

Research Article

New Iterative Manner Involving Sunny Nonexpansive Retractions for Pseudocontractive Mappings

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Iterative methods for pseudocontractions have been studied by many authors in the literature. In the present paper, we firstly propose a new iterative method involving sunny nonexpansive retractions for pseudocontractions in Banach spaces. Consequently, we show that the suggested algorithm converges strongly to a fixed point of the pseudocontractive mapping which also solves some variational inequality.

1. Introduction

Let C be a nonempty closed convex subset of a real Banach space E . A mapping $T : C \rightarrow C$ is said to be nonexpansive, if

$$\|Tx - Ty\| \leq \|x - y\|, \quad (1.1)$$

for all $x, y \in C$.

Now we know that the involved operators in the many practical applications can be reduced to the nonexpansive mappings, that is, there are a large number of applied areas which are closely related to the nonexpansive mappings, for example, inverse problem, partial differential equations, image recovery, and signal processing. Based on these facts, recently, iterative methods for finding fixed points of nonexpansive mappings have received vast investigations. For related works, please see [1–26] and the references therein.

In the present paper, we focus on a class of strictly pseudocontractive mappings which strictly includes the class of nonexpansive mappings. Recall that a mapping $T : C \rightarrow C$ is

said to be strictly pseudocontractive if there exists a constant $\lambda > 0$ and $j(x - y) \in J(x - y)$ such that

$$\langle Tx - Ty, j(x - y) \rangle \leq \|x - y\|^2 - \lambda \|x - y - (Tx - Ty)\|^2, \quad (1.2)$$

for all $x, y \in C$. We use $\text{Fix}(T)$ to denote the set of fixed points of T .

We know that the strict pseudocontractions have more powerful applications than nonexpansive mappings in solving inverse problems. There are some related references in the literature for strictly pseudocontractive mappings; see, for example, [27–30]. Motivated and inspired by the works in the literature, in the present paper, we firstly propose a new iterative method involving sunny nonexpansive retractions for pseudocontractions in Banach spaces. Consequently, we show that the suggested algorithm converges strongly to a fixed point of the pseudocontractive mapping which also solves some variational inequality.

2. Preliminaries

Let E^* be the dual space of a Banach space E . Let J_q ($q > 1$) be the generalized duality mapping from E into 2^{E^*} given by

$$J_q(x) = \{g \in E^* : \langle x, g \rangle = \|x\| \|g\|, \|g\| = \|x\|^{q-1}\}, \quad (2.1)$$

where $\langle \cdot, \cdot \rangle$ denotes the generalized duality pairing. In particular, J_2 is called the normalized duality mapping and it is usually denoted by j . It is well known that E is a uniformly smooth Banach space if and only if J_q is single valued and uniformly continuous on any bounded subset of E .

Let C be a nonempty closed convex subset of a Banach space E , and let D be a nonempty subset of C . Recall that a mapping $Q : C \rightarrow D$ is called a retraction from C onto D provided $Q(x) = x$ for all $x \in D$. A retraction $Q : C \rightarrow D$ is sunny provided $Q(x + t(x - Q(x))) = Q(x)$ for all $x \in C$ and $t \geq 0$ whenever $x + t(x - Q(x)) \in C$. A sunny nonexpansive retraction is a sunny retraction which is also nonexpansive. Sunny nonexpansive retractions are characterized as follows.

Lemma 2.1. *If E is a smooth Banach space, then $Q : C \rightarrow D$ is a sunny nonexpansive retraction if and only if there holds the inequality*

$$\langle x - Qx, J(y - Qx) \rangle \leq 0, \quad (2.2)$$

for all $x \in C$ and $y \in D$.

Lemma 2.2 (see [31]). *Let E be a real q -uniformly smooth Banach space, and let $1 < q \leq 2$. Then, one has*

$$\|x + y\|^q \leq \|x\|^q + q \langle y, J_q(x) \rangle + 2 \|Ky\|^q, \quad (2.3)$$

for all $x, y \in E$.

Lemma 2.3 (see [16]). Let $\{x_n\}$ and $\{z_n\}$ be two bounded sequences in Banach spaces, and let $\{\beta_n\}$ be a sequence in $[0, 1]$ with $0 < \liminf_{n \rightarrow \infty} \beta_n \leq \limsup_{n \rightarrow \infty} \beta_n < 1$. Suppose that $x_{n+1} = (1 - \beta_n)x_n + \beta_n z_n$ for all $n \geq 0$ and $\limsup_{n \rightarrow \infty} (\|z_{n+1} - z_n\| - \|x_{n+1} - x_n\|) \leq 0$. Then $\lim_{n \rightarrow \infty} \|z_n - x_n\| = 0$.

Lemma 2.4 (see [14]). Let C be a nonempty closed convex subset of a real q -uniformly smooth and uniformly convex Banach space E . Let $T : C \rightarrow C$ be a strictly pseudocontractive mapping. Then $I - T$ is demiclosed.

Lemma 2.5 (see [32]). Assume $\{a_n\}$ is a sequence of nonnegative real numbers such that $a_{n+1} \leq (1 - \gamma_n)a_n + \gamma_n \delta_n$ for $n \geq 0$ where $\{\gamma_n\}$ is a sequence in $(0, 1)$ and $\{\delta_n\}$ is a sequence in \mathbb{R} such that

- (i) $\sum_{n=0}^{\infty} \gamma_n = \infty$;
- (ii) $\limsup_{n \rightarrow \infty} \delta_n \leq 0$ or $\sum_{n=0}^{\infty} |\delta_n \gamma_n| < \infty$.

Then $\lim_{n \rightarrow \infty} a_n = 0$.

3. Main Results

In this section, we will give our main results. In the sequel, we assume the following:

- (C1) E is a uniformly convex and 2-uniformly smooth Banach space which admits a weakly sequentially continuous duality mapping j from E to E^* ;
- (C2) C is a nonempty closed convex subset of E ;
- (C3) Q_C is a sunny nonexpansive retraction from E onto C ;
- (C4) $T : C \rightarrow C$ is a λ -strict pseudocontraction;
- (C5) $\vartheta : E \rightarrow E$ is a ρ -contraction;
- (C6) $A : E \rightarrow E$ is strongly positive (i.e., $\langle Ax, j(x) \rangle \geq \gamma \|x\|^2$ for some $0 < \gamma < 1$) and linear bounded operator with $\|(1 - \zeta)I - \theta A\| \leq 1 - \zeta - \theta$ for all $\zeta > 0, \theta > 0$ and $0 < \zeta + \theta < 1$;
- (C7) $\text{Fix}(T) \neq \emptyset$.

First, we consider the following VI: finding $x^\dagger \in \text{Fix}(T)$ such that

$$\langle (A - \delta \vartheta)x^\dagger, j(\tilde{x} - x^\dagger) \rangle \geq 0, \quad \tilde{x} \in \text{Fix}(T). \quad (3.1)$$

The set of solutions of (3.1) is denoted by $VI(\text{Fix}(T), A)$. In the sequel, we assume that $VI(\text{Fix}(T), A) \neq \emptyset$. Note that (3.1) has the unique solution.

Next, we propose our algorithm.

Algorithm 3.1. For the initial point $x_0 \in C$, we generate a sequence $\{x_n\}$ via the following manner:

$$x_{n+1} = \alpha_n \delta \vartheta(x_n) + [(1 - \eta)I - \alpha_n A]x_n + \eta[(1 - k)Q_C x_n + kTQ_C x_n], \quad n \geq 0, \quad (3.2)$$

where $\{\alpha_n\}$ is a sequence in $(0, 1)$ and $\delta > 0$, $0 < \eta < 1$, $\delta \rho < \gamma$, $0 < k < \lambda/K^2$.

Theorem 3.2. *If the sequence $\{\alpha_n\}$ satisfies $\lim_{n \rightarrow \infty} \alpha_n = 0$ and $\sum_{n=0}^{\infty} \alpha_n = \infty$, then the sequence $\{x_n\}$ generated by (3.2) converges strongly to the unique solution x^\dagger of VI (3.1).*

Proof. First, by using Lemma 2.2, we know that $(1-k)I + kT$ is nonexpansive. Q_C is sunny nonexpansive. Thus, $(1-k)Q_C + kTQ_C$ is nonexpansive. Let $x^\dagger \in \text{Fix}(T)$. From (3.2), we have

$$\begin{aligned}
\|x_{n+1} - x^\dagger\| &= \|\alpha_n \delta \vartheta(x_n) + [(1-\eta)I - \alpha_n A]x_n + \eta[(1-k)Q_C x_n + kTQ_C x_n] - x^\dagger\| \\
&= \|\alpha_n (\delta \vartheta(x_n) - Ax^\dagger) + [(1-\eta)I - \alpha_n A](x_n - x^\dagger) \\
&\quad + \eta[(1-k)Q_C x_n + kTQ_C x_n - x^\dagger]\| \\
&\leq \alpha_n \|\delta \vartheta(x_n) - Ax^\dagger\| + \|(1-\eta)I - \alpha_n A\| \|x_n - x^\dagger\| \\
&\quad + \eta \|(1-k)Q_C x_n + kTQ_C x_n - x^\dagger\| \\
&\leq \alpha_n \delta \|\vartheta(x_n) - \vartheta(x^\dagger)\| + \alpha_n \|\delta \vartheta(x^\dagger) - Ax^\dagger\| + (1-\eta-\alpha_n \gamma) \|x_n - x^\dagger\| \\
&\quad + \eta \|x_n - x^\dagger\| \\
&\leq \alpha_n \delta \rho \|x_n - x^\dagger\| + \alpha_n \|\delta \vartheta(x^\dagger) - Ax^\dagger\| + (1-\eta-\alpha_n \gamma) \|x_n - x^\dagger\| \\
&\quad + \eta \|x_n - x^\dagger\| \\
&= [1 - (\gamma - \delta \rho) \alpha_n] \|x_n - x^\dagger\| + \alpha_n \|\delta \vartheta(x^\dagger) - Ax^\dagger\|.
\end{aligned} \tag{3.3}$$

Thus,

$$\|x_{n+1} - x^\dagger\| \leq \max \left\{ \|x_n - x^\dagger\|, \frac{\|\delta \vartheta(x^\dagger) - Ax^\dagger\|}{\gamma - \delta \rho} \right\} \leq \max \left\{ \|x_0 - x^\dagger\|, \frac{\|\delta \vartheta(x^\dagger) - Ax^\dagger\|}{\gamma - \delta \rho} \right\}. \tag{3.4}$$

This indicates that $\{x_n\}$ is bounded.

We write $x_{n+1} = (1-\eta)x_n + \eta u_n$ for all $n \geq 0$. So,

$$\begin{aligned}
u_n &= \frac{x_{n+1} - (1-\eta)x_n}{\eta} \\
&= \frac{\alpha_n \delta \vartheta(x_n)}{\eta} - \frac{\alpha_n A x_n}{\eta} + (1-k)Q_C x_n + kTQ_C x_n \\
&= \alpha_n \frac{\delta \vartheta - A}{\eta} x_n + [(1-k)Q_C + kTQ_C]x_n.
\end{aligned} \tag{3.5}$$

Hence,

$$\begin{aligned} u_{n+1} - u_n &= \alpha_{n+1} \frac{\delta\vartheta - A}{\eta} x_{n+1} + [(1-k)Q_C + kTQ_C]x_{n+1} - \alpha_n \frac{\delta\vartheta - A}{\eta} x_n \\ &\quad - [(1-k)Q_C + kTQ_C]x_n. \end{aligned} \quad (3.6)$$

It follows that

$$\begin{aligned} \|u_{n+1} - u_n\| &\leq \alpha_{n+1} \left\| \frac{\delta\vartheta - A}{\eta} x_{n+1} \right\| + \alpha_n \left\| \frac{\delta\vartheta - A}{\eta} x_n \right\| \\ &\quad + \|[(1-k)Q_C + kTQ_C]x_{n+1} - [(1-k)Q_C + kTQ_C]x_n\| \\ &\leq (\alpha_n + \alpha_{n+1})M + \|x_{n+1} - x_n\|, \end{aligned} \quad (3.7)$$

where $M > 0$ is a constant satisfying $\sup\{\|((\delta\vartheta - A)/\eta)x_n\|\} \leq M$. This implies that

$$\limsup_{n \rightarrow \infty} (\|u_{n+1} - u_n\| - \|x_{n+1} - x_n\|) \leq 0. \quad (3.8)$$

By Lemma 2.3, we deduce

$$\lim_{n \rightarrow \infty} \|u_n - x_n\| = 0. \quad (3.9)$$

Therefore,

$$\lim_{n \rightarrow \infty} \|x_{n+1} - x_n\| = \lim_{n \rightarrow \infty} \eta \|u_n - x_n\| = 0. \quad (3.10)$$

Note that $\text{Fix}(T) = \text{Fix}(TQ_C)$. As a matter of fact, if $p \in \text{Fix}(T)$, that is $p = Tp$, then $p \in \text{Fix}(TQ_C)$. Since T is a self-mapping, it is clear that $p \in C$. So, $Q_C p = p$. Therefore, $TQ_C p = Tp = p$. Conversely, if $q \in \text{Fix}(TQ_C)$, that is $q = TQ_C q$, we also have $q \in C$. Thus, $q = Tq$. Set $S = (1-k)I + kT$. We observe that $\text{Fix}(S) = \text{Fix}(T) = \text{Fix}(TQ_C) = \text{Fix}(SQ_C)$. Next, we estimate $\|x_n - SQ_C x_n\|$.

Since

$$\begin{aligned} \|x_n - SQ_C x_n\| &\leq \|x_n - x_{n+1}\| + \|x_{n+1} - SQ_C x_n\| \\ &\leq \|x_n - x_{n+1}\| + (1-\eta)\|x_n - SQ_C x_n\| + \eta\|u_n - SQ_C x_n\| \\ &\leq \|x_n - x_{n+1}\| + (1-\eta)\|x_n - SQ_C x_n\| + \alpha_n \|\delta\vartheta(x_n) - Ax_n\|, \end{aligned} \quad (3.11)$$

we have

$$\|x_n - SQ_C x_n\| \leq \frac{1}{\eta} \|x_{n+1} - x_n\| + \frac{\alpha_n}{\eta} \|\delta\vartheta(x_n) - Ax_n\| \longrightarrow 0. \quad (3.12)$$

Next, we show

$$\limsup_{n \rightarrow \infty} \langle \delta\vartheta(x^\dagger) - Ax^\dagger, j(x_{n+1} - x^\dagger) \rangle \leq 0, \quad (3.13)$$

where $x^\dagger \in VI(\text{Fix}(T), A)$.

First, we have

$$\limsup_{n \rightarrow \infty} \langle \delta\vartheta(x^\dagger) - Ax^\dagger, j(x_{n+1} - x^\dagger) \rangle = \lim_{i \rightarrow \infty} \langle \delta\vartheta(x^\dagger) - Ax^\dagger, j(x_{n_i+1} - x^\dagger) \rangle. \quad (3.14)$$

Since the sequence $\{x_n\}$ is bounded, hence $\{x_{n_i}\}$ is bounded. Thus, we can take a subsequence $\{x_{n_{i_j}}\}$ of $\{x_{n_i}\}$ such that $x_{n_{i_j}} \rightarrow \tilde{x}$ weakly. Without loss of generality, we may assume that $x_{n_i} \rightarrow \tilde{x}$ weakly. Note that SQ_C is nonexpansive and $\|x_{n_i} - SQ_C x_{n_i}\| \rightarrow 0$. By using the demiclosed principle of nonexpansive mappings (see Lemma 2.4), we get $\tilde{x} \in \text{Fix}(SQ_C) = \text{Fix}(T)$. At the same time, j is weakly sequentially continuous. Therefore,

$$\begin{aligned} \limsup_{n \rightarrow \infty} \langle \delta\vartheta(x^\dagger) - Ax^\dagger, j(x_{n+1} - x^\dagger) \rangle &= \lim_{i \rightarrow \infty} \langle \delta\vartheta(x^\dagger) - Ax^\dagger, j(x_{n_i+1} - x^\dagger) \rangle \\ &= \langle \delta\vartheta(x^\dagger) - Ax^\dagger, j(\tilde{x} - x^\dagger) \rangle \leq 0. \end{aligned} \quad (3.15)$$

Finally we show that $x_n \rightarrow x^\dagger$. From (3.2), we have

$$\begin{aligned} \|x_{n+1} - x^\dagger\|^2 &= \|\alpha_n(\delta\vartheta(x_n) - Ax^\dagger) + ((1-\eta)I - \alpha_n A)(x_n - x^\dagger) \\ &\quad + \eta((1-k)Q_C x_n + kTQ_C x_n - x^\dagger)\|^2 \\ &\leq \|((1-\eta)I - \alpha_n A)(x_n - x^\dagger) + \eta((1-k)Q_C x_n + kTQ_C x_n - x^\dagger)\|^2 \\ &\quad + 2\alpha_n \langle \delta\vartheta(x_n) - Ax^\dagger, j(x_{n+1} - x^\dagger) \rangle \\ &\leq \left(\|((1-\eta)I - \alpha_n A)(x_n - x^\dagger)\|^2 + \|\eta((1-k)Q_C x_n + kTQ_C x_n - x^\dagger)\|^2 \right) \\ &\quad + 2\alpha_n \delta \langle \vartheta(x_n) - \vartheta(x^\dagger), j(x_{n+1} - x^\dagger) \rangle + 2\alpha_n \langle \delta\vartheta(x^\dagger) - Ax^\dagger, j(x_{n+1} - x^\dagger) \rangle \\ &\leq [(1-\eta-\alpha_n\gamma)\|x_n - x^\dagger\| + \eta\|x_n - x^\dagger\|]^2 + 2\alpha_n \delta \rho \|x_n - x^\dagger\| \|x_{n+1} - x^\dagger\| \\ &\quad + 2\alpha_n \langle \delta\vartheta(x^\dagger) - Ax^\dagger, j(x_{n+1} - x^\dagger) \rangle \\ &\leq (1-\alpha_n\gamma)^2 \|x_n - x^\dagger\|^2 + \alpha_n \delta \rho \left(\|x_n - x^\dagger\|^2 + \|x_{n+1} - x^\dagger\|^2 \right) \\ &\quad + 2\alpha_n \langle \delta\vartheta(x^\dagger) - Ax^\dagger, j(x_{n+1} - x^\dagger) \rangle. \end{aligned} \quad (3.16)$$

It follows that

$$\begin{aligned}
\|x_{n+1} - x^\dagger\|^2 &\leq \frac{1 - 2\alpha_n\gamma + \alpha_n^2\gamma^2 + \alpha_n\delta\rho}{1 - \alpha_n\delta\rho} \|x_n - x^\dagger\|^2 \\
&\quad + \frac{2\alpha_n}{1 - \alpha_n\delta\rho} \langle \delta\vartheta(x^\dagger) - Ax^\dagger, j(x_{n+1} - x^\dagger) \rangle \\
&= \left[1 - \frac{2(\gamma - \delta\rho)\alpha_n}{1 - \alpha_n\delta\rho} \right] \|x_n - x^\dagger\|^2 + \frac{\alpha_n^2\gamma^2}{1 - \alpha_n\delta\rho} \|x_n - x^\dagger\|^2 \\
&\quad + \frac{2\alpha_n}{1 - \alpha_n\delta\rho} \langle \delta\vartheta(x^\dagger) - Ax^\dagger, j(x_{n+1} - x^\dagger) \rangle \\
&= \left[1 - \frac{2(\gamma - \delta\rho)\alpha_n}{1 - \alpha_n\delta\rho} \right] \|x_n - x^\dagger\|^2 + \frac{2(\gamma - \delta\rho)\alpha_n}{1 - \alpha_n\delta\rho} \\
&\quad \times \left\{ \frac{\alpha_n\gamma^2 \|x_n - x^\dagger\|^2}{2(\gamma - \delta\rho)} + \frac{1}{\gamma - \delta\rho} \langle \delta\vartheta(x^\dagger) - Ax^\dagger, j(x_{n+1} - x^\dagger) \rangle \right\}.
\end{aligned} \tag{3.17}$$

It can be checked easily that $\sum_n (2(\gamma - \delta\rho)\alpha_n / (1 - \alpha_n\delta\rho)) = \infty$ and $\limsup_{n \rightarrow \infty} (\alpha_n\gamma^2 \|x_n - x^\dagger\|^2 / 2(\gamma - \delta\rho)) + (1 / (\gamma - \delta\rho)) \langle \delta\vartheta(x^\dagger) - Ax^\dagger, j(x_{n+1} - x^\dagger) \rangle \leq 0$. From Lemma 2.5, we deduce $x_n \rightarrow x^\dagger$. This completes the proof. \square

Algorithm 3.3. For the initial point $x_0 \in C$, we generate a sequence $\{x_n\}$ via the following manner:

$$x_{n+1} = \alpha_n \delta\vartheta(x_n) + (1 - \eta - \alpha_n)x_n + \eta[(1 - k)Q_C x_n + kTQ_C x_n], \quad n \geq 0, \tag{3.18}$$

where $\{\alpha_n\}$ is a sequence in $(0, 1)$ and $\delta > 0$, $0 < \eta < 1$, $\delta\rho < 1$, $0 < k < \lambda/K^2$.

Corollary 3.4. *If the sequence $\{\alpha_n\}$ satisfies $\lim_{n \rightarrow \infty} \alpha_n = 0$ and $\sum_{n=0}^{\infty} \alpha_n = \infty$, then the sequence $\{x_n\}$ generated by (3.18) converges strongly to the unique solution x^\dagger of VI: finding $x^\dagger \in \text{Fix}(T)$ such that*

$$\langle (I - \delta\vartheta)x^\dagger, j(\tilde{x} - x^\dagger) \rangle \geq 0, \quad \tilde{x} \in \text{Fix}(T). \tag{3.19}$$

Algorithm 3.5. For the initial point $x_0 \in C$ and $u \in E$, we generate a sequence $\{x_n\}$ via the following manner:

$$x_{n+1} = \alpha_n u + (1 - \eta - \alpha_n)x_n + \eta[(1 - k)Q_C x_n + kTQ_C x_n], \quad n \geq 0, \tag{3.20}$$

where $\{\alpha_n\}$ is a sequence in $(0, 1)$ and $0 < \eta < 1$, $0 < k < \lambda/K^2$.

Corollary 3.6. *If the sequence $\{\alpha_n\}$ satisfies $\lim_{n \rightarrow \infty} \alpha_n = 0$ and $\sum_{n=0}^{\infty} \alpha_n = \infty$, then the sequence $\{x_n\}$ generated by (3.20) converges strongly to the unique solution x^\dagger of VI: finding $x^\dagger \in \text{Fix}(T)$ such that*

$$\langle (I - u)x^\dagger, j(\tilde{x} - x^\dagger) \rangle \geq 0, \quad \tilde{x} \in \text{Fix}(T). \quad (3.21)$$

Algorithm 3.7. For the initial point $x_0 \in C$, we generate a sequence $\{x_n\}$ via the following manner:

$$x_{n+1} = (1 - \eta - \alpha_n)x_n + \eta[(1 - k)Q_C x_n + kTQ_C x_n], \quad n \geq 0, \quad (3.22)$$

where $\{\alpha_n\}$ is a sequence in $(0, 1)$ and $0 < \eta < 1$, $0 < k < \lambda/K^2$.

Corollary 3.8. *If the sequence $\{\alpha_n\}$ satisfies $\lim_{n \rightarrow \infty} \alpha_n = 0$ and $\sum_{n=0}^{\infty} \alpha_n = \infty$, then the sequence $\{x_n\}$ generated by (3.22) converges strongly to $Q_{\text{Fix}(T)}(0)$.*

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References

- [1] F. E. Browder and W. V. Petryshyn, "Construction of fixed points of nonlinear mappings in Hilbert space," *Journal of Mathematical Analysis and Applications*, vol. 20, pp. 197–228, 1967.
- [2] B. Halpern, "Fixed points of nonexpanding maps," *Bulletin of the American Mathematical Society*, vol. 73, pp. 957–961, 1967.
- [3] Z. Opial, "Weak convergence of the sequence of successive approximations for nonexpansive mappings," *Bulletin of the American Mathematical Society*, vol. 73, pp. 591–597, 1967.
- [4] P.-L. Lions, "Approximation de points fixes de contractions," *Comptes Rendus de l'Académie des Sciences*, vol. 284, no. 21, pp. A1357–A1359, 1977.
- [5] R. Wittmann, "Approximation of fixed points of nonexpansive mappings," *Archiv der Mathematik*, vol. 58, no. 5, pp. 486–491, 1992.
- [6] S. Reich, "Weak convergence theorems for nonexpansive mappings in Banach spaces," *Journal of Mathematical Analysis and Applications*, vol. 67, no. 2, pp. 274–276, 1979.
- [7] B. E. Rhoades, "Fixed point iterations using infinite matrices," *Transactions of the American Mathematical Society*, vol. 196, pp. 161–176, 1974.
- [8] B. E. Rhoades, "Comments on two fixed point iteration methods," *Journal of Mathematical Analysis and Applications*, vol. 56, no. 3, pp. 741–750, 1976.
- [9] L. C. Ceng, P. Cubiotti, and J. C. Yao, "Strong convergence theorems for finitely many nonexpansive mappings and applications," *Nonlinear Analysis A*, vol. 67, no. 5, pp. 1464–1473, 2007.
- [10] L.-C. Ceng, S.-M. Guu, and J.-C. Yao, "Hybrid viscosity-like approximation methods for nonexpansive mappings in Hilbert spaces," *Computers & Mathematics with Applications*, vol. 58, no. 3, pp. 605–617, 2009.
- [11] J.-P. Chancelier, "Iterative schemes for computing fixed points of nonexpansive mappings in Banach spaces," *Journal of Mathematical Analysis and Applications*, vol. 353, no. 1, pp. 141–153, 2009.
- [12] S.-S. Chang, "Viscosity approximation methods for a finite family of nonexpansive mappings in Banach spaces," *Journal of Mathematical Analysis and Applications*, vol. 323, no. 2, pp. 1402–1416, 2006.
- [13] C. E. Chidume and C. O. Chidume, "Iterative approximation of fixed points of nonexpansive mappings," *Journal of Mathematical Analysis and Applications*, vol. 318, no. 1, pp. 288–295, 2006.

- [14] K. Goebel and W. A. Kirk, *Topics in Metric Fixed Point Theory*, vol. 28 of *Cambridge Studies in Advanced Mathematics*, Cambridge University Press, Cambridge, UK, 1990.
- [15] K. Shimoji and W. Takahashi, "Strong convergence to common fixed points of infinite nonexpansive mappings and applications," *Taiwanese Journal of Mathematics*, vol. 5, no. 2, pp. 387–404, 2001.
- [16] T. Suzuki, "Strong convergence theorems for infinite families of nonexpansive mappings in general Banach spaces," *Fixed Point Theory and Applications*, no. 1, pp. 103–123, 2005.
- [17] W. Takahashi and M. Toyoda, "Weak convergence theorems for nonexpansive mappings and monotone mappings," *Journal of Optimization Theory and Applications*, vol. 118, no. 2, pp. 417–428, 2003.
- [18] A. Moudafi, "Viscosity approximation methods for fixed-points problems," *Journal of Mathematical Analysis and Applications*, vol. 241, no. 1, pp. 46–55, 2000.
- [19] H.-K. Xu, "Viscosity approximation methods for nonexpansive mappings," *Journal of Mathematical Analysis and Applications*, vol. 298, no. 1, pp. 279–291, 2004.
- [20] P.-E. Maingé, "Approximation methods for common fixed points of nonexpansive mappings in Hilbert spaces," *Journal of Mathematical Analysis and Applications*, vol. 325, no. 1, pp. 469–479, 2007.
- [21] G. Lopez, V. Martin, and H.-K. Xu, "Perturbation techniques for nonexpansive mappings with applications," *Nonlinear Analysis*, vol. 10, no. 4, pp. 2369–2383, 2009.
- [22] Y. Yao, R. Chen, and J.-C. Yao, "Strong convergence and certain control conditions for modified Mann iteration," *Nonlinear Analysis A*, vol. 68, no. 6, pp. 1687–1693, 2008.
- [23] Y. Yao, Y.-C. Liou, and J.-C. Yao, "An iterative algorithm for approximating convex minimization problem," *Applied Mathematics and Computation*, vol. 188, no. 1, pp. 648–656, 2007.
- [24] Y. Yao, Y.-C. Liou, and J.-C. Yao, "An extragradient method for fixed point problems and variational inequality problems," *Journal of Inequalities and Applications*, vol. 2007, Article ID 38752, 12 pages, 2007.
- [25] Y. Yao, M. Aslam Noor, K. Inayat Noor, Y.-C. Liou, and H. Yaqoob, "Modified extragradient methods for a system of variational inequalities in Banach spaces," *Acta Applicandae Mathematicae*, vol. 110, no. 3, pp. 1211–1224, 2010.
- [26] Y. Yao, M. A. Noor, and Y.-C. Liou, "Strong convergence of a modified extragradient method to the minimum-norm solution of variational inequalities," *Abstract and Applied Analysis*, vol. 2012, Article ID 817436, 9 pages, 2012.
- [27] G. L. Acedo and H.-K. Xu, "Iterative methods for strict pseudo-contractions in Hilbert spaces," *Nonlinear Analysis A*, vol. 67, no. 7, pp. 2258–2271, 2007.
- [28] G. Marino and H.-K. Xu, "Weak and strong convergence theorems for strict pseudo-contractions in Hilbert spaces," *Journal of Mathematical Analysis and Applications*, vol. 329, no. 1, pp. 336–346, 2007.
- [29] M. O. Osilike, "Implicit iteration process for common fixed points of a finite family of strictly pseudocontractive maps," *Journal of Mathematical Analysis and Applications*, vol. 294, no. 1, pp. 73–81, 2004.
- [30] L.-C. Zeng, N.-C. Wong, and J.-C. Yao, "Strong convergence theorems for strictly pseudocontractive mappings of Browder-Petryshyn type," *Taiwanese Journal of Mathematics*, vol. 10, no. 4, pp. 837–849, 2006.
- [31] H. K. Xu, "Inequalities in Banach spaces with applications," *Nonlinear Analysis A*, vol. 16, no. 12, pp. 1127–1138, 1991.
- [32] H.-K. Xu, "Iterative algorithms for nonlinear operators," *Journal of the London Mathematical Society*, vol. 66, no. 1, pp. 240–256, 2002.