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Research Article

Common Fixed Point Results for Four Mappings on Partial Metric Spaces

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We give fixed point results for four mappings which satisfy almost generalized contractive condition on partial metric space and we support the results with an example.

1. Introduction and Preliminaries

Partial metric spaces, introduced by Matthews [1, 2], are a generalization of the notion of the metric space in which in definition of metric, the condition $d(x,x) = 0$ is replaced by the condition $d(x,x) \leq d(x,y)$.

In [1], Matthews discussed some properties of convergence of sequence and proved the fixed point theorems for contractive mapping on partial metric spaces: any mapping T of a complete partial metric space X into itself that satisfies, where $0 \leq k < 1$, the inequality $d(Tx,Ty) \leq kd(x,y)$ for all $x, y \in X$, has a unique fixed point. Recently, many authors (see [3–15]) have focused on this subject and generalized some fixed point theorems from the class of metric spaces.

The definition of partial metric space is given by Matthews (see [2]) as follows.

Definition 1.1. Let X be a nonempty set and let $p : X \times X \to \mathbb{R}_0^+$ satisfy

(PM1) $x = y \Leftrightarrow p(x,x) = p(y,y) = p(x,y)$,

(PM2) $p(x,x) \leq p(x,y)$,

(PM3) $p(x,y) = p(y,x)$,

(PM4) $p(x,y) \leq p(x,z) + p(z,y) - p(z,z)$,

for all x, y and $z \in X$, where $\mathbb{R}_0^+ = [0,\infty)$. Then the pair (X,p) is called a partial metric space (in short PMS) and p is called a partial metric on X.

Let (X,p) be a PMS. Then, the functions $p^s, p^w : X \times X \to \mathbb{R}_0^+$ given by

$$\begin{aligned} p^s(x,y) &= 2p(x,y) - p(x,x) - p(y,y) \\ p^w(x,y) &= p(x,y) - \min\{p(x,x), p(y,y)\} \end{aligned} \tag{1.1}$$

are ordinary equivalent metrics on X. Each partial metric p on X generates a T_0 topology τ_p on X with a base of the family of open p-balls $\{B_p(x,\varepsilon) : x \in X, \varepsilon > 0\}$, where $B_p(x,\varepsilon) = \{y \in X : p(x,y) < p(x,x) + \varepsilon\}$ for all $x \in X$ and $\varepsilon > 0$.

Example 1.2 (see [1, 2]). Let $X = \{[a,b] : a,b \in \mathbb{R}, a \leq b\}$ and define

$$p([a,b],[c,d]) = \max\{b,d\} - \min\{a,c\}. \tag{1.2}$$

Then (X,p) is a partial metric space.

We give same topological definitions on partial metric spaces.

Definition 1.3 (see [1, 2, 4]).

(i) A sequence $\{x_n\}$ in a PMS (X,p) converges to $x \in X$ if and only if $p(x,x) = \lim_{n\to\infty} p(x,x_n)$.

(ii) A sequence $\{x_n\}$ in a PMS (X,p) is called a Cauchy sequence if and only if $\lim_{n,m\to\infty} p(x_n,x_m)$ exists (and finite).

(iii) A PMS (X,p) is said to be complete if every Cauchy sequence $\{x_n\}$ in X converges, with respect to τ_p, to a point $x \in X$ such that $p(x,x) = \lim_{n,m\to\infty} p(x_n,x_m)$.

(iv) A mapping $f : X \to X$ is said to be continuous at $x_0 \in X$ if for every $\varepsilon > 0$, there exists $\delta > 0$ such that $f(B(x_0,\delta)) \subset B(f(x_0),\varepsilon)$.

Lemma 1.4 (see [1, 2, 4]).

(A) *A sequence $\{x_n\}$ is Cauchy in a PMS (X,p) if and only if $\{x_n\}$ is Cauchy in a metric space (X,p^s).*

(B) *A PMS (X,p) is complete if and only if the metric space (X,p^s) is complete. Moreover,*

$$\lim_{n\to\infty} p^s(x,x_n) = 0 \Longleftrightarrow p(x,x) = \lim_{n\to\infty} p(x,x_n) = \lim_{n,m\to\infty} p(x_n,x_m), \tag{1.3}$$

where x is a limit of $\{x_n\}$ in (X,p^s).

Remark 1.5 (see [11]). Let (X, p) be a PMS. Therefore,

(A) if $p(x, y) = 0$, then $x = y$;

(B) if $x \neq y$, then $p(x, y) > 0$.

Lemma 1.6 (see [10]). *Assume $x_n \to z$ as $n \to \infty$ in a PMS (X, p) such that $p(z, z) = 0$. Then $\lim_{n\to\infty} p(x_n, y) = p(z, y)$ for every $y \in X$.*

On the other hand, Kannan [16] proved a fixed point theorem for a map satisfying a contractive condition that did not require continuity at each point. Afterward Sessa [17] introduced the notion of weakly commuting maps, which generalized the concept of commuting maps. Then Jungck generalized this idea, first to compatible mappings [18] and then to weakly compatible mappings [19].

A pair (f, T) of self-mappings on X is said to be weakly compatible if they commute at their coincidence point (i.e., $fTx = Tfx$ whenever $fx = Tx$). A point $y \in X$ is called point of coincidence of a family T_j, $j \in J$, of self-mappings on X if there exists a point $x \in X$ such that $y = T_j x$ for all $j \in J$.

The concept of almost contraction property was given to as follows by Berinde.

Definition 1.7 (see [20, 21]). Let (X, d) be a metric space. A map $f : X \to X$ is called an almost contraction if there exist a constant $\delta \in [0, 1[$ and some $L \geq 0$ such that for all $x, y \in X$

$$d(fx, fy) \leq \delta d(x, y) + Ld(fx, y). \tag{1.4}$$

Berinde called this as "weak contraction" in [20], then he renamed it as "almost contraction" in [21, 22], also Berinde [21] proved some fixed point theorems for almost contraction in complete metric space. Definition 1.7 is a special case of the following definition (choose $g = I_X$, I_X is the identity map on X).

Definition 1.8 (see [7]). Let (X, d) be a metric space. A map $f : X \to X$ is called an almost contraction with respect to a mapping $g : X \to X$ if there exist a constant $\delta \in [0, 1[$ and some $L \geq 0$ such that for all x, $y \in X$

$$d(fx, fy) \leq \delta d(gx, gy) + Ld(fx, gy). \tag{1.5}$$

Babu et al. [23] considered the class of mappings that satisfy "condition (B)."

Let (X, d) be a metric space. A map $T : X \to X$ is said to satisfy "condition (B)" if there exist a constant $\delta \in [0, 1[$ and some $L \geq 0$ such that for all $x, y \in X$,

$$d(fx, fy) \leq \delta d(x, y) + L \min\{p(x, fx), p(y, fy), p(x, fy), p(y, fx)\}. \tag{1.6}$$

Afterward, Berinde [21], Abbas and Ilić [24], and Ćirić et al. [7] generalized the above definition and proved some fixed point results.

In recent paper, Altun and Acar [25] introduced the notion of (δ, L) weak contraction in the sense of Berinde in partial metric space.

Definition 1.9 (see [25]). Let (X, p) be a partial metric space. A map $T : X \to X$ is called (δ, L)-weak contraction if there exist a $\delta \in [0, 1)$ and some $L \geq 0$ such that

$$p(Tx, Ty) \leq \delta p(x, y) + Lp^w(y, Tx), \tag{1.7}$$

for all $x, y \in X$.

In this paper, we give a fixed point theorem for four mappings satisfying almost generalized contractive condition in [26] on partial metric spaces.

2. Main Results

Theorem 2.1. *Let (X, p) be a complete partial metric space and f, g, S and T be self maps on X, with $f(X) \subseteq T(X)$ and $g(X) \subseteq S(X)$. If there exists $\delta \in [0, 1[$ and $L \geq 0$ with such that*

$$p(fx, gy) \leq \delta M(x, y) + LN(x, y), \tag{2.1}$$

for any x, $y \in X$, where,

$$\begin{gathered} M(x, y) = \max\left\{p(Sx, Ty), p(fx, Sx), p(gy, Ty), \frac{p(Sx, gy) + p(fx, Ty)}{2}\right\}, \\ N(x, y) = \min\{p^w(fx, Sx), p^w(gy, Ty), p^w(Sx, gy), p^w(fx, Ty)\}. \end{gathered} \tag{2.2}$$

If $\{f, S\}$ and $\{g, T\}$ are weakly compatible and one of $f(X), g(X), S(X)$, and $T(X)$ is a complete subspace of X, then f, g, S, and T have a common fixed point.

Proof. Let x_0 be an arbitrary point in X. Since $f(X) \subseteq T(X)$, we can find $x_1 \in X$ such that $fx_0 = Tx_1$ and also, as $gx_1 \in S(X)$, there exist $x_2 \in X$ such that $gx_1 = Sx_2$. In general, $x_{2n+1} \in X$ is chosen such that $fx_{2n} = Tx_{2n+1}$ and $x_{2n+2} \in X$ such that $gx_{2n+1} = Sx_{2n+2}$, we obtain a sequences $\{y_n\}$ in X such that

$$y_{2n} = fx_{2n} = Tx_{2n+1}, \quad y_{2n+1} = gx_{2n+1} = Sx_{2n+2}, \quad \forall n \geq 0. \tag{2.3}$$

Suppose $y_{2m} = y_{2m+1}$ for some m. Thus, g and T have a coincidence point. Due to (2.1), we have

$$\begin{aligned} p(y_{2m+2}, y_{2m+1}) &= p(fx_{2m+2}, gx_{2m+1}) \\ &\leq \delta M(x_{2m+2}, x_{2m+1}) + LN(x_{2m+2}, x_{2m+1}), \end{aligned} \tag{2.4}$$

where

$$
\begin{aligned}
N(x_{2m+2}, x_{2m+1}) &= \min\left\{\begin{matrix} p^w(fx_{2m+2}, Sx_{2m+2}), p^w(gx_{2m+1}, Tx_{2m+1}), \\ p^w(Sx_{2m+2}, gx_{2m+1}), p^w(fx_{2m+2}, Tx_{2m+1}) \end{matrix}\right\} \\
&= \min\left\{\begin{matrix} p^w(y_{2m+2}, y_{2m+1}), p^w(y_{2m+1}, y_{2m}), \\ p^w(y_{2m+1}, y_{2m+1}), p^w(y_{2m+2}, y_{2m}) \end{matrix}\right\} \\
&= 0, \\
M(x_{2m+2}, x_{2m+1}) &= \max\left\{\begin{matrix} p(Sx_{2m+2}, Tx_{2m+1}), p(fx_{2m+2}, Sx_{2m+2}), \\ p(gx_{2m+1}, Tx_{2m+1}), \\ \dfrac{p(Sx_{2m+2}, gx_{2m+1}) + p(fx_{2m+2}, Tx_{2m+1})}{2} \end{matrix}\right\} \\
&= \max\left\{\begin{matrix} p(y_{2m+1}, y_{2m}), p(y_{2m+2}, y_{2m+1}), \\ p(y_{2m+1}, y_{2m}), \\ \dfrac{p(y_{2m+1}, y_{2m+1}) + p(y_{2m+2}, y_{2m})}{2} \end{matrix}\right\} \\
&= p(y_{2m+2}, y_{2m+1}).
\end{aligned}
\tag{2.5}
$$

So,

$$
p(y_{2m+2}, y_{2m+1}) \le \delta p(y_{2m+2}, y_{2m+1}). \tag{2.6}
$$

Therefore, by $\delta \in [0, 1[$, we have $p(y_{2m+2}, y_{2m+1}) = 0$, that is, $y_{2m+1} = y_{2m+2}$. So, f and S have a coincidence point.

Suppose now that $y_n \ne y_{n+1}$ for all $n \ge 0$. From (2.1), we obtain

$$
p(y_{2n}, y_{2n+1}) = p(fx_{2n}, gx_{2n+1}) \le \delta M(x_{2n}, x_{2n+1}) + LN(x_{2n}, x_{2n+1}), \tag{2.7}
$$

where

$$
\begin{aligned}
N(x_{2n}, x_{2n+1}) &= \min\left\{\begin{matrix} p^w(fx_{2n}, Sx_{2n}), p^w(gx_{2n+1}, Tx_{2n+1}), \\ p^w(Sx_{2n}, gx_{2n+1}), p^w(fx_{2n}, Tx_{2n+1}) \end{matrix}\right\} \\
&= \min\left\{\begin{matrix} p^w(y_{2n}, y_{2n-1}), p^w(y_{2n+1}, y_{2n}), \\ p^w(y_{2n-1}, y_{2n+1}), p^w(y_{2n}, y_{2n}) \end{matrix}\right\} \\
&= 0, \\
M(x_{2n}, x_{2n+1}) &= \max\left\{\begin{matrix} p(Sx_{2n}, Tx_{2n+1}), p(fx_{2n}, Sx_{2n}), \\ p(gx_{2n+1}, Tx_{2n+1}), \dfrac{p(Sx_{2n}, gx_{2n+1}) + p(fx_{2n}, Tx_{2n+1})}{2} \end{matrix}\right\} \\
&= \max\left\{\begin{matrix} p(y_{2n-1}, y_{2n}), p(y_{2n}, y_{2n-1}), \\ p(y_{2n+1}, y_{2n}), \dfrac{p(y_{2n-1}, y_{2n+1}) + p(y_{2n}, y_{2n})}{2} \end{matrix}\right\}.
\end{aligned}
\tag{2.8}
$$

Due to (2.7), we have

$$p(y_{2n}, y_{2n+1}) \leq \delta M(x_{2n}, x_{2n+1}). \tag{2.9}$$

Due to PM4, we have

$$p(y_{2n-1}, y_{2n+1}) + p(y_{2n}, y_{2n}) \leq p(y_{2n-1}, y_{2n}) + p(y_{2n}, y_{2n+1}). \tag{2.10}$$

Hence, $M(x_{2n}, x_{2n+1}) = \max\{p(y_{2n}, y_{2n-1}), p(y_{2n+1}, y_{2n})\}$. If $M(x_{2n}, x_{2n+1}) = p(y_{2n+1}, y_{2n})$, then by (2.7)

$$p(y_{2n+1}, y_{2n}) \leq \delta p(y_{2n+1}, y_{2n}). \tag{2.11}$$

Since $\delta \in [0,1[$, the inequality (2.9) yields a contradiction. Hence, $M(x_{2n}, x_{2n+1}) = p(y_{2n}, y_{2n-1})$, then by (2.7) we have

$$p(y_{2n+1}, y_{2n}) \leq \delta p(y_{2n}, y_{2n-1}). \tag{2.12}$$

Thus, one can observe that

$$p(y_{n+1}, y_n) \leq \delta^n p(y_0, y_1), \quad \forall n = 0, 1, 2, \ldots. \tag{2.13}$$

Consider now

$$\begin{aligned} p^s(y_{n+2}, y_{n+1}) &= 2p(y_{n+2}, y_{n+1}) - p(y_{n+2}, y_{n+2}) - p(y_{n+1}, y_{n+1}) \\ &\leq 2p(y_{n+2}, y_{n+1}) \\ &\leq \delta^{n+1} p(y_0, y_1). \end{aligned} \tag{2.14}$$

Hence, regarding (2.13), we have

$$\lim_{n\to\infty} p^s(y_{n+2}, y_{n+1}) = 0. \tag{2.15}$$

Moreover,

$$\begin{aligned} p^s(y_{n+1}, y_{n+k}) &\leq p^s(y_{n+k-1}, y_{n+k}) + \cdots + p^s(y_{n+1}, y_{n+2}) \\ &\leq 2\delta^{n+k-1} p(y_0, y_1) + \cdots + 2\delta^{n+1} p(y_0, y_1). \end{aligned} \tag{2.16}$$

After standard calculation, we obtain that $\{y_n\}$ is a Cauchy sequence in (X, p^s), that is, $p^s(y_n, y_m) \to 0$ as $n, m \to \infty$. Since (X, p) is complete, by Lemma 1.4, (X, p^s) is complete and sequence $\{y_n\}$ is convergent in (X, p^s) to say $z \in X$. From Lemma 1.4,

$$p(z, z) = \lim_{n\to\infty} p(y_n, z) = \lim_{n,m\to\infty} p(y_n, y_m). \tag{2.17}$$

Since $\{y_n\}$ is a Cauchy sequence in (X, p^s), we have

$$\lim_{n,m\to\infty} p^s(y_n, y_m) = 0. \tag{2.18}$$

We assert that $\lim_{n,m\to\infty} p(y_n, y_m) = 0$. Without loss of generality, we assume that $n > m$,

$$\begin{aligned} p(y_{n+2}, y_n) &\leq p(y_{n+2}, y_{n+1}) + p(y_{n+1}, y_n) - p(y_{n+1}, y_{n+1}) \\ &\leq p(y_{n+2}, y_{n+1}) + p(y_{n+1}, y_n). \end{aligned} \tag{2.19}$$

Similarly,

$$\begin{aligned} p(y_{n+3}, y_n) &\leq p(y_{n+3}, y_{n+2}) + p(y_{n+2}, y_n) - p(y_{n+2}, y_{n+2}) \\ &\leq p(y_{n+3}, y_{n+2}) + p(y_{n+2}, y_n). \end{aligned} \tag{2.20}$$

Taking into account (2.20), the expression (2.19) yields

$$p(y_{n+3}, y_n) \leq p(y_{n+3}, y_{n+2}) + p(y_{n+2}, y_{n+1}) + p(y_{n+1}, y_n). \tag{2.21}$$

Inductively, we obtain

$$p(y_m, y_n) \leq p(y_m, y_{m+1}) + \cdots + p(y_{n-2}, y_{n-1}) + p(y_{n-1}, y_n). \tag{2.22}$$

Due to (2.13),

$$\begin{aligned} p(y_m, y_n) &\leq \delta^m p(y_0, y_1) + \cdots + \delta^{n-2} p(y_0, y_1) + \delta^{n-1} p(y_0, y_1) \\ &\leq \delta^m \left(1 + \delta + \cdots + \delta^{n-m-1}\right) p(y_0, y_1). \end{aligned} \tag{2.23}$$

Regarding $\delta \in [0,1[$, we can observe that $\lim_{n,m\to\infty} p(y_n, y_m) = 0$.

Since $y_n \to z$ in X, $\{fx_{2n}\}$, $\{Tx_{2n+1}\}$, $\{gx_{2n+1}\}$, $\{Sx_{2n+2}\}$ converge to z.

Now we show that z is the fixed point for maps g and T. Assume that $T(X)$ is complete, there exists $u \in X$ such that $z = Tu$. We will show that $gu = z$. On the contrary, assume that $gu \neq z$.

From, (2.1) we have

$$p(fx_{2n}, gu) \leq \delta M(x_{2n}, u) + LN(x_{2n}, u), \tag{2.24}$$

where

$$\begin{aligned}
N(x_{2n},u) &= \min\{p^w(fx_{2n},Sx_{2n}),p^w(gu,Tu),p^w(Sx_{2n},gu),p^w(fx_{2n},Tu)\}\\
&= \min\{p^w(fx_{2n},Sx_{2n}),p^w(gu,z),p^w(Sx_{2n},gu),p^w(fx_{2n},z)\},\\
M(x_{2n},u) &= \max\left\{\begin{array}{c} p(Sx_{2n},Tu),p(fx_{2n},Sx_{2n}),p(gu,Tu),\\ \dfrac{p(Sx_{2n},gu)+p(fx_{2n},Tu)}{2}\end{array}\right\}\\
&= \max\left\{\begin{array}{c} p(Sx_{2n},z),p(fx_{2n},Sx_{2n}),p(gu,z),\\ \dfrac{p(Sx_{2n},gu)+p(fx_{2n},z)}{2}\end{array}\right\}.
\end{aligned} \tag{2.25}$$

Since $\lim_{n\to\infty}M(x_{2n},u)=p(gu,z)$ and $\lim_{n\to\infty}N(x_{2n},u)=0$. We get

$$p(z,gu)\leq \delta p(gu,z). \tag{2.26}$$

Since $\delta\in[0,1[$, we get $p(z,gu)=0$. Therefore, $gu=Tu=z$. Since the maps g and T are weakly compatible, we have $gz=gTu=Tgu=Tz$. We will also show that $gz=z$. From (2.1), we have

$$p(fx_{2n},gz)\leq \delta M(x_{2n},z)+LN(x_{2n},z), \tag{2.27}$$

where

$$\begin{aligned}
N(x_{2n},z) &= \min\{p^w(fx_{2n},Sx_{2n}),p^w(gz,Tz),p^w(Sx_{2n},gz),p^w(fx_{2n},Tz)\},\\
M(x_{2n},z) &= \max\left\{\begin{array}{c} p(Sx_{2n},Tz),p(fx_{2n},Sx_{2n}),\\ p(gz,Tz),\dfrac{p(Sx_{2n},gz)+p(fx_{2n},Tz)}{2}\end{array}\right\}\\
&= \max\left\{\begin{array}{c} p(Sx_{2n},gz),p(fx_{2n},Sx_{2n}),\\ p(gz,gz),\dfrac{p(Sx_{2n},gz)+p(fx_{2n},Tz)}{2}\end{array}\right\}.
\end{aligned} \tag{2.28}$$

Since $\lim_{n\to\infty}M(x_{2n},z)=p(z,gz)$ and $\lim_{n\to\infty}N(x_{2n},z)=0$, then

$$p(z,gz)=\lim_{n\to\infty}p(fx_{2n},gz)\leq \delta p(z,gz). \tag{2.29}$$

Since $\delta\in[0,1[$, $p(z,gz)=0$. By Remark 1.5, we get $z=gz$.

Similarly, we show that z is also fixed point of f and S. Hence, $fz=gz=Tz=Sz=z$.

The proofs for the cases in which $S(X)$, $f(X)$, or $g(X)$ is complete are similar.

Last, we show z is unique. Suppose on the contrary that there is another common fixed point t of f, g, S, and T. Then

$$p(z,t)=p(fz,gt)\leq \delta M(z,t)+LN(z,t), \tag{2.30}$$

where

$$\begin{aligned} N(z,t) &= \min\{p^w(fz,Sz),p^w(gt,Tt),p^w(Sz,gt),p^w(fz,Tt)\} \\ &= 0, \\ M(z,t) &= \max\left\{\begin{array}{c} p(Sz,Tt),p(fz,Sz),p(gt,Tt), \\ \dfrac{p(Sz,gt)+p(fz,Tt)}{2} \end{array}\right\} \\ &= p(Sz,Tt) \\ &= p(z,t). \end{aligned} \tag{2.31}$$

Thus,

$$p(z,t) \le \delta p(z,t). \tag{2.32}$$

Therefore, $p(z,t) = 0$ and Remark 1.5 $z = t$. So, z is the unique common fixed point os f, g, S, and T. □

Example 2.2. Let $X = \{0,1,2\}$ endowed with the partial metric p given by $p(x,y) = \max\{x,y\}$ for all $x,y \in X$. It is clear that (X,p) is a complete partial metric space. Define the mappings $f,g,S,T : X \to X$ by

$$\begin{gathered} f = g, \qquad S = T, \\ f0 = f2 = 0, \qquad f1 = 1 \\ T0 = 0, \quad T1 = 2, \qquad T2 = 1. \end{gathered} \tag{2.33}$$

We have $f(X) \subseteq T(X) = X$. For $\delta = 1/2$, $L = 1$,

$$\begin{gathered} p(f0,f1) = 1 \le \delta.2 + L.1, \\ p(f2,f1) = 1 \le \delta.2 + L.0, \\ p(f2,f2) = p(f0,f0) = 0 \le \delta.0 + L.1, \\ p(f1,f1) = 1 \le \delta.2 + L.0. \end{gathered} \tag{2.34}$$

Then, the contractive condition (2.1) is satisfied for every x, $y \in X$. Moreover, $\{f,T\}$ is weakly compatible. So all conditions of Theorem 2.1 are satisfied. We deduce the existence and uniqueness of a common fixed point of f and T. Here, 0 is the unique common fixed point.

Corollary 2.3. *Let (X,p) is complete PMS and f and T be self maps on X, with $f(X) \subseteq T(X)$. If there exists $\delta \in [0,1[$ and $L \ge 0$ such that*

$$p(fx,fy) \le \delta M(x,y) + LN(x,y), \tag{2.35}$$

where,

$$M(x,y) = \max\left\{p(Tx,Ty), p(fx,Tx), p(fy,Ty), \frac{p(Tx,fy)+p(fx,Ty)}{2}\right\},$$
$$N(x,y) = \min\{p^w(fx,Tx), p^w(fy,Ty), p^w(Tx,fy), p^w(fx,Ty)\}, \tag{2.36}$$

for every $x, y \in X$. If $\{f,T\}$ is weakly compatible and one of $f(X)$ and $T(X)$ is a complete subspace of X, then f and T have a common fixed point.

Remark 2.4. It is easy to see that for every map $T : X \to X$, $\{T, I_X\}$ is weakly compatible, where I_X is identity map on X, so by taking $f = g = I_X$ in Theorem 2.1 we have the following results.

Corollary 2.5. *Let (X,p) is complete PMS and S and T be self maps on X. If there exists $\delta \in [0,1[$ and $L \geq 0$ such that*

$$p(x,y) \leq \delta M(x,y) + LN(x,y), \tag{2.37}$$

for every $x, y \in X$, where

$$M(x,y) = \max\left\{p(Sx,Ty), p(x,Sx), p(y,Ty), \frac{p(Sx,y)+p(x,Ty)}{2}\right\},$$
$$N(x,y) = \min\{p^w(x,Sx), p^w(y,Ty), p^w(Sx,y), p^w(x,Ty)\}. \tag{2.38}$$

Then S and T have a common fixed point.

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