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James type results for polynomials and symmetric multilinear forms

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Abstract. We prove versions of James' weak compactness theorem for polynomials and symmetric multilinear forms of finite type. We also show that a Banach space X is reflexive if and only if it admits an equivalent norm such that there exists $x_0 \neq 0$ in X and a weak-*-open subset A of the dual space, satisfying that $x^* \otimes x_0$ attains its numerical radius, for each x^* in A.

1. Introduction

The classical James' theorem states that a Banach space is reflexive if and only if each—bounded and linear—functional attains its norm [J]. On the other hand, there are results stating that in the non-reflexive case, the set of norm attaining functionals is small. For instance, if the unit ball of a separable Banach space is not dentable, then the set of norm attaining functionals is of the first Baire category, a result due to Bourgin and Stegall (see [B, Theorem 3.5.5 and Problem 3.5.6]). Kenderov, Moors and Sciffer proved that for any infinite compact Hausdorff space K, the space $C(K)$ satisfies the same property [KMS].

By using the weak-* topology in the dual space, Debs, Godefroy and Saint Raymond showed that a separable Banach space is reflexive provided that the set of norm attaining functionals has a non-empty weak-* interior [DGS, Lemma 11]. This result was generalized by Jiménez Sevilla and Moreno, who proved that the same assertion holds for any Banach space not necessarily separable [JiM, Proposition 3.2].

We shall use $A(X)$ to denote the set of norm attaining functionals on a Banach space X. For the norm topology, it is known that any Banach space is isomorphic to another one satisfying that the interior of the set of norm attaining functionals is non-empty [AR1, Corollary 2].

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However, in the non-reflexive case, if X satisfies that the set $A(X)$ has a non-empty interior, then the norm of X does not satisfy strong differentiability conditions. For instance, Jiménez Sevilla and Moreno proved that a Banach space X satisfying the Mazur intersection property and being such that $A(X)$ has non-empty interior, must be reflexive [JiM]. The parallel assertion also holds if the space is very smooth or Hahn–Banach smooth [AR1], [AR2]. For other results along the same line, but assuming that the space does not contain (an isomorphic copy of) l_1, see [AB1] and [AB2].

Here we shall show that an abundance of certain other functions on the Banach space that attain their suprema also implies reflexivity. To be more precise, we shall use as functions the simplest finite-type homogeneous polynomials, symmetric multilinear forms and numerical radii of rank-one operators.

In Section 2, we shall prove a characterization of reflexive spaces in terms of norm attaining n-homogeneous polynomials or norm attaining symmetric multilinear forms. For $n+1$ functionals $x_1^*, \dots, x_n^*, x^*$ in X^*, consider the polynomial given by

$$x \longmapsto x^*(x) \prod_{i=1}^{n} x_i^*(x).$$

We prove that a Banach space X is reflexive if and only if for n fixed non-zero functionals $x_1^*, \dots, x_n^*$, the set of elements x^* in X^* such that the above polynomial attains its norm, contains a (non-empty) weak-*-open set. We also provide a parallel result by considering the symmetrization of the n-linear forms given by

$$(x_1, \dots, x_n) \longmapsto \prod_{i=1}^{n} x_i^*(x_i).$$

Section 3 is devoted to the numerical radius case. Here we show that a space X is reflexive as soon as, for some non-trivial x_0 in X, it holds that the set of elements x^* in the dual of X satisfying that the operator $x^* \otimes x_0$ attains its numerical radius (see the definition below) has a non-empty weak-* interior. Moreover, we show that any reflexive space admits an equivalent norm such that in this new norm rank-one operators whose image is contained in a fixed one-dimensional subspace attain their numerical radii. Therefore, we get a characterization of reflexive spaces in these terms. A Banach space is known to have finite dimension if and only if for every equivalent norm, each rank-one operator attains its numerical radius [AR3], [AR4]. For $X = l_p$ $(1<p<\infty)$, every compact operator satisfies the previous condition for the usual norm.

2. Versions of James' theorem for polynomials and symmetric multilinear forms

Hereafter, X will be a Banach space over the scalar field $\mathbf{K}$ ($\mathbf{R}$ or $\mathbf{C}$) and X^* its topological dual. By B_X and S_X we will denote the closed unit ball and unit sphere, respectively. The usual norm of a bounded multilinear form A on X is given by

$$\|A\| = \sup\{|A(x_1, \dots, x_n)| : \|x_i\| \leq 1,\ 1 \leq i \leq n\}.$$

If we fix n functionals $x_1^*, \dots, x_n^* \in X^*$, we consider the finite type n-linear form A given by

$$A(x_1, \dots, x_n) = \prod_{i=1}^{n} x_i^*(x_i), \quad x_i \in X,$$

and in this case, $\|A\|$ is just $\prod_{i=1}^{n} \|x_i^*\|$. It is also clear that *if A is non-trivial, then A attains its norm*, that is, there are $x_i \in S_X$, $1 \leq i \leq n$, with

$$|A(x_1, \dots, x_n)| = \|A\|,$$

if and only if every functional x_i^* attains its norm. Therefore, if we assume that for $n-1$ fixed functionals $x_1^*, \dots, x_{n-1}^* \in X^*$ the n-linear form A attains the norm, for any $x_n^* \in X^*$, then, by using James' theorem, the space is reflexive. For this reason, we shall use n-homogeneous polynomials of the form

$$x \longmapsto \prod_{i=1}^{n} x_i^*(x)$$

and symmetric multilinear forms deriving from A instead of n-linear forms, in order to obtain James-type results. We shall denote by $P_{x_1^* \dots x_n^*}$ the continuous n-homogeneous polynomial on X given by

$$P_{x_1^* \dots x_n^*}(x) := \prod_{i=1}^{n} x_i^*(x), \quad x \in X.$$

Let us recall that a continuous polynomial P on X *attains the norm* if the supremum defining the usual norm is a maximum, that is,

$$|P(x_0)| = \|P\| \quad \text{for some} \quad x_0 \in B_X.$$

By $S_{x_1^* \dots x_n^*}$ we denote the symmetrization of the continuous multilinear form

$$(x_1, \dots, x_n) \longmapsto x_1^*(x_1) \dots x_n^*(x_n), \quad x_1, \dots, x_n \in X,$$

so, $S_{x_1^*\dots x_n^*}$ is the continuous symmetric n-linear form given by

$$S_{x_1^*\dots x_n^*}(x_1,\dots,x_n) := \frac{1}{n!}\sum_{\sigma\in\Delta_n}\left(\prod_{i=1}^{n} x_i^*(x_{\sigma(i)})\right),$$

where Δ_n is the set of all permutations of n elements.

Some key references on the denseness of norm attaining polynomials and multilinear forms can be found in [AFW], [AAP], [CK], [Ch], [JP], [A2], [ACKP] and [PS].

Now we state the James-type result for polynomials.

Theorem 2.1. *A Banach space X is reflexive if and only if there are $n\geq 1$ and non-zero functionals $x_1^*,\dots,x_n^*\in X^*$ so that the weak-* interior of the set*

$$\{x^*\in X^* : P_{x_1^*\dots x_n^* x^*} \text{ attains its norm}\}$$

is non-empty.

Proof. Since each polynomial $P_{x_1^*\dots x_n^* x^*}$ is weakly continuous, on a reflexive space the finite type polynomials that we consider attain the norm.

On the other hand, let us observe that for any $x^*\in X^*$,

$$\|P_{x_1^*\dots x_n^* x^*}\| = \sup_{\|x\|\leq 1}|x_1^*(x)\dots x_n^*(x)x^*(x)| = \sup_{b\in B}|x^*(b)|,$$

where B is given by

$$B := \{x_1^*(x)\dots x_n^*(x)x : x\in B_X\}.$$

It is also clear that the polynomial $P_{x_1^*\dots x_n^* x^*}$ attains its norm if and only if the function $|x^*|$ attains its supremum on B. Thus we can reformulate the assumption by saying that the set

$$A := \{x^*\in X^* : |x^*| \text{ attains the supremum on } B\}$$

has non-empty weak-* interior.

Since we shall prove that X is reflexive, we can clearly assume that X is infinite-dimensional. Now we shall give an equivalent norm on X such that for this new norm A will be contained in the set of norm attaining functionals. For this purpose, let us note that any element $y\in X$ satisfying that $x_i^*(y)\neq 0$, $1\leq i\leq n$, can be expressed as

$$y = \frac{\|y\|^{n+1}}{\prod_{i=1}^{n} x_i^*(y)}\left[\prod_{i=1}^{n} x_i^*\left(\frac{y}{\|y\|}\right)\right]\frac{y}{\|y\|} \in \frac{\|y\|^{n+1}}{\prod_{i=1}^{n} x_i^*(y)}B.$$

In view of the continuity of the scalar-valued function f given by

$$f(y)=\frac{\|y\|^{n+1}}{\prod_{i=1}^{n} x_i^*(y)}, \quad y\in X \text{ and } x_i^*(y)\neq 0,$$

in order to prove that the set

$$\mathbf{D}B=\{\lambda b:\lambda\in\mathbf{K},\ |\lambda|\leq 1 \text{ and } b\in B\}$$

has a non-empty norm interior, it suffices to find some $y\in X$ satisfying $|f(y)|<1$. To this end, let us fix x_0 in S_X such that for all $i=1,\dots,n$, $x_i^*(x_0)\neq 0$, and a scalar satisfying $0<a<|\prod_{i=1}^{n} x_i^*(x_0)|$, and consider $y=ax_0$. Thus

$$|f(y)|=\frac{\|y\|^{n+1}}{|\prod_{i=1}^{n} x_i^*(y)|}=\frac{a^{n+1}}{a^n|\prod_{i=1}^{n} x_i^*(x_0)|}<1.$$

We write C for the closure of the convex hull of $\mathbf{D}B$. Therefore, C is bounded, closed, convex and balanced, with 0 belonging to the norm interior of it; and so, it is the unit ball of an equivalent norm on X.

Because of the assumption, the set A is contained in the set of norm attaining functionals for this new norm, and A has non-empty weak-* interior. Since a functional x^* attains its norm if and only if $\mathrm{Re}\, x^*$ attains its norm, the previous assumption implies that there is a weak-*-open set of $(X_{\mathbf{R}})^*$ of norm attaining functionals on $X_{\mathbf{R}}$. Hence, by using [JiM, Proposition 3.2], X is reflexive. □

The analogous version for symmetric multilinear forms can be stated as follows.

Theorem 2.2. *A Banach space X is reflexive if and only if there are $n\geq 1$ and $x_1^*,\dots,x_n^*\in X^*\setminus\{0\}$ so that the weak-* interior of the set*

$$\{x^*\in X^*: S_{x_1^*\dots x_n^* x^*} \text{ attains its norm}\}$$

is non-empty.

Proof. One can proceed as in Theorem 2.1, if we note that given a functional $x^*\in X^*$, the symmetric $(n+1)$-linear form $S_{x_1^*\dots x_n^* x^*}$ attains its norm if and only if the function $|x^*|$ attains the supremum on B, where

$$B:=\left\{\frac{1}{(n+1)!}\sum_{\sigma\in\Delta_{n+1}}\left(\prod_{i=1}^{n} x_i^*(x_{\sigma(i)})\right)x_{\sigma(n+1)}: x_i\in B_X\right\}.$$

The set B clearly contains the subset given by

$$\left\{\left(\prod_{i=1}^{n} x_i^*(x)\right)x: x\in B_X\right\}.$$

Again, we assume that X is infinite-dimensional. Now, it suffices to fix $x_0 \in S_X$ with $x_i^*(x_0) \neq 0$ for all i and $0 < a < |\prod_{i=1}^n x_i^*(x_0)|$. The element $y = ax_0$ belongs to the norm interior of $\mathbf{D}B$ since

$$y = \frac{a}{\prod_{i=1}^n x_i^*(x_0)} \frac{\prod_{i=1}^n x_i^*(y)}{\|y\|^{n+1}} y \in \frac{a}{\prod_{i=1}^n x_i^*(x_0)} B.$$

We finish by using the argument in the proof of Theorem 2.1. □

There are simple examples of spaces and polynomials of the type we considered here, such that the polynomial attains its norm but not all the functionals involved attain the norm (see, for instance [Ru, Example 4]). One can define analogous examples of symmetric multilinear forms [Ru]. Therefore, James' theorem cannot be directly applied in the proofs of Theorems 2.1 and 2.2. Some parallel results on the reflexivity of the space of all homogeneous polynomials have been studied by several authors (see [R], [AAD], [MV] and [JM]).

Remark. Following similar arguments as in Theorems 2.1 and 2.2, one can prove that a Banach space X is reflexive provided that there are n elements $x_1^*, \ldots, x_n^* \in X^* \backslash \{0\}$ and a bounded, closed and convex subset $A \subset X$ with non-empty norm interior satisfying that

$$\{x^* \in X^* : |P_{x_1^* \ldots x_n^* x^*}| \text{ attains its supremum on } A\}$$

or

$$\{x^* \in X^* : |S_{x_1^* \ldots x_n^* x^*}| \text{ attains the supremum on } A\}$$

contains a (non-void) weak-*-open subset of X^*.

3. The numerical radius type result

In order to state the third version of James' theorem, let us recall that the *numerical range* of an operator $T \in L(X)$ (the Banach algebra of all bounded and linear operators on X) is the bounded set of scalars

$$V(T) := \{x^*(Tx) : (x, x^*) \in \Pi(X)\},$$

where $\Pi(X) := \{(x, x^*) \in S_X \times S_{X^*} : x^*(x) = 1\}$, and the *numerical radius* of T is given by

$$v(T) := \sup\{|\mu| : \mu \in V(T)\}$$

(see [BD]). The operator T is said to *attain its numerical radius* when the supremum defining $v(T)$ actually is a maximum. For some known results about numerical radius attaining operators, one can see [BS], [C], [P], [A1] and [AP].

In the version of James' theorem posed here, we shall use only rank-one operators. If $x\in X$ and $x^*\in X^*$, then $x^*\otimes x$ will denote the operator in X defined by

$$(x^*\otimes x)(y):=x^*(y)x, \quad y\in X.$$

Theorem 3.1. *Let X be a Banach space and suppose that there exists $x_0\in X\backslash\{0\}$ such that the subset*

$$\{x^*\in X^*: x^*\otimes x_0 \text{ attains its numerical radius}\}$$

has a non-empty weak- interior. Then X is reflexive.*

Proof. If one writes B for the set

$$B:=\{y^*(x_0)y:(y,y^*)\in\Pi(X)\},$$

then for all $x^*\in X^*$ we have that

$$v(x^*\otimes x_0)=\sup_{(y,y^*)\in\Pi(X)}|x^*(y)y^*(x_0)|=\sup_{b\in B}|x^*(b)|$$

and the rank-one operator $x^*\otimes x_0$ attains its numerical radius if and only if the function $|x^*|$ attains the supremum on B. According to the assumption, the set

$$\{x^*\in X^*:|x^*| \text{ attains the supremum on } B\}$$

has a non-empty weak-* interior.

We shall prove that the set $\mathbf{D}B$ contains an open ball. If this condition holds, then the closed convex hull of $\mathbf{D}B$, let us say C, is closed, convex, bounded and contains a ball centered at zero. Hence, C is the closed unit ball of an equivalent norm on X. For this new norm, by using the assumption, the set of norm attaining functionals contains a weak-*-open set, and so, by using again that this condition implies the corresponding assumption for $X_{\mathbf{R}}$, it follows from [JiM, Proposition 3.2] that X is reflexive.

For $0<r<\frac{1}{10}$ let us check that

$$\tfrac{1}{2}x_0+rB_X\subset\mathbf{D}B.$$

Each element $y\in\frac{1}{2}x_0+rB_X$ satisfies

$$0<\tfrac{1}{2}-r\le\|y\|\le\tfrac{1}{2}+r. \tag{1}$$

Let us choose $y^* \in X^*$ so that $(y/\|y\|, y^*) \in \Pi(X)$. Then

$$y^*(x_0)\frac{y}{\|y\|} \in B.$$

Furthermore, as a result of the choice of y, r and (1), we have the inequality

$$|y^*(x_0)| \geq 2(\|y\| - r) \geq 1 - 4r > 0. \tag{2}$$

The element y can be trivially decomposed in the form

$$y = \frac{\|y\|}{y^*(x_0)} y^*(x_0) \frac{y}{\|y\|} \in \frac{\|y\|}{y^*(x_0)} B. \tag{3}$$

Finally, by using (1) and (2) it holds that

$$\frac{\|y\|}{|y^*(x_0)|} \leq \frac{\frac{1}{2} + r}{1 - 4r} < 1$$

and so, in view of (3), y belongs to $\mathbf{D}B$. □

Let us finally observe that the previous result generalizes, see [AR3, Theorem 1] where it was proven that a Banach space is reflexive if every rank-one operator attains its numerical radius. Moreover, the proof given here is much simpler.

We shall now obtain a partial converse of the previous result, which is sharp, as we will observe later.

Proposition 3.2. *Let X be a reflexive Banach space and $x_0 \in X \backslash \{0\}$. Then there is an equivalent norm on X such that, for any element x^* in X^*, the operator $x^* \otimes x_0$ attains its numerical radius.*

Proof. We can clearly assume that $\dim X \geq 2$. Hence, we can decompose $X = \mathbf{K}x_0 \oplus M$ for some closed linear subspace $M \neq \{0\}$ of X. Let us consider the space Y isomorphic to X given by

$$Y = \mathbf{K}x_0 \oplus_1 M$$

whose dual can be identified as

$$Y^* = \mathbf{K}x_0^* \oplus_\infty M^*,$$

where $x_0^* \in X^*$ satisfies $x_0^*(x_0) = 1$. We shall check that for any $x^* \in X^*$, the operator $x^* \otimes x_0$ attains its numerical radius on Y.

Let us consider the set

$$B := \{y^*(x_0)y : (y, y^*) \in \Pi(Y)\}$$

and assume that x_0 is in the unit sphere of Y. Any element y in the unit sphere of Y can be written as

$$y = sx_0 + m$$

for some scalar s and some element $m \in M$ satisfying $|s| + \|m\| = 1$. The element

$$y^* = \lambda x_0^* + m^*,$$

where $\lambda \in \mathbf{K}$, $m^* \in M^*$,

$$|\lambda| = \|m^*\| = 1, \quad \lambda s = |s| \quad \text{and} \quad m^*(m) = \|m\|,$$

satisfies that $(y, y^*) \in \Pi(Y)$ and $|y^*(x_0)| = 1$, so $y \in \mathbf{D}B$.

We have checked that $S_Y \subset \mathbf{D}B$ and so $B_Y = \mathbf{D}B$. Therefore,

$$v(x^* \otimes x_0) = \sup_{b \in B} |x^*(b)| = \|x^*\|$$

and $x^* \otimes x_0$ attains its numerical radius if and only if x^* attains its norm. Since Y is reflexive, it follows that any operator of the form $x^* \otimes x_0$ attains its numerical radius. □

By using Theorem 3.1 and Proposition 3.2, we arrive, in fact, at the following characterization.

Corollary 3.3. *A Banach space is reflexive if and only if for some $0 \neq x_0 \in X$, there is an equivalent norm on X such that $x^* \otimes x_0$ attains its numerical radius for any element x^* in the dual space of X.*

Let us note that in any infinite-dimensional Banach space there is an equivalent norm such that at least one rank-one operator does not attain its numerical radius (see Theorem 3.1, [AR3, Example] and [AR4, Theorem 3]). Therefore, if the Banach space is reflexive but not finite-dimensional, renorming may be necessary in order to obtain the statement in Proposition 3.2.

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