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Editorial

Recent Advances in Oscillation Theory

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The Theory of Oscillations is an important branch of the Applied Theory of Differential Equations related with the study of oscillatory phenomena in technology and natural and social sciences. Fundamental problems of the classical Theory of Oscillations consist in proving the existence or nonexistence of oscillatory (periodic, almost-periodic, etc.) solutions to a given equation or system, and, in simpler cases, funding such solutions. Furthermore, behavior of other solutions in relation to a given oscillatory (nonoscillatory) solution is often investigated.

Every year, hundreds of papers on theoretical aspects of oscillation theory for various classes of equations including ordinary and functional differential equations as well as difference, dynamic, impulsive, and partial differential equations are published. In fact, search for key words "oscillation" or "oscillatory" in the Mathematical Reviews database of the American Mathematical Society returns nearly fourteen thousand results! Oscillation of solutions can simply be one of the intrinsic features of a given system, as for the mathematical pendulum or the van der Pol oscillator. On the other hand, it is well known that oscillations can also be induced or destroyed by introducing, for instance, nonlinearity, delay, or stochasticity in a nonoscillatory system.

Remarkably, this important branch of research is not purely theoretical and has very important applications. For example, recent studies suggest that many animal and plant populations oscillate in synchrony because of interactions such as predation and competition. Coupled oscillating biological populations can give rise to potentially important effects such as "synchronized chaos." Presence of oscillatory solutions characterizes voltage-controlled oscillator neuron models in neuroscience and engineering, and so forth. Therefore, through the study of oscillations, one gets deeper insights into the dynamics of solutions to equations modeling applied problems arising in engineering, technology, and natural sciences.

The papers in this special issue address new challenging problems related to nonlinear oscillations, describe novel techniques and approaches to classical problems in the Theory of Oscillations, and deal with oscillation of solutions in complex biological systems.

This collection of papers opens with the contribution by John A. D. Appleby regarding the behavior of solutions to a class of stochastic Volterra convolution integrodifferential equations. The principal goal is to determine how a linear state-dependent, instantaneous and equilibrium preserving stochastic perturbation affects the zero-crossing and positivity properties of solutions to this equation. It is interesting to note that the change in sign of solutions is similar to that observed for the corresponding deterministic equations: at the first zero, the sample path of the solution is differentiable and the derivative is negative, although the sample path of the solution of the stochastic Volterra convolution integrodifferential equation is not differentiable at any other point. Hence, "oscillation" is not a consequence of the lack of regularity in the sample path of the nondifferentiable Brownian motion, but it is rather induced by fluctuation properties of its increments; the presence of delay in the equation is important too.

Guihong Fan and Gail S. K. Wolkowicz study the system describing a food chain in the chemostat where predators feed on microorganisms which, in turn, consume a nonreproducing nutrient whose growth is limited at low concentrations. A delayed argument is introduced in the system to model the time elapsed between the capture of the prey and its conversion to viable biomass. The purpose of the paper is to demonstrate that the presence of delay in the system with the response functions of the Holling type I can induce nontrivial periodic solutions. This effect is impossible in the model without delay which always possesses a globally asymptotically stable equilibrium. The authors analyze the stability of equilibria and prove that the coexistence equilibrium can undergo Hopf bifurcations. Numerical simulations confirm bifurcation of a stable periodic solution from the coexistence equilibrium as the delay parameter increases from zero. As the delay parameter increases further, this periodic orbit can disappear through a secondary Hopf bifurcation.

José M. Ferreira and Sandra Pinelas investigate the oscillatory behavior of a difference equation representing the nonautonomous case of the so-called Lucas sequences. First, oscillatory nature of the equation is established. Then, the oscillatory properties of solutions are characterized in more detail in what regards the number of consecutive terms of the same sign and the amplitude of oscillation.

In a survey paper by Luljeta K. Kikina and Ioannis P. Stavroulakis, a number of interesting recent oscillation results for a second-order linear delay differential equation—its discrete analogue—a second-order difference equation, and the second-order functional equation are presented. The analysis of the oscillation criteria is accompanied with numerous comments and illustrated with a selection of examples.

The paper by Jing Shao and Fanwei Meng is concerned with the forced second-order neutral nonlinear differential equations with delayed argument and provides several oscillation theorems of the so-called "interval" type.

Jurang Yan and Weiping Yan study oscillatory properties of solutions to a delay neutral differential equation with positive and alternating coefficients. First, a comparison theorem is established. Then it is then used to derive a number of explicit oscillation criteria.

The paper by Norio Yoshida deals with an important class of half-linear elliptic equations with a forcing term. The main purpose is to establish a Picone-type inequality for these equations and use it for deriving new oscillation results. After multidimensional oscillation problems are reduced to one-dimensional oscillation problems for ordinary half-linear differential equations, several Leighton-Wintner- and Hille-Nehari-type oscillation results are derived.

In the contribution of Siyu Zhang and Fanwei Meng which concludes the issue, integral averaging technique and a generalized Riccati transformation are employed to obtain new oscillation criteria for a class of even-order neutral delay differential equations.

The eight papers published in this special issue represent only a few directions of research conducted in the Theory of Oscillations, reflecting both theoretical and applied sides. Nevertheless, they deal with several useful classes of ordinary differential equations, partial differential equations, as well as difference, stochastic, integrodifferential, functional, and functional differential equations, demonstrating thus the breadth of the research field and unveiling oscillatory nature of solutions to a wide variety of problems, quite often in a somewhat unexpected manner. During the last decades, the Oscillation Theory of ordinary, functional, neutral, partial, and impulsive differential equations as well as their discrete versions has inspired many scholars. We hope that this collection of papers will attract interest of researchers working in related areas and will stimulate further progress in this important branch of the Qualitative Theory of Differential Equations.

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