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Research Article

Travelling Wave Solutions of the Schrödinger-Boussinesq System

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We establish exact solutions for the Schrödinger-Boussinesq System $iu_t + u_{xx} - auv = 0$, $v_{tt} - v_{xx} + v_{xxxx} - b(|u|^2)_{xx} = 0$, where a and b are real constants. The (G'/G)-expansion method is used to construct exact periodic and soliton solutions of this equation. Our work is motivated by the fact that the (G'/G)-expansion method provides not only more general forms of solutions but also periodic and solitary waves. As a result, hyperbolic function solutions and trigonometric function solutions with parameters are obtained. These solutions may be important and of significance for the explanation of some practical physical problems.

1. Introduction

It is well known that the nonlinear Schrödinger (NLS) equation models a wide range of physical phenomena including self-focusing of optical beams in nonlinear media, the modulation of monochromatic waves, propagation of Langmuir waves in plasmas, and so forth. The nonlinear Schrödinger equations play an important role in many areas of applied physics, such as nonrelativistic quantum mechanics, laser beam propagation, Bose-Einstein condensates, and so on (see [1]). Some properties of solutions for the nonlinear Schrödinger equations on $\mathbb{R}^n$ have been extensively studied in the last two decades (e.g., see [2]).

The Boussinesq-type equations are essentially a class of models appearing in physics and fluid mechanics. The so-called Boussinesq equation was originally derived by Boussinesq [3] to describe two-dimensional irrotational flows of an inviscid liquid in a uniform rectangular channel. It also arises in a large range of physical phenomena including the propagation of ion-sound waves in a plasma and nonlinear lattice waves. The study on the soliton solutions for various generalizations of the Boussinesq equation has recently attracted considerable attention from many mathematicians and physicists (see [4]). We should remark that it was the first equation proposed in the literature to describe this kind of physical phenomena. This equation was also used by Zakharov [5] as a model of nonlinear string and by Falk et al. [6] in their study of shape-memory alloys.

In the laser and plasma physics, the Schrödinger-Boussinesq system (hereafter referred to as the SB-system) has been raised. Consider the SB-system

$$\begin{aligned} &iu_t + u_{xx} - auv = 0, \\ &v_{tt} - v_{xx} + v_{xxxx} - b\left(|u|^2\right)_{xx} = 0, \end{aligned} \tag{1.1}$$

where $t > 0$, $x \in [0, L]$, for some $L > 0$, and a, b are real constants. Here u and v are, respectively, a complex-valued and a real-valued function defined in space-time $[0, L] \times \mathbb{R}$. The SB-system is considered as a model of interactions between short and intermediate long waves, which is derived in describing the dynamics of Langmuir soliton formation and interaction in a plasma [7] and diatomic lattice system [8]. The short wave term $u(x, t) : [0, L] \times \mathbb{R} \to \mathbb{C}$ is described by a Schrödinger type equation with a potential $v(x, t) : [0, L] \times \mathbb{R} \to \mathbb{R}$ satisfying some sort of Boussinesq equation and representing the intermediate long wave. The SB-system also appears in the study of interaction of solitons in optics. The solitary wave solutions and integrability of nonlinear SB-system has been considered by several authors (see [7, 8]) and the references therein.

In the literature, there is a wide variety of approaches to nonlinear problems for constructing travelling wave solutions, such as the inverse scattering method [9], Bcklund transformation [10], Hirota bilinear method [11], Painlevé expansion methods [12], and the Wronskian determinant technique [13].

With the help of the computer software, most of mentioned methods are improved and many other algebraic method, proposed, such as the tanh/coth method [14], the Exp-function method [15], and first integral method [16]. But, most of the methods may sometimes fail or can only lead to a kind of special solution and the solution procedures become very complex as the degree of nonlinearity increases.

Recently, the (G'/G)-expansion method, firstly introduced by Wang et al. [17], has become widely used to search for various exact solutions of NLEEs [17–19].

The main idea of this method is that the traveling wave solutions of nonlinear equations can be expressed by a polynomial in (G'/G), where $G = G(\xi)$ satisfies the second order linear ordinary differential equation $G''(\xi) + \lambda G'(\xi) + \mu G(\xi) = 0$, where $\xi = kx + \omega t$ and k, ω are arbitrary constants. The degree of this polynomial can be determined by considering the homogeneous balance between the highest order derivatives and the non-linear terms appearing in the given non-linear equations.

Our first interest in the present work is in implementing the (G'/G)-expansion method to stress its power in handling nonlinear equations, so that one can apply it to models of various types of nonlinearity. The next interest is in the determination of exact traveling wave solutions for the SB-system (1.1).

2. Description of the (G'/G)-Expansion Method

The objective of this section is to outline the use of the (G'/G)-expansion method for solving certain nonlinear partial differential equations (PDEs). Suppose that a nonlinear equation, say in two independent variables x and t, is given by

$$P(u, u_x, u_t, u_{xx}, u_{x,t}, u_{tt}, \ldots) = 0, \tag{2.1}$$

where $u(x,t)$ is an unknown function, P is a polynomial in $u(x,t)$ and its various partial derivatives, in which the highest order derivatives and nonlinear terms are involved. The main steps of the (G'/G)-expansion method are the following:

Step 1. Combining the independent variables x and t into one variable $\xi = kx+\omega t$, we suppose that

$$u(x,t) = U(\xi), \qquad \xi = kx + \omega t. \tag{2.2}$$

The travelling wave variable (2.2) permits us to reduce (2.1) to an ODE for $u(x,t) = U(\xi)$, namely,

$$P\Big(U, kU', \omega U', k^2U'', k\omega U'', \omega^2 U'', \ldots\Big) = 0, \tag{2.3}$$

where prime denotes derivative with respect to ξ.

Step 2. We assume that the solution of (2.3) can be expressed by a polynomial in (G'/G) as follows:

$$U(\xi) = \sum_{i=1}^{m} \alpha_i \left(\frac{G'}{G}\right)^i + \alpha_0, \quad \alpha_m \neq 0, \tag{2.4}$$

where m is called the balance number, α_0, and α_i, are constants to be determined later, $G(\xi)$ satisfies a second order linear ordinary differential equation (LODE):

$$\frac{d^2G(\xi)}{d\xi^2} + \lambda \frac{dG(\xi)}{d\xi} + \mu G(\xi) = 0, \tag{2.5}$$

where λ and μ are arbitrary constants. The positive integer m can be determined by considering the homogeneous balance between the highest order derivatives and nonlinear terms appearing in ODE (2.3).

Step 3. By substituting (2.4) into (2.3) and using the second order linear ODE (2.5), collecting all terms with the same order of (G'/G) together, the left-hand side of (2.3) is converted into another polynomial in (G'/G). Equating each coefficient of this polynomial to zero yields a set of algebraic equations for $k, \omega, \lambda, \mu, \alpha_0, \alpha_1, \ldots, \alpha_m$.

Step 4. Assuming that the constants $k, \omega, \lambda, \mu, \alpha_0, \alpha_1, \ldots, \alpha_m$ can be obtained by solving the algebraic equations in Step 3, since the general solutions of the second order linear ODE (2.5) is well known for us, then substituting $k, \omega, \lambda, \mu, \alpha_0, \ldots, \alpha_m$ and the general solutions of (2.5) into (2.4) we have more travelling wave solutions of the nonlinear evolution (2.1).

In the subsequent section, we will illustrate the validity and reliability of this method in detail with complex model of Schrödinger-Boussinesq System (1.1).

3. Application

To look for the traveling wave solution of the Schrödinger-Boussinesq System (1.1), we use the gauge transformation:

$$\begin{aligned} u(x,t) &= U(\xi)e^{i\eta}, \\ v(x,t) &= V(\xi), \end{aligned} \tag{3.1}$$

where $\xi = kx + \omega t$, $\eta = px + qt$, and p, q, ω are constants and $i = \sqrt{-1}$. We substitute (3.1) into (1.1) to obtain nonlinear ordinary differential equation

$$k^2 U'' - i(2kp + \omega)U' - aUV - \left(p^2 + q\right)U = 0, \tag{3.2}$$

$$\left(\omega^2 - k^2\right)V'' + k^4 V^{(4)} - bk^2\left(U^2\right)'' = 0. \tag{3.3}$$

In order to simplify, integrating (3.3) twice and taking integration constant to zero, the system (3.2)-(3.3) reduces to the following system:

$$\begin{aligned} k^2 U'' - i(2kp + \omega)U' - aUV - \left(p^2 + q\right)U &= 0, \\ \left(\omega^2 - k^2\right)V + k^4 V'' - bk^2 U^2 &= 0. \end{aligned} \tag{3.4}$$

Suppose that the solution of the nonlinear ordinary differential system (3.4) can be expressed by a polynomial in (G'/G) as follows:

$$\begin{aligned} U(\xi) &= \sum_{i=1}^{m} \alpha_i \left(\frac{G'}{G}\right)^i + \alpha_0, \quad \alpha_m \neq 0, \\ V(\xi) &= \sum_{i=1}^{n} \beta_i \left(\frac{G'}{G}\right)^i + \beta_0, \quad \beta_n \neq 0, \end{aligned} \tag{3.5}$$

where m, n are called the balance number, α_i, $(i = 0, 1, \ldots, m)$ and β_j, $(j = 0, 1, \ldots, n)$ are constants to be determined later, $G(\xi)$ satisfies a second order linear ordinary differential equation (2.5). The integers m, n can be determined by considering the homogeneous balance between the highest order derivatives and nonlinear terms appearing in nonlinear ordinary differential system (3.4) as follow:

$$
\begin{aligned}
m+n &= m+2,\\
2m &= n+2,
\end{aligned}
\tag{3.6}
$$

so that $m = n = 2$. We then suppose that (3.4) has the following formal solutions:

$$
\begin{aligned}
U(\xi) &= \alpha_2\left(\frac{G'}{G}\right)^2 + \alpha_1\left(\frac{G'}{G}\right) + \alpha_0, \quad \alpha_2 \neq 0,\\
V(\xi) &= \beta_2\left(\frac{G'}{G}\right)^2 + \beta_1\left(\frac{G'}{G}\right) + \beta_0, \quad \beta_2 \neq 0.
\end{aligned}
\tag{3.7}
$$

Substituting (3.7) along with (2.5) into (3.4) and collecting all the terms with the same power of (G'/G) together, equating each coefficient to zero, yields a set of simultaneous algebraic equations for k, ω, λ, μ, α_j, and β_j, $(j = 0, 1, 2)$, as follows:

$$
\begin{aligned}
&2k^2\alpha_2\mu^2 + i\omega\alpha_1\mu + k^2\alpha_1\lambda\mu - a\alpha_0\beta_0 - p^2\alpha_0 - q\alpha_0 + 2ikp\alpha_1\mu = 0,\\
&\omega^2\beta_0 + 2k^4\beta_2\mu^2 - k^2\left(-\mu\lambda\beta_1 k^2 + b\alpha_0{}^2 + \beta_0\right) = 0,\\
&k^2\alpha_1\lambda^2 - p^2\alpha_1 + (4ik\alpha_2\mu + 2ik\alpha_1\lambda)p + (2i\alpha_2\mu + i\alpha_1\lambda)\omega - q\alpha_1\\
&\quad - a\alpha_0\beta_1 + 2k^2\alpha_1\mu - a\alpha_1\beta_0 + 6k^2\alpha_2\lambda\mu = 0,\\
&\omega^2\beta_1 + 6k^4\beta_2\lambda\mu - k^2\left(-k^2\lambda^2\beta_1 - 2k^2\mu\beta_1 + \beta_1 + 2b\alpha_1\alpha_0\right) = 0,\\
&(4ik\alpha_2\lambda + 2ik\alpha_1)p + (2i\alpha_2\lambda + i\alpha_1)\omega - p^2\alpha_2 + \left(4\lambda^2k^2 - q - a\beta_0 + 8\mu k^2\right)\alpha_2\\
&\quad + 3k^2\alpha_1\lambda - a\alpha_1\beta_1 - a\alpha_0\beta_2 = 0,\\
&\omega^2\beta_2 + k^2\left(4\lambda^2k^2 + 8\mu k^2 - 1\right)\beta_2 - k^2\left(2b\alpha_2\alpha_0 + b\alpha_1{}^2 - 3\lambda\beta_1k^2\right) = 0,\\
&4ikp\alpha_2 + 2i\omega\alpha_2 + \left(10\lambda k^2 - a\beta_1\right)\alpha_2 - \alpha_1\left(-2k^2 + a\beta_2\right) = 0,\\
&10k^4\beta_2\lambda - 2k^2\left(-k^2\beta_1 + b\alpha_2\alpha_1\right) = 0,\\
&\left(6k^2 - a\beta_2\right)\alpha_2 = 0,\\
&6k^4\beta_2 - bk^2\alpha_2{}^2 = 0.
\end{aligned}
\tag{3.8}
$$

Then, explicit and exact wave solutions can be constructed through our ansatz (3.7) via the associated solutions of (2.5).

In the process of constructing exact solutions to the Schrödinger-Boussinesq system (1.1), (2.5) is often viewed as a key auxiliary equation, and the types of its solutions determine the solutions for the original system (1.1) indirectly. In order to seek more new solutions to (1.1), we here combine the solutions to (2.5) which were listed in [18, 19]. Our computation results show that this combination is an efficient way to obtain more diverse families of explicit exact solutions.

Solving (3.8) by use of Maple, we get the following reliable results:

$$\begin{aligned} &\Bigg\{\lambda = \pm\frac{1}{3}\frac{\sqrt{3\omega^2-3k^2}}{k^2},\ \mu = \frac{1}{6}\frac{k^2-\omega^2}{k^4},\ p = -\frac{1}{2}\frac{\omega}{k},\\ &q = \frac{1}{24}\frac{24k^4+30\omega^4-54k^2\omega^2}{k^2(k^2-\omega^2)},\ \alpha_0 = 0,\\ &\alpha_1 = \pm2\sqrt{\frac{1}{ab}}\sqrt{3\omega^2-3k^2},\ \alpha_2 = 6k^2\sqrt{\frac{1}{ab}},\\ &\beta_0 = 0,\ \beta_1 = \pm2\frac{\sqrt{3\omega^2-3k^2}}{a},\ \beta_2 = \frac{6k^2}{a}\Bigg\}, \end{aligned} \tag{3.9}$$

where k and ω are free constant parameters and $k \neq \pm\omega$. Therefore, substitute the above case in (3.7), we get

$$\begin{aligned} U(\xi) &= 6k^2\sqrt{\frac{1}{ab}}\left(\frac{G'}{G}\right)^2 \pm 2\sqrt{\frac{1}{ab}}\sqrt{3\omega^2-3k^2}\left(\frac{G'}{G}\right),\\ V(\xi) &= \frac{6k^2}{a}\left(\frac{G'}{G}\right)^2 \pm 2\frac{\sqrt{3\omega^2-3k^2}}{a}\left(\frac{G'}{G}\right). \end{aligned} \tag{3.10}$$

Substituting the general solutions of ordinary differential equation (2.5) into (3.10), we obtain two types of traveling wave solutions of (1.1) in view of the positive and negative of $\lambda^2-4\mu$.

When $\mathfrak{D} = \lambda^2-4\mu = (\omega^2-k^2)/k^4 > 0$, using the general solutions of ordinary differential equation (2.5) and relationships (3.10), we obtain hyperbolic function solutions $u_{\mathscr{H}}(x,t)$ and $v_{\mathscr{H}}(x,t)$ of the Schrödinger-Boussinesq system (1.1) as follows:

$$\begin{aligned} u_{\mathscr{H}}(x,t) = 2\sqrt{\frac{1}{ab}}\Bigg[3k^2\Bigg(&\frac{\sqrt{\mathfrak{D}}}{2}\left(\frac{C_1\sinh\left(\left(\sqrt{\mathfrak{D}}/2\right)\xi\right)+C_2\cosh\left(\left(\sqrt{\mathfrak{D}}/2\right)\xi\right)}{C_1\cosh\left(\left(\sqrt{\mathfrak{D}}/2\right)\xi\right)+C_2\sinh\left(\left(\sqrt{\mathfrak{D}}/2\right)\xi\right)}\right)\\ &\mp\frac{1}{6}\frac{\sqrt{3\omega^2-3k^2}}{k^2}\Bigg)^2 \end{aligned}$$

$$\pm\sqrt{3\omega^2-3k^2}\left(\frac{\sqrt{\mathfrak{D}}}{2}\left(\frac{C_1\sinh\left(\left(\sqrt{\mathfrak{D}}/2\right)\xi\right)+C_2\cosh\left(\left(\sqrt{\mathfrak{D}}/2\right)\xi\right)}{C_1\cosh\left(\left(\sqrt{\mathfrak{D}}/2\right)\xi\right)+C_2\sinh\left(\left(\sqrt{\mathfrak{D}}/2\right)\xi\right)}\right)\right.$$

$$\left.\left.\mp\frac{1}{6}\frac{\sqrt{3\omega^2-3k^2}}{k^2}\right)\right]e^{i\eta},$$

$$v_{\mathscr{H}}(x,t)=\frac{6k^2}{a}\left(\frac{\sqrt{\mathfrak{D}}}{2}\left(\frac{C_1\sinh\left(\left(\sqrt{\mathfrak{D}}/2\right)\xi\right)+C_2\cosh\left(\left(\sqrt{\mathfrak{D}}/2\right)\xi\right)}{C_1\cosh\left(\left(\sqrt{\mathfrak{D}}/2\right)\xi\right)+C_2\sinh\left(\left(\sqrt{\mathfrak{D}}/2\right)\xi\right)}\right)\mp\frac{1}{6}\frac{\sqrt{3\omega^2-3k^2}}{k^2}\right)^2$$

$$\pm 2\frac{\sqrt{3\omega^2-3k^2}}{a}\left(\frac{\sqrt{\mathfrak{D}}}{2}\left(\frac{C_1\sinh\left(\left(\sqrt{\mathfrak{D}}/2\right)\xi\right)+C_2\cosh\left(\left(\sqrt{\mathfrak{D}}/2\right)\xi\right)}{C_1\cosh\left(\left(\sqrt{\mathfrak{D}}/2\right)\xi\right)+C_2\sinh\left(\left(\sqrt{\mathfrak{D}}/2\right)\xi\right)}\right)\right.$$

$$\left.\mp\frac{1}{6}\frac{\sqrt{3\omega^2-3k^2}}{k^2}\right),\tag{3.11}$$

where $\mathfrak{D}=(\omega^2-k^2)/k^4>0$, $\xi=kx+\omega t$, $\eta=-(1/2)(\omega x/k)+(1/24)((24k^4+30\omega^4-54k^2\omega^2)/k^2(k^2-\omega^2))t$, and k,ω,C_1, and C_2 are arbitrary constants and $k\neq\pm\omega$.

It is easy to see that the hyperbolic solutions (3.11) can be rewritten at $C_1^2>C_2^2$, as follows:

$$u_{\mathscr{H}}(x,t)=\frac{1}{2}\sqrt{\frac{1}{ab}\frac{\omega^2-k^2}{k^2}}\left(3\tanh^2\left(\frac{1}{2}\sqrt{\frac{\omega^2-k^2}{k^4}}\,\xi+\rho_{\mathscr{H}}\right)-1\right)e^{i\eta},\tag{3.12a}$$

$$v_{\mathscr{H}}(x,t)=\frac{1}{2}\frac{\omega^2-k^2}{ak^2}\left(3\tanh^2\left(\frac{1}{2}\sqrt{\frac{\omega^2-k^2}{k^4}}\,\xi+\rho_{\mathscr{H}}\right)-1\right),\tag{3.12b}$$

while at $C_1^2<C_2^2$, one can obtain

$$u_{\mathscr{H}}(x,t)=\frac{1}{2}\sqrt{\frac{1}{ab}\frac{\omega^2-k^2}{k^2}}\left(3\coth^2\left(\frac{1}{2}\sqrt{\frac{\omega^2-k^2}{k^4}}\,\xi+\rho_{\mathscr{H}}\right)-1\right)e^{i\eta},\tag{3.12c}$$

$$v_{\mathscr{H}}(x,t)=\frac{1}{2}\frac{\omega^2-k^2}{ak^2}\left(3\coth^2\left(\frac{1}{2}\sqrt{\frac{\omega^2-k^2}{k^4}}\,\xi+\rho_{\mathscr{H}}\right)-1\right),\tag{3.12d}$$

where $\xi=kx+\omega t$, $\eta=-(1/2)(\omega x/k)+(1/24)((24k^4+30\omega^4-54k^2\omega^2)/k^2(k^2-\omega^2))t$, $\rho_{\mathscr{H}}=\tanh^{-1}(C_1/C_2)$, and k,ω are arbitrary constants.

Now, when $\mathfrak{D} = \lambda^2 - 4\mu = (\omega^2 - k^2)/k^4 < 0$, we obtain trigonometric function solutions u_τ and v_τ of Schrödinger-Boussinesq system (1.1) as follows:

$$\begin{aligned} u_\tau(x,t) = 2\sqrt{\frac{1}{ab}}\Bigg[3k^2\Bigg(\frac{\sqrt{-\mathfrak{D}}}{2}\left(\frac{-C_1\sin\left(\left(\sqrt{-\mathfrak{D}}/2\right)\xi\right)+C_2\cos\left(\left(\sqrt{-\mathfrak{D}}/2\right)\xi\right)}{C_1\cos\left(\left(\sqrt{-\mathfrak{D}}/2\right)\xi\right)+C_2\sin\left(\left(\sqrt{-\mathfrak{D}}/2\right)\xi\right)}\right) \\ \mp\frac{1}{6}\frac{\sqrt{3\omega^2-3k^2}}{k^2}\Bigg)^2 \\ \pm\sqrt{3\omega^2-3k^2}\Bigg(\frac{\sqrt{-\mathfrak{D}}}{2}\left(\frac{-C_1\sin\left(\left(\sqrt{-\mathfrak{D}}/2\right)\xi\right)+C_2\cos\left(\left(\sqrt{\mathfrak{D}}/2\right)\xi\right)}{C_1\cos\left(\left(\sqrt{-\mathfrak{D}}/2\right)\xi\right)+C_2\sin\left(\left(\sqrt{-\mathfrak{D}}/2\right)\xi\right)}\right) \\ \mp\frac{1}{6}\frac{\sqrt{3\omega^2-3k^2}}{k^2}\Bigg)\Bigg]e^{i\eta}, \\ v_\tau(x,t) = \frac{6k^2}{a}\left(\frac{\sqrt{-\mathfrak{D}}}{2}\left(\frac{-C_1\sin\left(\left(\sqrt{-\mathfrak{D}}/2\right)\xi\right)+C_2\cos\left(\left(\sqrt{-\mathfrak{D}}/2\right)\xi\right)}{C_1\cos\left(\left(\sqrt{-\mathfrak{D}}/2\right)\xi\right)+C_2\sin\left(\left(\sqrt{-\mathfrak{D}}/2\right)\xi\right)}\right)\mp\frac{1}{6}\frac{\sqrt{3\omega^2-3k^2}}{k^2}\right)^2 \\ \pm 2\frac{\sqrt{3\omega^2-3k^2}}{a}\Bigg(\frac{\sqrt{-\mathfrak{D}}}{2}\left(\frac{-C_1\sin\left(\left(\sqrt{-\mathfrak{D}}/2\right)\xi\right)+C_2\cos\left(\left(\sqrt{-\mathfrak{D}}/2\right)\xi\right)}{C_1\cos\left(\left(\sqrt{-\mathfrak{D}}/2\right)\xi\right)+C_2\sin\left(\left(\sqrt{-\mathfrak{D}}/2\right)\xi\right)}\right) \\ \mp\frac{1}{6}\frac{\sqrt{3\omega^2-3k^2}}{k^2}\Bigg), \end{aligned} \tag{3.13}$$

where $\mathfrak{D} = (\omega^2 - k^2)/k^4 < 0$, $\xi = kx + \omega t$, $\eta = -(1/2)(\omega x/k) + (1/24)((24k^4 + 30\omega^4 - 54k^2\omega^2)/k^2(k^2 - \omega^2))t$, and k, ω, C_1, and C_2 are arbitrary constants and $k \neq \pm\omega$. Similarity, the trigonometric solutions (3.13) can be rewritten at $C_1^2 > C_2^2$, and $C_1^2 < C_2^2$, as follows:

$$u_\tau(x,t) = \frac{1}{2}\sqrt{\frac{1}{ab}}\frac{k^2-\omega^2}{k^2}\left(3\tan^2\left(\frac{1}{2}\sqrt{\frac{k^2-\omega^2}{k^4}}\,\xi + \rho_\tau\right) + 1\right)e^{i\eta}, \tag{3.14a}$$

$$v_\tau(x,t) = \frac{1}{2}\frac{k^2-\omega^2}{ak^2}\left(3\tan^2\left(\frac{1}{2}\sqrt{\frac{k^2-\omega^2}{k^4}}\,\xi + \rho_\tau\right) + 1\right), \tag{3.14b}$$

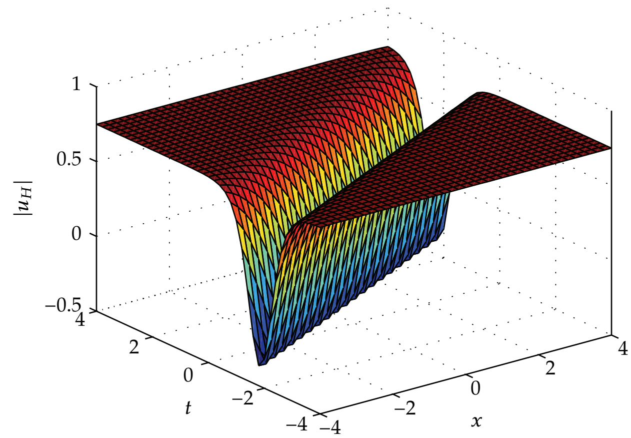

Figure 1: Soliton solution $|u_{\mathscr{L}}(x,t)|$ of the Schrödinger-Boussinesq system, (3.12a), for $\alpha = 1$, $\beta = 16$, $k = 1$, $\omega = -2$, and $\rho_{\mathscr{L}} = \tanh^{-1}(1/4)$.

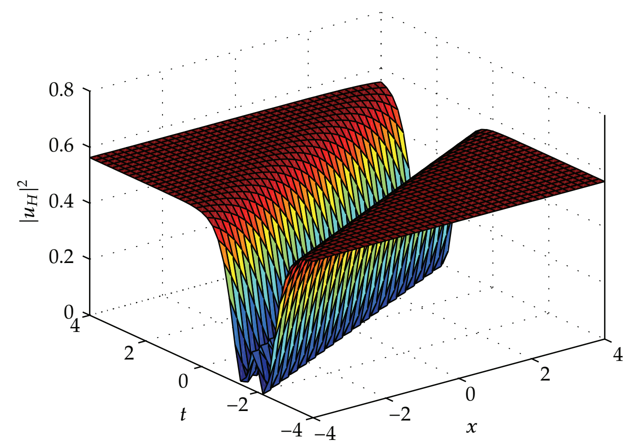

Figure 2: Soliton solution $|u_{\mathscr{L}}(x,t)|^2$ of the Schrödinger-Boussinesq system, (3.12a), for $\alpha = 1$, $\beta = 16$, $k = 1$, $\omega = -2$, and $\rho_{\mathscr{L}} = \tanh^{-1}(1/4)$.

$$u_{\mathcal{T}}(x,t) = \frac{1}{2}\sqrt{\frac{1}{ab}}\frac{k^2-\omega^2}{k^2}\left(3\cot^2\left(\frac{1}{2}\sqrt{\frac{k^2-\omega^2}{k^4}}\;\xi + \rho_{\mathcal{T}}\right)+1\right)e^{i\eta}, \tag{3.14c}$$

$$v_{\mathcal{T}}(x,t) = \frac{1}{2}\frac{k^2-\omega^2}{ak^2}\left(3\cot^2\left(\frac{1}{2}\sqrt{\frac{k^2-\omega^2}{k^4}}\;\xi + \rho_{\mathcal{T}}\right)+1\right), \tag{3.14d}$$

where $\xi = kx + \omega t$, $\eta = -(1/2)(\omega x/k) + (1/24)((24k^4 + 30\omega^4 - 54k^2\omega^2)/k^2(k^2-\omega^2))t$, $\rho_{\mathcal{T}} = \tan^{-1}(C_1/C_2)$, and k, ω are arbitrary constants.

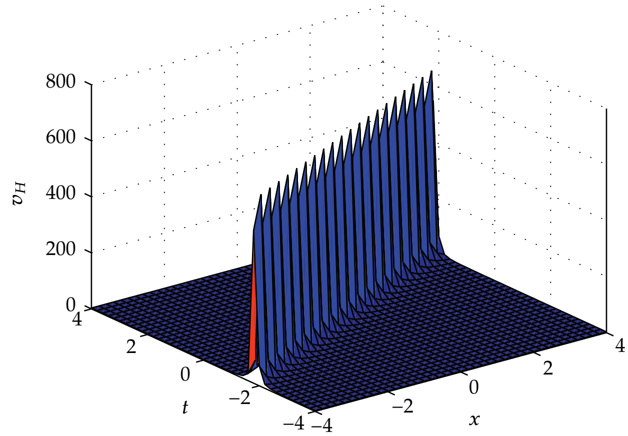

Figure 3: Soliton solution $|v_{\mathscr{L}}(x,t)|$ of the Schrödinger-Boussinesq system, (3.12b), for $\alpha = 1$, $\beta = 16$, $k = 1$, $\omega = -2$, and $\rho_{\mathscr{L}} = \tanh^{-1}(1/4)$.

4. Conclusions

This study shows that the (G'/G)-expansion method is quite efficient and practically well suited for use in finding exact solutions for the Schrödinger-Boussinesq system. With the aid of `Maple`, we have assured the correctness of the obtained solutions by putting them back into the original equation. To illustrate the obtained solutions, the hyperbolic type of obtained solutions, (3.12a) and (3.12b), are attached as Figures 1, 2, and 3. We hope that they will be useful for further studies in applied sciences.

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