

Research Article

Differential Subordinations for Certain Meromorphically Multivalent Functions Defined by Dziok-Srivastava Operator

Ying Yang,¹ Yu-Qin Tao,¹ and Jin-Lin Liu²

¹ Department of Mathematics, Maanshan Teacher's College, Maanshan 243000, China

² Department of Mathematics, Yangzhou University, Yangzhou 225002, China

Correspondence should be addressed to Jin-Lin Liu, jlliu@yzu.edu.cn

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By making use of the Dziok-Srivastava operator, we introduce a new class of meromorphically multivalent functions. Some inclusion properties of functions belonging to this class are derived.

1. Introduction

Let $\Sigma(p)$ denote the class of functions of the form

$$f(z) = z^{-p} + \sum_{n=1}^{\infty} a_n z^{n-p} \quad (p \in \mathbb{N} := \{1, 2, 3, \dots\}), \quad (1.1)$$

which are analytic in the punctured open unit disk $\mathbb{U}_0 = \{z : 0 < |z| < 1\}$ with a pole at $z = 0$. Also let the Hadamard product (or convolution) of the following functions:

$$f_j(z) = z^{-p} + \sum_{n=1}^{\infty} a_{n,j} z^{n-p} \quad (j = 1, 2) \quad (1.2)$$

be given by

$$(f_1 * f_2)(z) := z^{-p} + \sum_{n=1}^{\infty} a_{n,1} a_{n,2} z^{n-p} = (f_2 * f_1)(z). \quad (1.3)$$

Given two functions $f(z)$ and $g(z)$, which are analytic in $\mathbb{U} = \mathbb{U}_0 \cup \{0\}$, we say that the function $g(z)$ is subordinate to $f(z)$ and write $g < f$ or (more precisely) $g(z) < f(z)$ ($z \in \mathbb{U}$), if there exists a Schwarz function $w(z)$, analytic in \mathbb{U} with $w(0) = 0$ and $|w(z)| < 1$ ($z \in \mathbb{U}$) such that $g(z) = f(w(z))$ ($z \in \mathbb{U}$). In particular, if $f(z)$ is univalent in \mathbb{U} , we have the following equivalence:

$$g(z) < f(z) \quad (z \in \mathbb{U}) \iff g(0) = f(0), \quad g(\mathbb{U}) \subset f(\mathbb{U}). \quad (1.4)$$

Let A be the class of functions of the form

$$f(z) = z + \sum_{n=2}^{\infty} b_n z^n, \quad (1.5)$$

which are analytic in \mathbb{U} . A function $f(z) \in A$ is said to be in the class $S^*(\alpha)$ if

$$\operatorname{Re} \left\{ \frac{zf'(z)}{f(z)} \right\} > \alpha \quad (z \in \mathbb{U}) \quad (1.6)$$

for some α ($\alpha < 1$). When $0 \leq \alpha < 1$, $S^*(\alpha)$ is the class of starlike functions of order α in \mathbb{U} . A function $f(z) \in A$ is said to be prestarlike of order α in \mathbb{U} if

$$\frac{z}{(1-z)^{2(1-\alpha)}} * f(z) \in S^*(\alpha) \quad (\alpha < 1), \quad (1.7)$$

where the symbol $*$ means the familiar Hadamard product (or convolution) of two analytic functions in \mathbb{U} . We denote this class by $R(\alpha)$ (see [1]). Clearly a function $f(z) \in A$ is in the class $R(0)$ if and only if $f(z)$ is convex univalent in \mathbb{U} and $R(1/2) = S^*(1/2)$.

For complex parameters

$$\alpha_1, \dots, \alpha_q \text{ and } \beta_1, \dots, \beta_s \quad (\beta_j \notin \mathbb{Z}_0^- := \{0, -1, -2, \dots\}; j = 1, 2, \dots, s), \quad (1.8)$$

we define the generalized hypergeometric function ${}_qF_s(\alpha_1, \dots, \alpha_q; \beta_1, \dots, \beta_s; z)$ by

$${}_qF_s(\alpha_1, \dots, \alpha_q; \beta_1, \dots, \beta_s; z) := \sum_{n=0}^{\infty} \frac{(\alpha_1)_n \cdots (\alpha_q)_n}{(\beta_1)_n \cdots (\beta_s)_n} \cdot \frac{z^n}{n!} \quad (1.9)$$

$$(q \leq s + 1; q, s \in \mathbb{N}_0 := \mathbb{N} \cup \{0\}; z \in \mathbb{U}),$$

where $(x)_n$ is the Pochhammer symbol defined, in terms of the Gamma function $\Gamma(z)$, by

$$(x)_n := \frac{\Gamma(x+n)}{\Gamma(x)} = \begin{cases} 1 & (n = 0; x \in \mathbb{C} \setminus \{0\}) \\ x(x+1) \cdots (x+n-1) & (n \in \mathbb{N}; x \in \mathbb{C}). \end{cases} \quad (1.10)$$

Corresponding to a function $h_p(\alpha_1, \dots, \alpha_q; \beta_1, \dots, \beta_s; z)$ defined by

$$h_p(\alpha_1, \dots, \alpha_q; \beta_1, \dots, \beta_s; z) = z^{-p} {}_qF_s(\alpha_1, \dots, \alpha_q; \beta_1, \dots, \beta_s; z), \tag{1.11}$$

we now consider a linear operator

$$H_p(\alpha_1, \dots, \alpha_q; \beta_1, \dots, \beta_s) : \Sigma(p) \longrightarrow \Sigma(p), \tag{1.12}$$

defined by means of the Hadamard product (or convolution) as follows:

$$H_p(\alpha_1, \dots, \alpha_q; \beta_1, \dots, \beta_s) f(z) := h_p(\alpha_1, \dots, \alpha_q; \beta_1, \dots, \beta_s; z) * f(z). \tag{1.13}$$

For convenience, we write

$$H_{p,q,s}(\alpha_1) := H_p(\alpha_1, \dots, \alpha_q; \beta_1, \dots, \beta_s). \tag{1.14}$$

Thus, after some calculations, we have

$$z(H_{p,q,s}(\alpha_1)f(z))' = \alpha_1 H_{p,q,s}(\alpha_1 + 1)f(z) - (\alpha_1 + p)H_{p,q,s}(\alpha_1)f(z). \tag{1.15}$$

The operator $H_{p,q,s}(\alpha_1)$ is popularly known as the generalized Dziok-Srivastava operator. Many interesting subclasses of multivalent functions, associated with the operator $H_{p,q,s}(\alpha_1)$ and its various special cases, were investigated recently by (e.g.) Dziok and Srivastava [2–4], Liu [5], Liu and Srivastava [6, 7], Patel et al. [8], Wang et al. [9], and others.

Let P be the class of functions $h(z)$ with $h(0) = 1$, which are analytic and convex univalent in \mathbb{U} .

Definition 1.1. A function $f(z) \in \Sigma(p)$ is said to be in the class $T_{p,q,s}(\alpha_1, \lambda; h)$ if it satisfies the subordination condition

$$\frac{(\lambda - 1)}{p} z^{p+1} (H_{p,q,s}(\alpha_1)f(z))' + \frac{\lambda}{p(p+1)} z^{p+2} (H_{p,q,s}(\alpha_1)f(z))'' < h(z), \tag{1.16}$$

where λ is a complex number and $h(z) \in P$.

The main object of this paper is to present a systematic investigation of the class $T_{p,q,s}(\alpha_1, \lambda; h)$ defined above by means of the generalized Dziok-Srivastava operator $H_{p,q,s}(\alpha_1)$.

For our purpose, we shall need the following lemmas to derive our main results for the class $T_{p,q,s}(\alpha_1, \lambda; h)$.

Lemma 1.2 (see [10]). *Let $g(z)$ be analytic in \mathbb{U} and $h(z)$ be analytic and convex univalent in \mathbb{U} with $h(0) = g(0)$. If*

$$g(z) + \frac{1}{\mu} z g'(z) < h(z), \tag{1.17}$$

where $\operatorname{Re} \mu > 0$, then

$$g(z) \prec \tilde{h}(z) = \mu z^{-\mu} \int_v^z t^{\mu-1} h(t) dt \prec h(z) \quad (1.18)$$

and $\tilde{h}(z)$ is the best dominant of (1.17).

Lemma 1.3 (see [1]). Let $\alpha < 1$, $f(z) \in S^*(\alpha)$ and $g(z) \in R(\alpha)$. Then, for any analytic function $F(z)$ in \mathbb{U} ,

$$\frac{g * (fF)}{g * f}(\mathbb{U}) \subset \overline{\operatorname{co}}(F(\mathbb{U})), \quad (1.19)$$

where $\overline{\operatorname{co}}(F(\mathbb{U}))$ denotes the closed convex hull of $F(\mathbb{U})$.

2. Properties of the Class $T_{p,q,s}(\alpha_1, \lambda; h)$

Theorem 2.1. Let $\lambda_1 < \lambda_2 \leq 0$. Then $T_{p,q,s}(\alpha_1, \lambda_1; h) \subset T_{p,q,s}(\alpha_1, \lambda_2; h)$.

Proof. Let $\lambda_1 < \lambda_2 \leq 0$ and suppose that

$$g(z) = -\frac{z^{p+1}(H_{p,q,s}(\alpha_1)f(z))'}{p} \quad (2.1)$$

for $f(z) \in T_{p,q,s}(\alpha_1, \lambda_1; h)$. Then the function $g(z)$ is analytic in \mathbb{U} with $g(0) = 1$. Differentiating both sides of (2.1) with respect to z and using (1.16), we have

$$\frac{(\lambda_1 - 1)}{p} z^{p+1}(H_{p,q,s}(\alpha_1)f(z))' + \frac{\lambda_1}{p(p+1)} z^{p+2}(H_{p,q,s}(\alpha_1)f(z))'' = g(z) - \frac{\lambda_1}{p+1} z g'(z) \prec h(z). \quad (2.2)$$

Hence an application of Lemma 1.2 yields

$$g(z) \prec h(z). \quad (2.3)$$

Noting that $0 < \lambda_2/\lambda_1 < 1$ and that $h(z)$ is convex univalent in \mathbb{U} , it follows from (2.1) to (2.3) that

$$\begin{aligned} & \frac{(\lambda_2 - 1)}{p} z^{p+1}(H_{p,q,s}(\alpha_1)f(z))' + \frac{\lambda_2}{p(p+1)} z^{p+2}(H_{p,q,s}(\alpha_1)f(z))'' \\ &= \frac{\lambda_2}{\lambda_1} \left(\frac{(\lambda_1 - 1)}{p} z^{p+1}(H_{p,q,s}(\alpha_1)f(z))' + \frac{\lambda_1}{p(p+1)} z^{p+2}(H_{p,q,s}(\alpha_1)f(z))'' \right) \\ &+ \left(1 - \frac{\lambda_2}{\lambda_1} \right) g(z) \prec h(z). \end{aligned} \quad (2.4)$$

Thus $f(z) \in T_{p,q,s}(\alpha_1, \lambda_2; h)$ and the proof of Theorem 2.1 is completed. \square

Theorem 2.2. Let $0 < b_1 < b_2$. Then $T_{p,q,s}(b_2, \lambda; h) \subset T_{p,q,s}(b_1, \lambda; h)$.

Proof. Define a function $g(z)$ by

$$g(z) = z + \sum_{n=1}^{\infty} \frac{(b_1)_n}{(b_2)_n} z^{n+1} \quad (z \in \mathbb{U}; 0 < b_1 < b_2). \quad (2.5)$$

Then

$$z^{p+1} h_p(b_1, \alpha_2, \dots, \alpha_s, 1; b_2, \alpha_2, \dots, \alpha_s; z) = g(z) \in A, \quad (2.6)$$

where

$$h_p(b_1, \alpha_2, \dots, \alpha_s, 1; b_2, \alpha_2, \dots, \alpha_s; z) \quad (2.7)$$

is defined as in (1.11), and

$$\frac{z}{(1-z)^{b_2}} * g(z) = \frac{z}{(1-z)^{b_1}}. \quad (2.8)$$

By (2.8), we see that

$$\frac{z}{(1-z)^{b_2}} * g(z) \in S^* \left(1 - \frac{b_1}{2}\right) \subset S^* \left(1 - \frac{b_2}{2}\right) \quad (0 < b_1 < b_2), \quad (2.9)$$

which implies that

$$g(z) \in R \left(1 - \frac{b_2}{2}\right). \quad (2.10)$$

Let $f(z) \in T_{p,q,s}(b_2, \lambda; h)$. It is easy to verify that

$$z^{p+1} (H_{p,q,s}(b_1) f(z))' = (z^p h_p(b_1, \alpha_2, \dots, \alpha_s, 1; b_2, \alpha_2, \dots, \alpha_s; z)) * (z^{p+1} (H_{p,q,s}(b_2) f(z))') \quad (2.11)$$

$$z^{p+2} (H_{p,q,s}(b_1) f(z))'' = (z^p h_p(b_1, \alpha_2, \dots, \alpha_s, 1; b_2, \alpha_2, \dots, \alpha_s; z)) * (z^{p+2} (H_{p,q,s}(b_2) f(z))''). \quad (2.12)$$

From (2.11), (2.12), and (2.6), we deduce that

$$\begin{aligned} & \frac{(\lambda-1)}{p} z^{p+1} (H_{p,q,s}(b_1) f(z))' + \frac{\lambda}{p(p+1)} z^{p+2} (H_{p,q,s}(b_1) f(z))'' \\ &= \frac{g(z)}{z} * w(z) = \frac{g(z) * (zw(z))}{g(z) * z}, \end{aligned} \quad (2.13)$$

where

$$w(z) := \frac{(\lambda - 1)}{p} z^{p+1} (H_{p,q,s}(b_2)f(z))' + \frac{\lambda}{p(p+1)} z^{p+2} (H_{p,q,s}(b_2)f(z))'' < h(z). \quad (2.14)$$

Since the function z belongs to the function class $S^*(1 - b_2/2)$ and $h(z)$ is convex univalent in \mathbb{U} , it follows from (2.12), (2.13), (2.14), and Lemma 1.3 that

$$\frac{(\lambda - 1)}{p} z^{p+1} (H_{p,q,s}(b_1)f(z))' + \frac{\lambda}{p(p+1)} z^{p+2} (H_{p,q,s}(b_1)f(z))'' < h(z). \quad (2.15)$$

Thus $f(z) \in T_{p,q,s}(b_1, \lambda; h)$ and the proof of Theorem 2.2 is completed. \square

Theorem 2.3. Let $f(z) \in T_{p,q,s}(\alpha_1, \lambda; h)$, $g(z) \in \Sigma(p)$ and

$$\operatorname{Re}\{z^p g(z)\} > \frac{1}{2} \quad (z \in \mathbb{U}). \quad (2.16)$$

Then

$$(f * g)(z) \in T_{p,q,s}(\alpha_1, \lambda; h). \quad (2.17)$$

Proof. For $f(z) \in T_{p,q,s}(\alpha_1, \lambda; h)$ and $g(z) \in \Sigma(p)$, we have

$$\begin{aligned} & \frac{(\lambda - 1)}{p} z^{p+1} (H_{p,q,s}(\alpha_1)(f * g)(z))' + \frac{\lambda}{p(p+1)} z^{p+2} (H_{p,q,s}(\alpha_1)(f * g)(z))'' \\ &= \frac{(\lambda - 1)}{p} (z^p g(z)) * (z^{p+1} (H_{p,q,s}(\alpha_1)f(z))') + \frac{\lambda}{p(p+1)} (z^p g(z)) * (z^{p+2} (H_{p,q,s}(\alpha_1)f(z))'') \\ &= (z^p g(z)) * \psi(z), \end{aligned} \quad (2.18)$$

where

$$\psi(z) = \frac{(\lambda - 1)}{p} z^{p+1} (H_{p,q,s}(\alpha_1)f(z))' + \frac{\lambda}{p(p+1)} z^{p+2} (H_{p,q,s}(\alpha_1)f(z))''. \quad (2.19)$$

In view of (2.16), the function $z^p g(z)$ has the Herglotz representation

$$z^p g(z) = \int_{|x|=1} \frac{d\mu(x)}{1 - xz} \quad (z \in \mathbb{U}), \quad (2.20)$$

where $\mu(x)$ is a probability measure defined on the unit circle $|x| = 1$ and

$$\int_{|x|=1} d\mu(x) = 1. \quad (2.21)$$

Since $h(z)$ is convex univalent in \mathbb{U} , it follows from (2.18) to (2.20) that

$$\begin{aligned} & \frac{(\lambda - 1)}{p} z^{p+1} (H_{p,q,s}(\alpha_1)(f * g)(z))' + \frac{\lambda}{p(p+1)} z^{p+2} (H_{p,q,s}(\alpha_1)(f * g)(z))'' \\ & = \int_{|x|=1} \psi(xz) d\mu(x) < h(z). \end{aligned} \tag{2.22}$$

This shows that $(f * g)(z) \in T_{p,q,s}(\alpha_1, \lambda; h)$ and the theorem is proved. \square

Theorem 2.4. Let $f(z) \in T_{p,q,s}(\alpha_1, \lambda; h)$, $g(z) \in \Sigma(p)$ and

$$z^{p+1}g(z) \in R(\alpha) \quad (\alpha < 1). \tag{2.23}$$

Then

$$(f * g)(z) \in T_{p,q,s}(\alpha_1, \lambda; h). \tag{2.24}$$

Proof. For $f(z) \in T_{p,q,s}(\alpha_1, \lambda; h)$ and $g(z) \in \Sigma(p)$, from (2.18) we have

$$\begin{aligned} & \frac{(\lambda - 1)}{p} z^{p+1} (H_{p,q,s}(f * g)(z))' + \frac{\lambda}{p(p+1)} z^{p+2} (H_{p,q,s}(f * g)(z))'' \\ & = \frac{(z^{p+1}g(z)) * (z\psi(z))}{(z^{p+1}g(z)) * z} \quad (z \in \mathbb{U}), \end{aligned} \tag{2.25}$$

where $\psi(z)$ is defined as in (2.19).

Since $h(z)$ is convex univalent in \mathbb{U} ,

$$\psi(z) < h(z), \quad z^{p+1}g(z) \in R(\alpha), \quad z \in S^*(\alpha) \quad (\alpha < 1), \tag{2.26}$$

it follows from (2.25) and Lemma 1.3 the desired result. \square

Theorem 2.5. Let $\lambda < 0$, $\beta > 0$ and $f(z) \in T_{p,q,s}(\lambda; \beta h + 1 - \beta)$. If $\beta \leq \beta_0$, where

$$\beta_0 = \frac{1}{2} \left(1 + \frac{p+1}{\lambda} \int_0^1 \frac{u^{-(p+1)/\lambda-1}}{1+u} du \right)^{-1}, \tag{2.27}$$

then $f(z) \in T_{p,q,s}(0; h)$. The bound β_0 is sharp when $h(z) = 1/(1-z)$.

Proof. Let us define

$$g(z) = -\frac{z^{p+1}(H_{p,q,s}(\alpha_1)f(z))'}{p} \tag{2.28}$$

for $f(z) \in T_{p,q,s}(\lambda; \beta h + 1 - \beta)$ with $\lambda < 0$ and $\beta > 0$. Then we have

$$g(z) - \frac{\lambda}{p+1} z g'(z) = \frac{(\lambda-1)}{p} z^{p+1} (H_{p,q,s}(\alpha_1) f(z))' + \frac{\lambda}{p(p+1)} z^{p+2} (H_{p,q,s}(\alpha_1) f(z))'' < \beta h(z) + 1 - \beta. \quad (2.29)$$

Hence an application of Lemma 1.2 yields

$$g(z) < -\frac{\beta(p+1)}{\lambda} z^{(p+1)/\lambda} \int_0^z t^{-((p+1)/\lambda)-1} h(t) dt + 1 - \beta = (h * \psi)(z), \quad (2.30)$$

where

$$\psi(z) = -\frac{\beta(p+1)}{\lambda} z^{(p+1)/\lambda} \int_0^z \frac{t^{-((p+1)/\lambda)-1}}{1-t} dt + 1 - \beta. \quad (2.31)$$

If $0 < \beta \leq \beta_0$, where $\beta_0 (> 1)$ is given by (2.27), then it follows from (2.31) that

$$\begin{aligned} \operatorname{Re} \psi(z) &= -\frac{\beta(p+1)}{\lambda} \int_0^1 u^{-((p+1)/\lambda)-1} \operatorname{Re} \left(\frac{1}{1-uz} \right) du + 1 - \beta \\ &> -\frac{\beta(p+1)}{\lambda} \int_0^1 \frac{u^{-((p+1)/\lambda)-1}}{1+u} du + 1 - \beta \\ &\geq \frac{1}{2} \quad (z \in \mathbb{U}; \lambda < 0). \end{aligned} \quad (2.32)$$

Now, by using the Herglotz representation for $\psi(z)$, from (2.28) and (2.30), we arrive at

$$-\frac{z^{p+1} (H_{p,q,s}(\alpha_1) f(z))'}{p} < (h * \psi)(z) < h(z) \quad (2.33)$$

because $h(z)$ is convex univalent in \mathbb{U} . This shows that $f(z) \in T_{p,q,s}(0; h)$.

For $h(z) = 1/(1-z)$ and $f(z) \in \Sigma(p)$ defined by

$$-\frac{z^{p+1} (H_{p,q,s}(\alpha_1) f(z))'}{p} = -\frac{\beta(p+1)}{\lambda} z^{(p+1)/\lambda} \int_0^z \frac{t^{-((p+1)/\lambda)-1}}{1-t} dt + 1 - \beta, \quad (2.34)$$

it is easy to verify that

$$\frac{(\lambda-1)}{p} z^{p+1} (H_{p,q,s}(\alpha_1) f(z))' + \frac{\lambda}{p(p+1)} z^{p+2} (H_{p,q,s}(\alpha_1) f(z))'' = \beta h(z) + 1 - \beta. \quad (2.35)$$

Thus $f(z) \in T_{p,q,s}(\lambda; \beta h + 1 - \beta)$. Also, for $\beta > \beta_0$, we have

$$\operatorname{Re} \left\{ -\frac{z^{p+1} (H_{p,q,s}(\alpha_1) f(z))'}{p} \right\} \rightarrow -\frac{\beta(p+1)}{\lambda} \int_0^1 \frac{u^{-((p+1)/\lambda)-1}}{1+u} du + 1 - \beta < \frac{1}{2} \quad (z \rightarrow -1), \quad (2.36)$$

which implies that $f(z) \notin T_{p,q,s}(0; h)$. Hence the bound β_0 cannot be increased when $h(z) = 1/(1-z)$. \square

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