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A smooth pseudoconvex domain in $\mathbf{C}^2$ for which L^∞-estimates for $\bar\partial$ do not hold

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Let $\mathcal{D}$ be a smoothly bounded domain in $\mathbf{C}^n$. It is well known (see [HL] and [Ø]) that if $\mathcal{D}$ is strictly pseudoconvex then we can solve the $\bar\partial$-equation with estimates in L^p for any $1 \le p \le \infty$. It has also been known for some time that this is no longer true if $\mathcal{D}$ is merely pseudoconvex. Namely, Sibony [S2] found an example of such a domain in $\mathbf{C}^3$ where L^∞-estimates do not hold. The reader should also consult the paper [FS1] which contains a discussion of L^p-estimates in general and many counterexamples to this type of questions. However, all counterexamples known seem to treat the case $n \ge 3$ and L^p-estimates for $p > 2$.

In this paper we shall prove

Theorem 1. *There is a smoothly bounded Hartogs domain in $\mathbf{C}^2$, and a $\bar\partial$-closed $(0,1)$-form g in $\mathcal{D}$, which extends continuously to $\overline{\mathcal{D}}$, such that the equation $\bar\partial u = g$ has no bounded solution.*

Recall that a Hartogs domain is a domain of the form

$$\mathcal{D} = \{(z,w); |w| < e^{-\varphi(z)}\} \tag{1}$$

where φ is subharmonic. If e.g. φ is smooth in the disk and

$$\varphi = \frac{1}{2} \log \frac{1}{1-|z|^2}$$

near the boundary of the disk, then $\partial\mathcal{D}$ will be smooth.

There is a special reason why we are interested in the case $n=2$. The form g in Theorem 1 extends continuously to $\partial\mathcal{D}$. So, the same example shows that we don't have L^∞-estimates for $\bar\partial_b$ either. But in $\mathbf{C}^2$ there is a duality between $\bar\partial_b$ in L^∞ and in L^1. Therefore we get

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Theorem 2. *There is a sequence of functions g_n on $\partial\mathcal{D}$ such that*
(i) $\|g_n\|_{L^1}\leq 1$,
(ii) *there is u_n in L^1 such that $\bar\partial_b u_n = g_n$,*
(iii) *if $\bar\partial v_n = g_n$ then $\|v_n\|_{L^1}\to\infty$.*

In other words, we do not have L^1 estimates for $\bar\partial_b$ either. Another way of expressing the conclusion of Theorem 2 is that the closed and densely defined operator $\bar\partial_b\colon L^1\to L^1$ does not have closed range.

Whether one can solve the $\bar\partial$-equation in $\mathcal{D}$ with L^1-estimates is another question, which I do not know the answer to. In a recent paper by Bonneau and Diederich ([BD]), L^1-estimates with a logarithmic loss are proved. One should also compare the results by Feffermann–Kohn, Christ and Nagel–Rosay–Stein–Wainger (see [FK], [C], [NRSW]), which contain sup-norm estimates, and even Hölder estimates for $\bar\partial_b$ in domains of finite type in $\mathbf{C}^2$ (thus in particular for domains with real-analytic boundary). Recently a more elementary proof of a slightly weaker result was obtained by Range ([R]).

Our construction is quite different from the one in [S2] (it is actually more similar to the earlier one in [S1]). It is based on the relation between estimates for the $\bar\partial$-equation in domains of the form (1), and estimates for the one dimensional $\bar\partial$-equation in the disk with weight $e^{-n\varphi}$ where $n\in N$.

It is well known (see [FS1], [FS2] or [B]) that if φ is an arbitrary subharmonic function in the disk, then one can in general not solve the equation

$$\frac{\partial u}{\partial\bar z}=f \tag{2}$$

in the disk with estimates

$$\sup_\Delta |u|e^{-\varphi}\leq C\sup_\Delta |f|e^{-\varphi}. \tag{3}$$

The analogous question for smooth φ's is whether one can solve (2) with the estimate

$$\sup_\Delta |u|e^{-n\varphi}\leq C\sup_\Delta |f|e^{-n\varphi}. \tag{4}$$

where C is a constant that does not depend on n (nor f of course). It turns out (see Section 2) that if we have L^∞-estimates in a domain $\mathcal{D}$ of type (1), then one can solve the $\bar\partial$-equation in the disk with the estimate (4). Hence, all we need to do to prove Theorem 1, is to find a subharmonic function $\varphi\in\mathcal{C}^\infty(\bar\Delta)$ for which this is impossible. This is the object of Section 1. In Section 2 we show how this implies Theorems 1 and 2.

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Section 1

In this section we shall study estimates of the form

$$\sup|v|e^{-\varphi}\le C\sup|f|e^{-\varphi} \tag{1.1}$$

for solutions to the equation

$$\frac{\partial v}{\partial \bar z}=f$$

in the unit disk, Δ. We are interested in for which functions φ such an estimate holds for all f in, say, $C_c^\infty(\Delta)$, and also for which functions ψ (1.1) holds with a fixed constant for $\varphi=n\psi$, $n=1,2,3\,\dots$.

Proposition 1.2. *We can solve the $\bar\partial$-equation in Δ with the estimate* (1.1) *if and only if the following inequality holds*

$$\int_\Delta |\alpha|e^{\varphi}\le C\int_\Delta\left|\frac{\partial\alpha}{\partial\bar z}\right|e^{\varphi}\quad \forall\alpha\in C_c^\infty(\Delta). \tag{1.3}$$

The best constants in (1.1) *and* (1.3) *are the same.*

Proof. The fact that $\partial v/\partial\bar z=f$ is equivalent to

$$\int_\Delta f\alpha=-\int v\frac{\partial\alpha}{\partial\bar z}\quad \forall\alpha\in C_c^\infty(\Delta).$$

If v satisfies (1.1) we get

$$\left|\int f\alpha\right|\le C\sup(|f|e^{-\varphi})\int\left|\frac{\partial\alpha}{\partial\bar z}\right|e^{\varphi}.$$

Taking the supremum over all f with $|f|\le e^{\varphi}$ we get (1.3).

If, on the other hand (1.3) holds then

$$\left|\int f\alpha\right|\le C\sup(|f|e^{-\varphi})\int\left|\frac{\partial\alpha}{\partial\bar z}\right|e^{\varphi}. \tag{1.4}$$

Let

$$F=\left\{\frac{\partial\alpha}{\partial\bar z}e^{\varphi}\,;\,\alpha\in C_c^\infty\right\}\subseteq L^1(\mathcal{D}).$$

Define a linear functional

$$T\colon F\to\mathbf{C}$$

by

$$T\left(\frac{\partial\alpha}{\partial\bar z}e^{\varphi}\right)=\int f\alpha.$$

(1.4) implies that T is well defined and that $\|T\|\le C\sup|f|e^{-\varphi}$.

We can extend T to a linear operator

$$T\colon L^1(\Delta)\to\mathbf{C}$$

with the same norm. Hence there is a function $u\in L^\infty(\Delta)$ such that

$$\int f\alpha=\int u\frac{\partial\alpha}{\partial\bar z}e^{\varphi}$$

and $\|u\|_\infty=\|T\|$. Letting $v=-ue^{\varphi}$ we have a solution to $\bar\partial v=f$ which satisfies (1.1). The proof is complete. □

Note that the question whether (1.3) holds depends only on $\Delta\varphi$. In other words, if h is harmonic and (1.3) holds for φ, then it holds with φ replaced by $\varphi+h$ (just multiply α by $e^{h+i\tilde h}$ where $\tilde h$ is the harmonic conjugate to h).

We also remark that if (1.3) holds for all $\alpha\in\mathcal{C}_c^\infty(\Delta)$, then it actually holds for all α in L^1 with compact support, which are such that $\partial\alpha/\partial\bar z$ is a finite measure. We will use this remark at several points.

Proposition 1.5. *Assume that* (1.3) *holds with a fixed constant for* $\varphi=n\psi$ *where* $n\in N$. *Then* ψ *is subharmonic.*

Proof. Let $\Delta'\subset\subset\Delta$ be a disk and let $\alpha=X_{\Delta'}$. Then (1.3) implies

$$\int_{\Delta'}e^{n\psi}\le C\int_{\partial\Delta'}e^{n\psi}\,|dz|.$$

Hence, if $\psi\le 0$ on $\partial\Delta'$ then $\psi\le 0$ in Δ'. Since we can change ψ to $\psi-h$ where h is any harmonic polynomial, we see that if $\psi\le h$ on $\partial\Delta'$, then $\psi\le h$ in Δ'. This means that ψ is subharmonic. □

A similar argument shows that if the analog of (1.3) in L^2-norm

$$\int|\alpha|^2e^{\varphi}\le C\int\left|\frac{\partial\alpha}{\partial\bar z}\right|^2e^{\varphi} \tag{1.6}$$

holds for $\varphi=n\psi$ then ψ is also subharmonic. In this case, the converse is also true. This follows from the inequality used in the proof of Hörmander's theorem (see [H]). We shall now see that in L^1-norm the situation is quite different.

Proposition 1.7. *Let φ be any subharmonic function in Δ with the property that $\varphi=-\infty$ on some set with an interior accumulation point. Then* (1.3) *does not hold with any constant C.*

Proof. Assume that $a\in\Delta$ and that $\varphi(a)=-\infty$. Then it follows from (1.3) that

$$\int |\alpha|\frac{e^{\varphi}}{|z-a|}\le C\int\left|\frac{\partial\alpha}{\partial\bar z}\right|\frac{e^{\varphi}}{|z-a|}.$$

To see this consider (1.3) with α replaced by $\alpha/(z-a)$. Then

$$\frac{\partial\alpha}{\partial z}/(z-a)=\frac{\partial\alpha}{\partial\bar z}/(z-a)+\pi\alpha\delta_a,$$

but the last term gives no contribution since $e^{\varphi(a)}=0$. Iterating this observation we see that if $\varphi(a_1)=\varphi(a_2)=...=\varphi(a_n)=-\infty$ then

$$\int |\alpha|\frac{e^{\varphi}}{\prod_1^n|z-a_j|}\le C\int\left|\frac{\partial\alpha}{\partial\bar z}\right|\frac{e^{\varphi}}{\prod_1^n|z-a_j|}. \tag{1.8}$$

In particular we can take $\alpha=\chi_{\Delta'}$, where $\Delta'\subset\subset\Delta$ is a disk containing an accumulation point, p, of the set where $\varphi=-\infty$. Then (1.8) can clearly not hold in the limit as all a_j tend to p. Hence (1.3) cannot hold. □

The preceding proof shows a curious fact. If φ is given by

$$\varphi_a=\frac{1}{3}\sum_1^3\log|z-a_j|$$

then (1.3) cannot hold with a fixed constant as $a=(a_1,a_2,a_3)\to 0$. On the other hand, in the limit we get

$$\varphi=\log|z|,$$

which does satisfy (1.3) (just multiply α by z).

We are now ready to prove our main technical result.

Proposition 1.9. *There is a subharmonic function ψ in $C^\infty(\overline{\Delta})$ such that if C_n denotes the best constant in*

$$\int_\Delta |\alpha|e^{n\psi}\le C_n\int_\Delta\left|\frac{\partial\alpha}{\partial\bar z}\right|e^{n\psi}\quad\forall\alpha\in C_c^\infty, \tag{1.10}$$

then $\overline{\lim}\, C_n=\infty$.

Proof. Let φ be a function subharmonic in a neighborhood of $\overline{\Delta}$ which is harmonic near $\partial\Delta$ and which is such that (1.3) is violated. Then there is a sequence of smooth subharmonic functions φ_k in $\overline{\Delta}$, which are harmonic near $\partial\Delta$ such that $\varphi_k \downarrow \varphi$. Let $\|\varphi_k\|_{\mathcal{C}^k(\overline{\Delta})} = A_k$. Take a sequence of integers n_k such that $n_k/A_k \to \infty$, and put $\psi_k = \varphi_k/n_k$. Then we have a sequence of smooth subharmonic functions such that

(i) $\lim \|\psi_k\|_{\mathcal{C}^k} = 0$ and

(ii) if B_k is the best constant in

$$\int |\alpha| e^{n_k \psi_k} \leq B_k \int \left|\frac{\partial \alpha}{\partial \bar{z}}\right| e^{n_k \psi_k} \quad \forall \alpha \in \mathcal{C}_c^\infty$$

then $\lim B_k = \infty$.

Moreover, all ψ_k's are harmonic near $\partial\Delta$, so $\Delta\psi_k \in \mathcal{C}_c^\infty(\Delta)$. Now, if

$$\Delta' = \Delta(a, r) \subset\subset \Delta$$

is a small disk in Δ, and if ψ is one of our ψ_k's we transport ψ to Δ' by the following definition.

$$\tau(\Delta', \psi) = r^{-2} G\left[(\Delta\psi)\left(\frac{z-a}{r}\right)\right]$$

where G is the Green-potential in Δ.

Then $v = \tau(\Delta', \psi)$ is smooth and $\Delta v = \Delta(\psi((z-a)/r))$ in Δ' and $\Delta v = 0$ outside Δ'. Moreover

$$\|\Delta v\|_{\mathcal{C}^{k-2}} \leq r^{-k} \|\Delta\psi\|_{\mathcal{C}^{k-2}} \leq r^{-k} \|\psi\|_{\mathcal{C}^k}. \tag{1.11}$$

Choose a disjoint sequence of disks $\Delta_k = \Delta(a_k, r_k)$ in Δ, which converges to an interior point. By taking a sparse subsequence, renumbering and letting $\widetilde{\psi}_k = \tau(\Delta_k, \psi_k)$ we get that

(iii) $\sum r_k^{-k} \|\psi_k\|_{\mathcal{C}^k} < \infty$ and

(iv) $\lim r_k B_k = \infty$.

Then (iii) together with (1.11) shows that $\psi = \sum \widetilde{\psi}_k \in \mathcal{C}^\infty(\overline{\Delta})$.

Assume, to get a contradiction, that (1.10) holds with $C_n \leq C$. Take in particular $\alpha \in \mathcal{C}_c^\infty(\Delta_k)$. Then

$$\int_{\Delta_k} |\alpha| e^{n\widetilde{\psi}_k} \leq C \int_{\Delta_k} \left|\frac{\partial \alpha}{\partial \bar{z}}\right| e^{n\widetilde{\psi}_k}$$

since the other $\widetilde{\psi}_j$'s are harmonic in Δ_k. The change of variables $\tau = a_k + r_k\zeta$ transports this estimate to Δ. We then get

$$\int_{\Delta} |\alpha| e^{n\psi_k} \leq \frac{C}{r_k} \int_{\Delta} \left|\frac{\partial \alpha}{\partial \bar{z}}\right| e^{n\psi_k}$$

since $\Delta\psi_k = \Delta(\widetilde{\psi}_k(a + r\zeta))$. Taking in particular $n = n_k$ we obtain $B_k \leq C/r_k$, which contradicts $\lim r_k B_k = \infty$. $\square$

Section 2

We shall now see how we can use the function ψ from Proposition 1.9 to prove Theorem 1.

Proposition 2.1. *Let $\mathcal{D}$ be a domain of the form*

$$\mathcal{D}=\{(z,w)\,;\ |z|<1, |w|<e^{-\varphi(z)}\},$$

where φ is a smooth function. Assume that for any $\bar\partial$-closed form f which extends continuously to $\overline{\mathcal{D}}$ we can solve the equation $\bar\partial u=f$ with u bounded. Then there is a constant C such that for any $f\in\mathcal{C}_c^\infty(\Delta_{1/2})$ and any $n\in N$ we can solve $\partial u/\partial\bar z=f$ in $\Delta_{1/2}$ with

$$\sup_{\Delta_{1/2}} |u|e^{-n\varphi}\le C\sup_{\Delta_{1/2}} |f|e^{-n\varphi} \tag{2.2}$$

Proof. Assume the conclusion is false. Then there is a sequence $n_k\to\infty$ and a sequence of $f_k\in\mathcal{C}_c^\infty(\Delta_{1/2})$ such that

$$\sup|f_k|e^{-n_k\varphi}=:a_k\to 0$$

and if u_k are any solutions for

$$\frac{\partial u_k}{\partial\bar z}=f_k.$$

Then

$$\sup|u_k|e^{-n_k\varphi}\to\infty.$$

By choosing a sparse subsequence we can assume $\sum a_k<\infty$. Let

$$f=\sum_0^\infty f_k(z)w^{n_k}d\bar z.$$

The sum is absolutely and uniformly convergent in $\overline{\mathcal{D}}$. So, f is continuous on $\overline{\mathcal{D}}$ and $\bar\partial f=0$. By hypothesis we can find a function $u\in L^\infty(\mathcal{D})$ such that $\bar\partial u=f$. Since f has no $d\bar w$ component, u is holomorphic in w, and we can expand u in a power series

$$u(z,w)=\sum_0^\infty u_n(z)w^n.$$

Identifying coefficients of w^n in $\bar\partial u=f$ we get

$$\frac{\partial u_{n_k}}{\partial\bar z}=f_k.$$

But

$$u_n(z)r^n = \frac{1}{2\pi}\int_{-\pi}^{\pi} u(z, re^{i\theta})e^{-in\theta}\,d\theta$$

so if $r<e^{-\varphi}$ we get

$$\sup_{\Delta_{1/2}} |u_n(z)|r^n \le \|u\|_{L^\infty}.$$

Letting $n=n_k$ and $r\uparrow e^{-\varphi}$ we see that

$$\sup_{\Delta_{1/2}} |u_n|e^{-n_k\varphi} \le C$$

contradicting the choice of f_k. □

Theorem 1 is now a direct consequence. Starting with our function ψ from Proposition 1.9, we can scale it down to $\Delta_{1/2}$. Then we can extend ψ to a smooth subharmonic function φ in Δ such that $\varphi=\frac{1}{2}\log(1/(1-|z|^2))$ near $\partial\Delta$. By Proposition 1.2 the conclusion of Proposition 2.1 fails for φ. Hence, if we use φ to define $\mathcal{D}$ we get a smoothly bounded domain in $\mathbf{C}^2$ which satisfies the claim in Theorem 1.

Let us now briefly discuss $\bar\partial_b$ on $\partial\mathcal{D}$. Let $u(z,w)$ be a bounded function on $\partial\mathcal{D}$. We can then expand $u(z,w)$ in a Fourier series

$$u(z, e^{-\varphi+i\theta}) \sim \sum_{-\infty}^{\infty} u_n(z)e^{-|n|\varphi}e^{in\theta}.$$

We can always extend u smoothly to $\mathcal{D}$. Let on the other hand f be a $(0,1)$ form in $\mathcal{D}$ which extends continuously to $\overline{\mathcal{D}}$. Finally let ϱ be any smooth defining function for $\mathcal{D}$. We say $\bar\partial_b u=f$ if

$$\bar\partial u\wedge\bar\partial\varrho = f\wedge\bar\partial\varrho \quad \text{on } \partial\mathcal{D}$$

in the sense of distributions.

Now, let in particular f be the form in Theorem 1. By extending u to $\mathcal{D}$ by

$$u(z,w) = \sum_{0}^{\infty} u_n(z)w^n + \sum_{-\infty}^{-1} u_n(z)\bar w^{-n}$$

we see that if $\bar\partial_b u=f$ then

$$\frac{\partial u_{n_k}}{\partial\bar z} = f_k.$$

Again, by estimating Fourier coefficients we see that $\bar\partial_b u=f$ has no bounded solution.

We finally turn to the proof of Theorem 2. This follows in principle from what we just said, but it is probably more instructive to give a direct construction.

We know from the construction of φ that there is a sequence of functions $\alpha_k \in \mathcal{C}_c^\infty(\Delta_{1/2})$, and a sequence n_k such that

(a)
$$\int \left|\frac{\partial \alpha_k}{\partial \bar z}\right| e^{n_k \varphi} = 1$$

and

(b)
$$\int |\alpha_k| e^{n_k\varphi} \to \infty.$$

Put for $(z,w)\in\partial\mathcal{D}$
$$g_k = \frac{\partial\alpha_k}{\partial\bar z} w^{-n_k} d\bar z.$$

Since surface measure on $\partial\mathcal{D}$ is equivalent to $idz\wedge d\bar z\wedge d\theta$ (a) means that $\|g_k\|_{L^1(\partial D)} \le C$. Moreover $g_k = \bar\partial_b \alpha_k w^{-n_k}$. If now v_k is any solution to $\bar\partial_b v_k = g_k$ then

$$v_k - \alpha_k w^{-n_k} = h_k$$

has a holomorphic extension to $\mathcal{D}$. Hence

$$\frac{1}{2\pi}\int_{-\pi}^{\pi} v_k(z, we^{i\theta}) e^{in_k\theta}\, d\theta = \alpha_k w^{-n_k}.$$

Therefore
$$\|\alpha_k w^{-n_k}\|_{L^1} \le \|v_k\|_{L^1}.$$

But (b) says precisely that the left hand side here tends to infinity. Hence $\|v_k\|_{L^1}$ cannot be bounded, so we have proved Theorem 2.

Remark added March 31, 1993. The construction in this paper is based on the existence of a subharmonic function, φ, in the disk such that sup-norm estimates for $\bar\partial$ with the weight factor $e^{-\varphi}$ fail. Shortly after the paper was completed Fornaess and Sibony [FS2] noted that their construction from [FS1] of a function with similar properties for L^p-estimates, implies in the same way that there is a smoothly bounded Hartogs domain in $\mathbf{C}^2$ where L^p-estimates fail for any $p>2$. Feeding the same function into our construction for $\bar\partial_b$ one gets a smoothly bounded domain in $\mathbf{C}^2$ where $\bar\partial_b$ does not have closed range in any L^p-space except for $p=2$. A more detailed explanation of this together with further examples of the same kind can be found in [B2].

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