Hindawi Publishing Corporation
Abstract and Applied Analysis
Volume 2011, Article ID 126205, 11 pages
doi:10.1155/2011/126205

Research Article

Some Fixed Point Theorems in Ordered G-Metric Spaces and Applications

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Received 9 January 2011; Revised 22 March 2011; Accepted 19 April 2011

Academic Editor: D. Anderson

We study a number of fixed point results for the two weakly increasing mappings f and g with respect to partial ordering relation $\preceq$ in generalized metric spaces. An application to integral equation is given.

1. Introduction

The existence of fixed points in partially ordered sets has been at the center of active research. In fact, the existence of fixed point in partially ordered sets has been investigated in [1]. Moreover, Ran and Reurings [1] applied their results to matrix equations. Some generalizations of the results of [1] are given in [2–6]. In [6], O'Regan and Petruşel gave some existence results for the Fredholm and Volterra type.

The notion of G-metric space was introduced by Mustafa and Sims [7] as a generalization of the notion of metric spaces. Mustafa et al. studied many fixed point results in G-metric space [8–10] (also see [11–15]). In fact the study of common fixed points of mappings satisfying certain contractive conditions has been at the center of strong research activity. The following definition is introduced by Mustafa and Sims [7].

Definition 1.1 (see [7]). Let X be a nonempty set and let $G : X \times X \times X \to \mathbf{R}^+$ be a function satisfying the following properties:

(G_1) $G(x, y, z) = 0$ if $x = y = z$,

(G_2) $0 < G(x, x, y)$, for all $x, y \in X$ with $x \neq y$,

(G_3) $G(x, x, y) \leq G(x, y, z)$, for all $x, y, z \in X$ with $z \neq y$,

(G_4) $G(x,y,z) = G(x,z,y) = G(y,z,x) = \cdots$, symmetry in all three variables,

(G_5) $G(x,y,z) \le G(x,a,a) + G(a,y,z)$, for all $x,y,z,a \in X$.

Then the function G is called a generalized metric, or, more specifically, a G-metric on X, and the pair (X,G) is called a G-metric space.

Definition 1.2 (see [7]). Let (X,G) be a G-metric space, and let $\{x_n\}$ be a sequence of points of X, a point $x \in X$ is said to be the limit of the sequence $\{x_n\}$, if $\lim_{n,m\to+\infty} G(x,x_n,x_m) = 0$, and we say that the sequence $\{x_n\}$ is G-convergent to x or $\{x_n\}$ G-converges to x.

Thus, $x_n \to x$ in a G-metric space (X,G) if for any $\varepsilon > 0$, there exists $k \in \mathbf{N}$ such that $G(x,x_n,x_m) < \varepsilon$ for all $m,n \ge k$.

Proposition 1.3 (see [7]). *Let (X,G) be a G-metric space. Then the following are equivalent:*

(1) *$\{x_n\}$ is G-convergent to x;*

(2) *$G(x_n,x_n,x) \to 0$ as $n \to +\infty$;*

(3) *$G(x_n,x,x) \to 0$ as $n \to +\infty$;*

(4) *$G(x_n,x_m,x) \to 0$ as $n,m \to +\infty$.*

Definition 1.4 (see [7]). Let (X,G) be a G-metric space, a sequence $\{x_n\}$ is called G-Cauchy if for every $\varepsilon > 0$, there is $k \in \mathbf{N}$ such that $G(x_n,x_m,x_l) < \varepsilon$, for all $n,m,l \ge k$; that is $G(x_n,x_m,x_l) \to 0$ as $n,m,l \to +\infty$.

Proposition 1.5 (see [7]). *Let (X,G) be a G-metric space. Then the following are equivalent:*

(1) *the sequence $\{x_n\}$ is G-Cauchy;*

(2) *for every $\epsilon > 0$, there is $k \in \mathbf{N}$ such that $G(x_n,x_m,x_m) < \epsilon$, for all $n,m \ge k$.*

Definition 1.6 (see [7]). Let (X,G) and (X',G') be G-metric spaces, and let $f : (X,G) \to (X',G')$ be a function. Then f is said to be G-continuous at a point $a \in X$ if and only if for every $\varepsilon > 0$, there is $\delta > 0$ such that $x,y \in X$ and $G(a,x,y) < \delta$ implies $G'(f(a),f(x),f(y)) < \varepsilon$. A function f is G-continuous at X if and only if it is G-continuous at all $a \in X$.

Proposition 1.7 (see [7]). *Let (X,G) be a G-metric space. Then the function $G(x,y,z)$ is jointly continuous in all three of its variables.*

Every G-metric on X will define a metric d_G on X by

$$d_G(x,y) = G(x,y,y) + G(y,x,x), \quad \forall x,y \in X. \tag{1.1}$$

For a symmetric G-metric space,

$$d_G(x,y) = 2G(x,y,y), \quad \forall x,y \in X. \tag{1.2}$$

However, if G is not symmetric, then the following inequality holds:

$$\frac{3}{2}G(x,y,y) \le d_G(x,y) \le 3G(x,y,y), \quad \forall x,y \in X. \tag{1.3}$$

The following are examples of G-metric spaces.

Example 1.8 (see [7]). Let $(\mathbf{R}, d)$ be the usual metric space. Define G_s by

$$G_s(x, y, z) = d(x, y) + d(y, z) + d(x, z), \tag{1.4}$$

for all $x, y, z \in \mathbf{R}$. Then it is clear that $(\mathbf{R}, G_s)$ is a G-metric space.

Example 1.9 (see [7]). Let $X = \{a, b\}$. Define G on $X \times X \times X$ by

$$\begin{gathered} G(a, a, a) = G(b, b, b) = 0, \\ G(a, a, b) = 1, \qquad G(a, b, b) = 2 \end{gathered} \tag{1.5}$$

and extend G to $X \times X \times X$ by using the symmetry in the variables. Then it is clear that (X, G) is a G-metric space.

Definition 1.10 (see [7]). A G-metric space (X, G) is called G-complete if every G-Cauchy sequence in (X, G) is G-convergent in (X, G).

The notion of weakly increasing mappings was introduced in by Altun and Simsek [16].

Definition 1.11 (see [16]). Let $(X, \preceq)$ be a partially ordered set. Two mappings $F, G : X \to X$ are said to be weakly increasing if $Fx \preceq GFx$ and $Gx \preceq FGx$, for all $x \in X$.

Two weakly increasing mappings need not be nondecreasing.

Example 1.12 (see [16]). Let $X = \mathbf{R}$, endowed with the usual ordering. Let $F, G : X \to X$ defined by

$$\begin{gathered} Fx = \begin{cases} x, & 0 \leq x \leq 1, \\ 0, & 1 < x < +\infty, \end{cases} \\ gx = \begin{cases} \sqrt{x}, & 0 \leq x \leq 1, \\ 0, & 1 < x < +\infty. \end{cases} \end{gathered} \tag{1.6}$$

Then F and G are weakly increasing mappings. Note that F and G are not nondecreasing.

The aim of this paper is to study a number of fixed point results for two weakly increasing mappings f and g with respect to partial ordering relation $(\preceq)$ in a generalized metric space.

2. Main Results

Theorem 2.1. *Let $(X, \preceq)$ be a partially ordered set and suppose that there exists G-metric in X such that (X, G) is G-complete. Let $f, g : X \to X$ be two weakly increasing mappings with respect to $\preceq$. Suppose there exist nonnegative real numbers a, b, and c with $a + 2b + 2c < 1$ such that*

$$\begin{aligned} G(fx, gy, gy) \leq{}& aG(x, y, y) + b[G(x, fx, fx) + G(y, gy, gy)] \\ &+ c[G(x, gy, gy) + G(y, fx, fx)], \end{aligned} \tag{2.1}$$

$$\begin{aligned} G(gx, fy, fy) \leq{}& aG(x, y, y) + b[G(x, gx, gx) + G(y, fy, fy)] \\ &+ c[G(x, fy, fy) + G(y, gx, gx)], \end{aligned} \tag{2.2}$$

for all comparative $x, y \in X$. If f or g is continuous, then f and g have a common fixed point $u \in X$.

Proof. By inequality (2.2), we have

$$\begin{aligned} G(gy, fx, fx) \leq{}& aG(y, x, x) + b[G(y, gy, gy) + G(x, fx, fx)] \\ &+ c[G(y, fx, fx) + G(x, gy, gy)]. \end{aligned} \tag{2.3}$$

If X is a symmetric G-metric space, then by adding inequalities (2.1) and (2.3), we obtain

$$\begin{aligned} &G(fx, gy, gy) + G(gy, fx, fx) \\ &\quad\leq a[G(x, y, y) + G(y, x, x)] + 2b[G(x, fx, fx) + G(y, gy, gy)] \\ &\qquad+ 2c[G(x, gy, gy) + G(y, fx, fx)], \end{aligned} \tag{2.4}$$

which further implies that

$$d_G(fx, fy) \leq ad_G(x, y) + b[d_G(x, fx) + d_G(y, gy)] + c[d_G(x, gy) + d_G(y, fx)], \tag{2.5}$$

for all $x, y \in X$ with $0 \leq a + 2b + 2c < 1$ and the fixed point of f and g follows from [2].

Now if X is not a symmetric G-metric space. Then by the definition of metric (X, d_G) and inequalities (2.1) and (2.3), we obtain

$$\begin{aligned} d_G(fx, gy) &= G(fx, gy, gy) + G(gy, fx, fx) \\ &\leq a[G(x, y, y) + G(x, x, y)] + 2b[G(x, fx, fx) + G(y, gy, gy)] \\ &\quad+ 2c[G(x, gy, gy) + G(y, fx, fx)] \end{aligned}$$

$$\begin{aligned}
&\leq a d_G(x,y) + 2b\left[\frac{2}{3}d_G(x,fx) + \frac{2}{3}d_G(y,gy)\right] \\
&\quad + 2c\left[\frac{2}{3}d_G(x,gy) + \frac{2}{3}d_G(y,fx)\right] \\
&= a d_G(x,y) + \frac{4}{3}b\left[d_G(x,fx) + d_G(y,gy)\right] + \frac{4}{3}c\left[d_G(x,gy) + d_G(y,fx)\right],
\end{aligned} \tag{2.6}$$

for all $x \in X$. Here, the contractivity factor $a + (8/3)b + (8/3)c$ may not be less than 1.

Therefore metric gives no information. In this case, for given $x_0 \in X$, choose $x_1 \in X$ such that $x_1 = fx_0$. Again choose $x_2 \in X$ such that $gx_1 = x_2$. Also, we choose $x_3 \in X$ such that $x_3 = fx_2$. Continuing as above process, we can construct a sequence $\{x_n\}$ in X such that $x_{2n+1} = fx_{2n}$, $n \in \mathbf{N} \cup \{\mathbf{0}\}$ and $x_{2n+2} = gx_{2n+1}$, $n \in \mathbf{N} \cup \{\mathbf{0}\}$. Since f and g are weakly increasing with respect to $\preceq$, we have

$$x_1 = fx_0 \preceq g(fx_0) = gx_1 = x_2 \preceq f(gx_1) = fx_2 = x_3 \preceq g(fx_2) = gx_3 = x_4 \preceq \cdots. \tag{2.7}$$

Thus from (2.1), we have

$$\begin{aligned}
G(x_{2n+1}, x_{2n+2}, x_{2n+2}) &= G(fx_{2n}, gx_{2n+1}, gx_{2n+1}) \\
&\leq aG(x_{2n}, x_{2n+1}, x_{2n+1}) \\
&\quad + b\left[G(x_{2n}, fx_{2n}, fx_{2n}) + G(x_{2n+1}, gx_{2n+1}, gx_{2n+1})\right] \\
&\quad + c\left[G(x_{2n}, gx_{2n+1}, gx_{2n+1}) + G(x_{2n+1}, fx_{2n}, fx_{2n})\right] \\
&= aG(x_{2n}, x_{2n+1}, x_{2n+1}) \\
&\quad + b\left[G(x_{2n}, x_{2n+1}, x_{2n+1}) + G(x_{2n+1}, x_{2n+2}, x_{2n+2})\right] \\
&\quad + c\left[G(x_{2n}, x_{2n+2}, gx_{2n+2}) + G(x_{2n+1}, x_{2n+1}, x_{2n+1})\right] \\
&= (a+b)G(x_{2n}, x_{2n+1}, x_{2n+1}) + bG(x_{2n+1}, x_{2n+2}, x_{2n+2}) \\
&\quad + cG(x_{2n}, x_{2n+2}, x_{2n+2}).
\end{aligned} \tag{2.8}$$

By (G_5), we have

$$G(x_{2n+1}, x_{2n+2}, x_{2n+2}) \leq \frac{a+b+c}{1-b-c}G(x_{2n}, x_{2n+1}, x_{2n+1}). \tag{2.9}$$

Also, we have

$$\begin{aligned}
G(x_{2n}, x_{2n+1}, x_{2n+1}) &= G(gx_{2n-1}, fx_{2n}, fx_{2n}) \\
&\leq aG(x_{2n-1}, x_{2n}, x_{2n}) \\
&\quad + b\left[G(x_{2n-1}, gx_{2n-1}, gx_{2n-1}) + G(x_{2n}, fx_{2n}, fx_{2n})\right] \\
&\quad + c\left[G(x_{2n-1}, fx_{2n}, fx_{2n}) + G(x_{2n}, gx_{2n-1}, gx_{2n-1})\right]
\end{aligned}$$

$$\begin{aligned}
&= aG(x_{2n-1}, x_{2n}, x_{2n}) \\
&\quad + b[G(x_{2n-1}, x_{2n}, x_{2n}) + G(x_{2n}, x_{2n+1}, x_{2n+1})] \\
&\quad + c\left[G(x_{2n-1}, x_{2n+1}, gx_{2n+1}) + G(x_{2n}, x_{2n}, x_{2n})\right] \\
&= (a+b)G(x_{2n-1}, x_{2n}, x_{2n}) + bG(x_{2n}, x_{2n+1}, x_{2n+1}) \\
&\quad + cG(x_{2n-1}, x_{2n+1}, x_{2n+1}).
\end{aligned} \tag{2.10}$$

By (G_5), we get

$$G(x_{2n}, x_{2n+1}, x_{2n+1}) \le \frac{a+b+c}{1-b-c} G(x_{2n-1}, x_{2n}, x_{2n}). \tag{2.11}$$

Let

$$k = \frac{a+b+c}{1-b-c}. \tag{2.12}$$

Then by (2.9) and (2.11), we have

$$G(x_n, x_{n+1}, x_{n+1}) \le kG(x_{n-1}, x_n, x_n), \quad \forall n \in \mathbf{N}. \tag{2.13}$$

Thus, if $x_0 = x_1$, we get $G(x_n, x_{n+1}, x_{n+1}) = 0$ for each $n \in \mathbf{N}$. Hence $x_n = x_0$ for each $n \in \mathbf{N}$. Therefore $\{x_n\}$ is G-Cauchy. So we may assume that $x_0 \neq x_1$. Let $n, m \in \mathbf{N}$ with $m > n$. By axiom (G_5) of the definition of G-metric space, we have

$$G(x_n, x_m, x_m) \le G(x_n, x_{n+1}, x_{n+1}) + G(x_{n+1}, x_{n+2}, x_{n+2}) + \cdots + G(x_{m-1}, x_m, x_m). \tag{2.14}$$

By (2.13), we get

$$\begin{aligned}
G(x_n, x_m, x_m) &\le k^n G(x_0, x_1, x_1) + k^{n+1} G(x_0, x_1, x_1) + \cdots + k^{m-1} G(x_0, x_1, x_1) \\
&\le \frac{k^n}{1-k} G(x_0, x_1, x_1).
\end{aligned} \tag{2.15}$$

On taking limit $m, n \to \infty$, we have

$$\lim_{m,n \to \infty} G(x_n, x_m, x_m) = 0. \tag{2.16}$$

So we conclude that (x_n) is a Cauchy sequence in X. Since X is G-complete, then it yields that (x_n) and hence any subsequence of (x_n) converges to some $u \in X$. So that, the subsequences $(x_{2n+1}) = (fx_{2n})$ and $(x_{2n+2}) = (gx_{2n+1})$ converge to u. First suppose that f is G-continuous. Since (x_{2n}) converges to u, we get (fx_{2n}) converges fu. By the uniqueness of limit we get $fu = u$. Claim: $gu = u$.

Since $u \preceq u$, by inequality (2.1), we have

$$\begin{aligned} G(u, gu, gu) &= G(fu, gu, gu) \\ &\leq aG(u, u, u) + b[G(u, fu, fu) + G(u, gu, gu)] \\ &\quad + c[G(u, gu, gu) + G(u, fu, fu)] \\ &\leq (b + c)G(u, gu, gu). \end{aligned} \tag{2.17}$$

Since $b + c < 1$, we get $G(u, gu, gu) = 0$. Hence $gu = u$. If g is G-continuous, by similar argument as above we show that g and f have a common fixed point. □

Theorem 2.2. *Let $(X, \preceq)$ be a partially ordered set and suppose that there exists G-metric in X such that (X, G) is G-complete. Let $f, g : X \to X$ be two weakly increasing mappings with respect to $\preceq$. Suppose there exist nonnegative real numbers a, b, and c with $a + 2b + 2c < 1$ such that*

$$\begin{aligned} G(fx, gy, gy) &\leq aG(x, y, y) + b[G(x, fx, fx) + G(y, gy, gy)] \\ &\quad + c[G(x, gy, gy) + G(y, fx, fx)], \\ G(gx, fy, fy) &\leq aG(x, y, y) + b[G(x, gx, gx) + G(y, fy, fy)] \\ &\quad + c[G(x, fy, fy) + G(y, gx, gx)], \end{aligned} \tag{2.18}$$

for all comparative $x, y \in X$. Assume that X has the following property:

(P) If (x_n) is an increasing sequence converges to x in X, then $x_n \preceq x$ for all $n \in \mathbf{N}$.

Then f and g have a common fixed point $u \in X$.

Proof. As in the proof of Theorem 3.1, we construct an increasing sequence (x_n) in X such that $x_{2n+1} = fx_{2n}$ and $x_{2n+2} = gx_{2n+1}$. Also, we can show (x_n) is G-Cauchy. Since X is G-complete, there is $u \in X$ such that (x_n) is converges to $u \in X$. Thus (x_{2n}), (x_{2n+1}), (fx_{2n}) and (gx_{2n+1}) converge to u. Since X satisfies property (P), we get that $x_n \preceq u$, for all $n \in \mathbf{N}$. Thus x_{2n} and u are comparative. Hence by inequality (2.1), we have

$$\begin{aligned} G(fx_{2n}, gu, gu) &\leq aG(x_{2n}, u, u) + b[G(x_{2n}, fx_{2n}, fx_{2n}) + G(u, gu, gu)] \\ &\quad + c[G(x_{2n}, gu, gu) + G(u, fx_{2n}, fx_{2n})]. \end{aligned} \tag{2.19}$$

On letting $n \to +\infty$, we get

$$G(u, gu, gu) \leq (b + c)G(u, gu, gu). \tag{2.20}$$

Since $b + c < 1$, we get $G(u, gu, gu) = 0$. Hence $gu = u$. By similar argument, we may show that $u = fu$. □

Corollary 2.3. *Let $(X, \preceq)$ be a partially ordered set, and suppose that (X, G) is a G-complete metric space. Let $f : X \to X$ be a continuous mapping such that $fx \preceq f(fx)$, for all $x \in X$. Suppose there exist nonnegative real numbers a, b and c with $a + 2b + 2c < 1$ such that*

$$\begin{aligned} G(fx, fy, fy) \leq\; & aG(x, y, y) + b[G(x, fx, fx) + G(y, fy, fy)] \\ & + c[G(x, fy, fy) + G(y, fx, fx)], \end{aligned} \tag{2.21}$$

for all comparative $x, y \in X$. Then f has a fixed point $u \in X$.

Proof. It follows from Theorem 2.1 by taking $g = f$. □

Corollary 2.4. *Let $(X, \preceq)$ be a partially ordered set and suppose that there exists G-metric in X such that (X, G) is G-complete. Let $f : X \to X$ be a mapping such that $fx \preceq f(fx)$ for all $x \in X$. Suppose there exist nonnegative real numbers a, b and c with $a + 2b + 2c < 1$ such that*

$$\begin{aligned} G(fx, fy, fy) \leq\; & aG(x, y, y) + b[G(x, fx, fx) + G(y, fy, fy)] \\ & + c[G(x, fy, fy) + G(y, fx, fx)] \end{aligned} \tag{2.22}$$

for all comparative $x, y \in X$. Assume that X has the following property:

(P) If (x_n) is an increasing sequence converges to x in X, then $x_n \preceq x$ for all $n \in \mathbf{N}$.

Then f has fixed point $u \in X$.

Proof. It follows from Theorem 2.2 by taking $g = f$. □

3. Application

Consider the integral equation:

$$u(t) = \int_0^T K(t, s, u(s))ds + g(t), \quad t \in [0, T], \tag{3.1}$$

where $T > 0$. The aim of this section is to give an existence theorem for a solution of the above integral equation using Corollary 2.4 This section is related to those [16–19].

Let $X = C([0, T])$ be the set of all continuous functions defined on $[0, T]$. Define

$$G : X \times X \times X \to \mathbf{R}^+ \tag{3.2}$$

by

$$G(x, y, z) = \sup_{t \in [0,T]} |x(t) - y(t)| + \sup_{t \in [0,T]} |x(t) - z(t)| + \sup_{t \in [0,T]} |y(t) - z(t)|. \tag{3.3}$$

Then (X, G) is a G-complete metric space. Define an ordered relation $\preceq$ on X by

$$x \preceq y \quad \text{iff } x(t) \leq y(t), \quad \forall t \in [0, T]. \tag{3.4}$$

Then $(X, \preceq)$ is a partially ordered set. The purpose of this section is to give an existence theorem for solution of integral equation on (3.1). This section is inspired in [17–19].

Theorem 3.1. *Suppose the following hypotheses hold:*

(1) $K : [0, T] \times [0, T] \times \mathbf{R} \to \mathbf{R}$ *and* $g : \mathbf{R} \to \mathbf{R}$ *are continuous,*

(2) *for each* $t, s \in [0, T]$, *one has*

$$K(t, s, u(t)) \leq K\left(t, s, \int_0^T K(s, \tau, u(\tau))d\tau + g(s)\right), \tag{3.5}$$

(3) *there exists a continuous function* $G : [0, T] \times [0, T] \to [0, +\infty]$ *such that*

$$|K(t, s, u) - K(t, s, v)| \leq G(t, s)|u - v|, \tag{3.6}$$

for each comparable $u, v \in \mathbf{R}$ *and each* $t, s \in [0, T]$,

(4) $\sup_{t\in[0,T]} \int_0^T G(t, s)ds \leq r$ *for some* $r < 1$.

Then the integral equation (3.1) *has a solution* $u \in C([0, T])$.

Proof. Define $S : C([0, T]) \to C([0, T])$ by

$$Sx(t) = \int_0^T K(t, s, x(s))ds + g(t), \quad t \in [0, T]. \tag{3.7}$$

Now, we have

$$\begin{aligned} Sx(t) &= \int_0^T K(t, s, x(s))ds + g(t) \\ &\leq \int_0^T K\left(t, s, \int_0^T K(s, \tau, x(\tau))d\tau + g(s)\right)ds + g(t) \\ &= \int_0^T K(t, s, Sx(s))ds + g(t) \\ &= S(Sx(t)). \end{aligned} \tag{3.8}$$

Thus, we have $Sx \preceq S(Sx)$, for all $x \in C([0, T])$.

For $x, y \in C([0,T])$ with $x \leq y$, we have

$$\begin{aligned}
G(Sx,Sy,Sy) &= 2\sup_{t\in[0,T]}\left|Sx(t)-Sy(t)\right| \\
&= 2\sup_{t\in[0,T]}\left|\int_0^T K(t,s,x(s))-K(t,s,y(s))ds\right| \\
&\leq 2\sup_{t\in[0,T]}\int_0^T \left|K(t,s,x(s))-K(t,s,y(s))\right|ds \\
&\leq 2\sup_{t\in[0,T]}\int_0^T G(t,s)\left|x(s)-y(s)\right|ds \\
&\leq 2\sup_{t\in[0,T]}\left|x(t)-y(t)\right|\sup_{t\in[0,T]}\int_0^T G(t,s)ds \\
&= G(x,y,y)\sup_{t\in[0,T]}\int_0^T G(t,s)ds.
\end{aligned} \tag{3.9}$$

By using hypotheses (4), there is $r \in [0,1)$ such that

$$\sup_{t\in[0,T]}\int_0^T G(t,s)ds < r. \tag{3.10}$$

Thus, we have $G(Sx,Sy,Sy) \leq rG(x,y,y)$. Thus all the required hypotheses of Corollary 2.4 are satisfied. Thus there exist a solution $u \in C([0,T])$ of the integral equation (3.1). $\square$

Acknowledgments

The author would like to thank the editor of the paper and the referees for their precise remarks to correct and improve the paper. Also, the author would like to thank the hashemite university for the financial assistant of this paper.

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