x\left(t_m^-\right) + \int_{t_m}^{t} \dot{x} ds, \end{aligned} \tag{3.6}$$

while $p' \to p$, $t_m \to t_f$. By virtue of implicit function theorem, we can easily obtain that $\Delta x(t_m^-) \to \Delta x(t_f)$.

Summing up the above, we can obtain our desired result. □

Table 1: Glycerol feeding strategy.

Time period	0 h–10 h	10 h–12 h	12 h–15 h	15 h–18 h	18 h–20 h
Feeding time (s/100 s)	0	1.25	1.61	1.65	2.22
Time period	20 h–22 h	22 h–24 h	24 h–26 h	26 h–28 h	28 h–30 h
Feeding time (s/100 s)	2.32	1.77	1.89	1.23	1.0
Time period	30 h–32 h	32 h–34 h	34 h–36 h	36 h–39 h	
Feeding time (s/100 s)	0.76	0.97	1.58	0.89	

Table 2: Parameters values of each reactant in the impulsive switching system.

Reactant	μ_m	k_s	x_i^*	m_i	Y_i	Δ_i	K_i
Biomass	0.67	0.28	10				
Glycerol			2039	2.20	0.0082	28.58	11.43
1,3-PD			1300	–2.69	67.69	26.59	15.50
Acetic acid			026	–0.97	33.07	5.74	85.71
Ethanol			360.9	–0.97	33.07	5.74	85.71

4. Numerical Simulation of Microbial Fed-Batch Culture

A fed-batch fermentation was carried out under anaerobic conditions at 37°C starting from the batch process (i.e., $j_0 = 0$) with initial continuous state vector $x_0 = (0.155, 434.783, 0.0,0.0, 0.0, 0.0, 2.0)$. The pH was controlled in 6.48–6.52, that is, $pH_* = 6.48$ and $pH^* = 6.52$. The total fermentation time was 39 hours, which was divided into time units of 100 seconds, that is, there were 39×36 time units. The flow rate of glycerol was $v = 0.80\,\mathrm{L/h}$, and the corresponding concentrations were $C_{s_0} = 12888\,\mathrm{mmol/L}$ and $\rho = 5000\,\mathrm{mmol/L}$, respectively. Glycerol feeding strategy is shown in Table 1.

The solution was sampled $l(= 18)$ times during the fermentation process to measure the concentrations of biomass, glycerol, 1,3-PD, acetic acid, and ethanol, accompanied with on-line measurement of the volume of the solution. The concentration of Na^+ ions was directly calculated from the mount of added NaOH and the volume of the solution.

The parameters d and γ were roughly estimated based on the experimental data. The other parameters were referred to [5] listed in Table 2. The impulsive switching system was numerical solved by using the Euler method. The step size of the Euler method was 1/72000 h, which was derived empirically after several times of numerical experiments.

The numbers of times and the volumes of feeding glycerol and alkali were calculated. The feeding time of glycerol is 0.4151 h. The numbers of times of feeding glycerol and alkali are 1044 and 6160, respectively. The feeding volumes of feeding glycerol and alkali are $0.3326\,\mathrm{L}^{-1}$ and $0.1887\,\mathrm{L}^{-1}$, respectively. Figures 2, 3, 4, 5, 6, 7, and 8 show the comparison of all components between experimental data and computational results, where the stars denote the experimental data and the solid lines denote the computational curves. Figure 9 shows that the pH is controlled within the desirable region during most of the fermentation time. There is no much difference between our results and previous ones in [13]. That is the total fermentation time was divided into time units of 100 seconds, it is not noticeable to see the impulsive effect from Figures 1–8. Therefore, for example, in the case of simulation

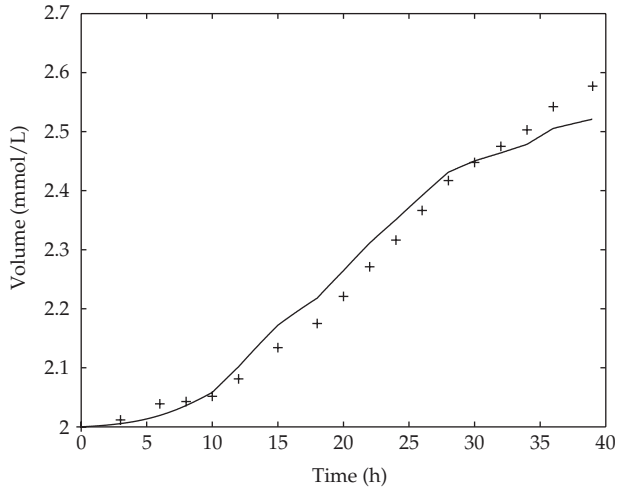

Figure 8: Comparison of the volume of solution between experimental data and computational results.

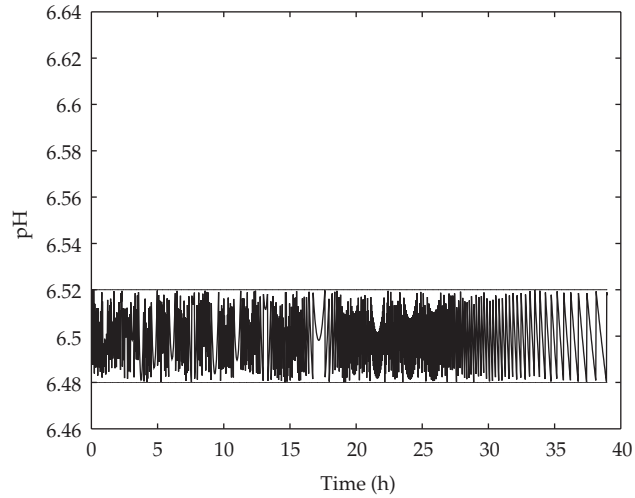

Figure 9: Simulation of the pH during 30–33 hours of fed-batch culture.

of the pH, we plot the value of pH during 30–33 hours on the step size of the Euler method to show the impulsive effect in Figure 10. It is essential difference between the two systems.

5. Discussions and Conclusions

In this paper, a nonlinear impulsive switching system based on [13] is developed to describe the fed-batch culture with open loop glycerol input and pH logic control. To be distinct from [13], the pH logic control, that is, the process of alkali feeding is described by impulsive process according to the factual fermentation. We consult the method of proof in [13] to prove some basic properties of the solution to the system, including the

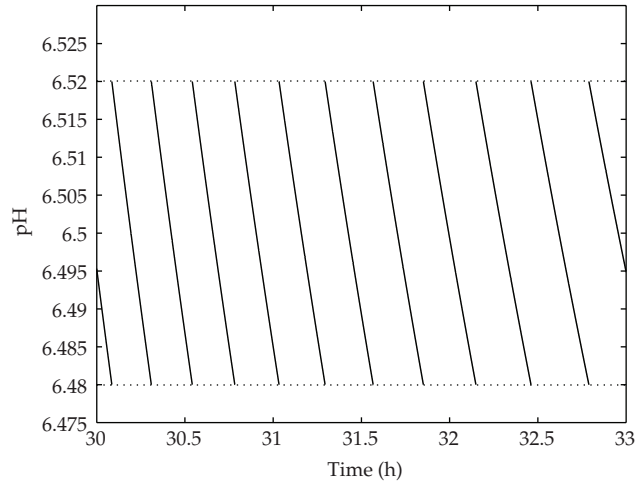

Figure 10: Simulation of the pH during 30–33 hours of fed-batch culture.

existence, uniqueness, boundedness, and continuous dependence with respect to initial-state-parameter pair. Numerical simulation of a factual experiment is carried out, illustrating that the proposed system can describe the concentrated fed-batch culture properly and the essential difference with the previous work.

Our current work accommodate the simulation of the fermentation process. In a future work, we will consider the stability and reachability of the impulsive switching model. Additionally, the objective of our efforts is to delve into the optimal control of the impulsive switching system.

Nomenclature

C_{s_0}: Concentration of glycerol in feed medium (mmol L^{-1})
d: Specific decay rate of cells (h^{-1})
F_G: Flow rates of glycerol ($L\,h^{-1}$))
k_s: Monod saturation constant for substrate (mmol L^{-1})
K_a: The averaged dissociation constant of acid byproducts
K_w^-: Dissociation constant of water
K_i^*: Saturation constants for substrate and product in kinetic equations with excess terms (mmol L^{-1}), $i = 2, 3, 4, 5$
m_i: Maintenance term of substrate consumption and product formation under substrate-limited conditions (mmol $g^{-1}\,h^{-1}$), $i = 2, 3, 4, 5$
x_1: Biomass concentration (g L^{-1})
x_2: Substrate (glycerol) concentration (mmol L^{-1})
x_i: Product (1,3-PD, EtOH, HAc) concentration (mmol L^{-1}), $i = 3, 4, 5$
x_6: Concentration of Na^+ ions coming from the added NaOH
x_7: Volume of feed medium (L)
x_1^*: Carrying capacity of the environment (g L^{-1})
x_2^*: Maximum residual substrate concentration (mmol L^{-1})
x_i^*: Maximum product (1,3-PD, EtOH, HAc) concentration (mmol L^{-1}), $i = 3, 4, 5$

x_6^*: Critical concentration of NaOH
x_7^*: Maximum working volume for the bioreactor
Y_2, Y_i: Maximum growth yield ($g\,mmol^{-1}$) and product yield ($mmol\,g^{-1}$) $i = 3, 4, 5$.

Greek Letters

μ, μ_m: Specific and maximum specific growth rates (h^{-1})
ρ: Concentration of NaOH in feed medium
γ: Ratio of acetic acid concentration to the total acid byproducts concentration
Δ_i: Maximum increment of substrate consumption rate and product formation rate under substrate-sufficient conditions ($mmol\,g^{-1})\,h^{-1}$), $i = 2, 3, 4, 5$.

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